香 港 考 試 局
HONG KONG EXAMINATIONS AUTHORITY

一 九 八 八 年 香 港 中 學 會 考
HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1988

附 加 數 學 (卷 一)
Additional Mathematics (Paper I)

評卷參考
Marking Scheme

請在學校任教之閱卷員特別留意

本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員在任何情況下披露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

Special Notes for Teacher Markers

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

Markers should therefore resist pleas from their students to have access to this documents. Making it available would constitute mis-conduct on the part of the marker and is, moreover in breach of the 1977 Hong Kong Examinations Authority Ordinance.

SOLUTIONS	MARKS	REMARKS
1. (a) $(\sqrt{x+1+\Delta x} - \sqrt{x+1})(\sqrt{x+1+\Delta x} + \sqrt{x+1})$		
$= (x+1+\Delta x) - (x+1)$		
$= \Delta x$	1A	
(b) $\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$	1A	
$= \lim_{\Delta x \to 0} \left(\frac{\sqrt{x+1+\Delta x} - \sqrt{x+1}}{\Delta x} \right)$	1A	
$= \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+1+\Delta x} + \sqrt{x+1}}$	1A	This can't be omitted.
$= \frac{1}{2\sqrt{x+1}}$	1A 5	
2. Differentiating $y^2 = x^2y + 2$		
$(2y)(y') = x^2y' + 2xy$	1M	
When $x = 1$, $y^2 = y + 2$		
$y = 2$ or -1		
At $(1, 2)$, $y' = \frac{4}{3}$	1A	
$4x - 3y + 2 = 0$	1A	
At $(1, -1)$, $y' = \frac{2}{3}$	1A	
$2x - 3y - 5 = 0$	1A 5	
3. (a) $z_1 = 1 + 2i$, $z_2 = 1 + i$, $z_3 = 3 + 2i$	1A	
$\frac{z_1z_2}{z_3} = \frac{-1+3i}{3+2i}$ or $\frac{7+4i}{13} \cdot (1+i)$	1A	For z_1z_2 or $\frac{z_1}{z_3}$
$= \frac{3}{13} + \frac{11}{13}i$	1A	Accept $\frac{3+11i}{13}$
(b) $\angle AOD + \angle BOD - \angle COD$		
$= \arg z_1 + \arg z_2 - \arg z_3$		
$= \arg\left(\frac{z_1z_2}{z_3}\right)$	1M	
$= \tan^{-1}\frac{11}{3}$		
$= 74.7°$ (or 1.30 rad.)	1A 5	

SOLUTIONS	MARKS	REMARKS
4. (a) $y' = \cos x + 2\sin x$	1A	
$y'' = -\sin x + 2\cos x$	1A	
(b) $y' = 0$	1M	
$\tan x = -\frac{1}{2}$		
$x = 2.68$ or 5.82	1A	Accept $x = 153°$ or $333°$ but pp-1.
Testing for min.	1M	
$x = 5.82$ corr. to a min.		
Minimum value of $y = -2.24$	1A	
	6	
5. $D = (4m)^2 - 4(4m + 15)$	1A	
$= 16m^2 - 16m - 60$	1A	
$f(x) > 0$ for all values of x		$f(x) = (x+2m)^2+(15+4m-4m^2) > 0$
$\therefore D < 0$	1M	$(15 + 4m - 4m^2) > 0$ 1
$16m^2 - 16m - 60 < 0$		
$(2m + 3)(2m - 5) < 0$	1A	
$-\frac{3}{2} < m < \frac{5}{2}$	1A	
	5	
6. $\underline{a} \cdot \underline{c} = 6 + 4k$	1A	If vector sign omitted, pp-
$\underline{b} \cdot \underline{c} = 16 + 6k$	1A	
Let θ be the angle between $\underline{a}$ and $\underline{c}$		
$\underline{a} \cdot \underline{c} = \lvert\underline{a}\rvert\lvert\underline{c}\rvert\cos\theta$		Alt. Solution (1):
$= 5\sqrt{4 + k^2}\cos\theta$	1A	
$\underline{b} \cdot \underline{c} = 10\sqrt{4 + k^2}\cos\theta$	1A	OA: $4x - 3y = 0$ OB: $6x - 8y = 0$ $3x - 4y = 0$
$\frac{6 + 4k}{5\sqrt{4 + k^2}} = \frac{16 + 6k}{10\sqrt{4 + k^2}}$	1M	$\frac{8 - 3k}{5} = \pm\frac{6 - 4k}{5}$ 1M+1
$k = 2$	1A	$k = \pm 2$ 1
	6	rejecting $k = -2$, $k = 2$ 1

Alt. Solution (2):

$\angle AOX = \tan^{-1}\frac{4}{3} = 53.13°$)
$\angle BOX = \tan^{-1}\frac{3}{4} = 36.87°$) 1A

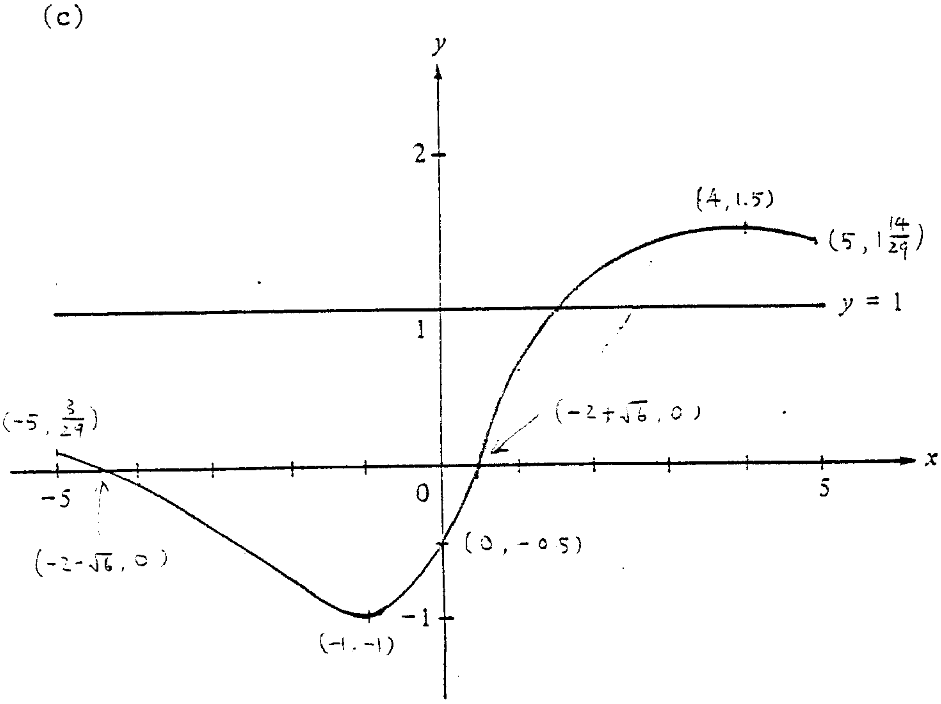
$\angle COX = \frac{\angle AOX + \angle BOX}{2}$ 1M

$= 45°$ 1A

$k = 2$ 1A

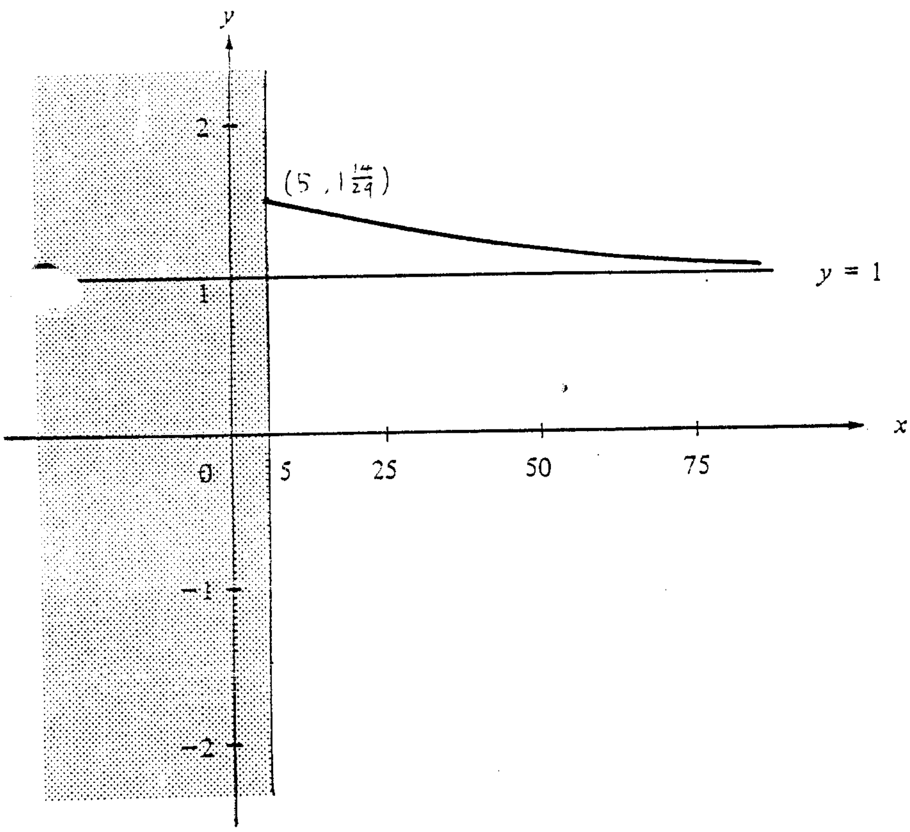
SOLUTIONS	MARKS	REMARKS
7. (i) $x \geq 3$, $\frac{x-3}{2x} < 1$		
$x - 3 < 2x$		
$x > -3$	1A	
Therefore, $x \geq 3$	1A	
(ii) $3 > x > 0$, $\frac{3-x}{2x} < 1$		
$3 - x < 2x$		
$x > 1$	1A	
Therefore, $3 > x > 1$	1A	
(iii) $x < 0$, $\frac{3-x}{2x} < 1$		
$3 - x > 2x$		
$1 > x$	1A	
Therefore, $x < 0$	1A	
Combining the 3 cases, $x < 0$ or $x > 1$	1A	
	7	

SOLUTIONS	MARKS	REMARKS
3. (a) Solving $y = \dfrac{x^2 + 4x - 2}{x^2 + 4}$ and $y = 1$,		
$x^2 + 4x - 2 = x^2 + 4$		
$4x = 6$		
$x = 1.5$		
P is the point (1.5, 1)	1A	
	1	
(b) (i) put $y = 0$		
$x^2 + 4x - 2 = 0$		
$x = -2 \pm \sqrt{6}$ (or -4.45, 0.45)	1A+1A	
put $x = 0$		
$y = -0.5$	1A	
(ii) $y' = \dfrac{(x^2 + 4)(2x + 4) - (x^2 + 4x - 2)(2x)}{(x^2 + 4)^2}$	1M	
$= \dfrac{-4x^2 + 12x + 16}{(x^2 + 4)^2}$	1A	
$= \dfrac{-4(x - 4)(x + 1)}{(x^2 + 4)^2}$		
< 0	1M	Putting $y' < 0$
$x > 4$ or $x < -1$	2A	
(iii) $y' = 0$	1M	
$x = 4$ or $x = -1$		
(4, 1.5) is a maximum point.	1A	
(−1, −1) is a minimum point.	1A	
	11	
(c)	2	Shape
	1A	intercepts, end-points (3 out of 5)
	1A	max. & min. points
	4	



RESTRICTED 內部文件

SOLUTIONS	MARKS	REMARKS
8. (d) (i) $y = 1 + \frac{4x-6}{x^2+4}$	1A	
If $x > 1\frac{1}{2}$, $4x - 6 > 0$ Therefore, $\frac{4x-6}{x^2+4} > 0$ .... $y > 1$	1	
(ii)	2	Curve cutting y = 1, deduc 1 mark. Wrong position of starting point, deduct 1 mark.
	4	



SOLUTIONS	MARKS	REMARKS
9. (a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$	1A	If vector sign ommitted, pp
$= -7\underline{i} - 4\underline{j}$	1A	Alt. Solution:
$\overrightarrow{AB} - \overrightarrow{BC} = -12\underline{i} + 6\underline{j}$		$\overrightarrow{AB} - \overrightarrow{BC}$
$\overrightarrow{BC} = (-7\underline{i} - 4\underline{j}) - (-12\underline{i} + 6\underline{j})$	1M	$= (\overrightarrow{OB} - \overrightarrow{OA}) - (\overrightarrow{OC} - \overrightarrow{OB})$ 1
$= 5\underline{i} - 10\underline{j}$		$= -12\underline{i} + 6\underline{j}$
$\overrightarrow{OC} - \overrightarrow{OB} = 5\underline{i} - 10\underline{j}$		$\overrightarrow{OC} = 2\overrightarrow{OB} - \overrightarrow{OA} - (-12\underline{i} + 6\underline{j})$
$\overrightarrow{OC} = -\underline{i} - 12\underline{j}$	2A / 5	$= -\underline{i} - 12\underline{j}$ ....
(b) (i) $\overrightarrow{AX} = k\overrightarrow{OX}$		
$\overrightarrow{OX} - \overrightarrow{OA} = k\overrightarrow{OX}$		
$(1 - k)\overrightarrow{OX} = \overrightarrow{OA}$	1A	
$k \neq 1$, $\overrightarrow{OX} = \frac{1}{1-k}(\underline{i} + 2\underline{j})$	1A	Accept omitting $k \neq 1$.
(ii) $\overrightarrow{BX} = \overrightarrow{OX} - \overrightarrow{OB}$		
$= \left(\frac{1}{1-k} + 6\right)\underline{i} + \left(\frac{2}{1-k} + 2\right)\underline{j}$	1A	
$OX \perp BX$		
$\frac{1}{1-k}\left(\frac{1}{1-k} + 6\right) + \frac{2}{1-k}\left(\frac{2}{1-k} + 2\right) = 0$	1M	
$(7 - 6k) + 2(4 - 2k) = 0$		
$k = 1\frac{1}{2}$	2A	
$\overrightarrow{OX} = -2\underline{i} - 4\underline{j}$	1A	Alt. Solution:
$\overrightarrow{AX} + \overrightarrow{BX} + \overrightarrow{CX}$		$\overrightarrow{AX} = -3\underline{i} - 6\underline{j}$)
$= (\overrightarrow{OX} - \overrightarrow{OA}) + (\overrightarrow{OX} - \overrightarrow{OB}) + (\overrightarrow{OX} - \overrightarrow{OC})$		) 1
		$\overrightarrow{BX} = 4\underline{i} - 2\underline{j}$)
$= 3\overrightarrow{OX} - (\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC})$	2A	$\overrightarrow{CX} = -\underline{i} + 8\underline{j}$ 1
$= (-6\underline{i} - 12\underline{j}) - (-6\underline{i} - 12\underline{j})$		$\overrightarrow{AX} + \overrightarrow{BX} + \overrightarrow{CX} = \vec{0}$ 1
$= \vec{0}$	1A	
$\overrightarrow{AC} = -2\underline{i} - 14\underline{j}$, $\overrightarrow{AB} = -7\underline{i} - 4\underline{j}$		Alt. Solution:
$\overrightarrow{AM} = \frac{1}{2}(\overrightarrow{AC} + \overrightarrow{AB})$	1M	M is the point $(-\frac{7}{2}, -7)$ 1
$= -\frac{9}{2}\underline{i} - 9\underline{j}$	1A	$\overrightarrow{AM} = -\frac{9}{2}\underline{i} - 9\underline{j}$ 1
$\overrightarrow{AX} = -3\underline{i} - 6\underline{j}$	1A	Alt. Solution: Slope of AX $= \frac{-6}{-3} = 2$
$= \frac{2}{3}(-\frac{9}{2}\underline{i} - 9\underline{j})$		Slope of AM $= \frac{-9}{\frac{-9}{2}} = 2$ 1
$= \frac{2}{3}\overrightarrow{AM}$	1A	
Therefore, X lies on AM.	1M / 15	Slope of AX = slope of AM
		∴ X lies on AM 1

SOLUTIONS	MARKS	REMARKS
10.(a) (i) $f(x) = 0$		Alt. Solution:
$x^2 + 2x - 1 = 0$	1M	$x^2 + 2x - 1 = 0$
$x = -1 \pm \sqrt{2}$ (Accept -2.41 or 0.41)	1A	$PQ = \lvert x_1 - x_2 \rvert$
$PQ = 2\sqrt{2}$ (Accept $\sqrt{8}$ or 2.83)	1A	$= \sqrt{(x_1 - x_2)^2}$
$g(x) = 0$		$= \sqrt{(x_1+x_2)^2 - 4x_1x_2}$
$-x^2 + 2kx - k^2 + 6 = 0$		$= \sqrt{(-2)^2 - 4(-1)}$
$x^2 - 2kx + k^2 - 6 = 0$		$= 2\sqrt{2}$ ....
$x = k \pm \sqrt{k^2 - (k^2 - 6)}$		$g(x) = 0$
$= k \pm \sqrt{6}$	1A	$RS = \sqrt{(2k)^2 - 4(k^2 - 6)}$
$RS = 2\sqrt{6}$ (Accept $\sqrt{24}$ or 4.90)	1A	$= 2\sqrt{6}$ ....
(ii) x-coordinate of the mid-point of RS		
$= \dfrac{(k + \sqrt{6}) + (k - \sqrt{6})}{2}$	1M	This can be omitted.
$= k$	1A	
x-coordinate of the mid-point of PQ $= -1$	1A	This can be omitted.
$k = -1$	1A	
	9	
(b) $x^2 + 2x - 1 = -x^2 + 2kx - k^2 + 6$	1	
$2x^2 + (2 - 2k)x + (k^2 - 7) = 0$		
$D = 0$		
$(2 - 2k)^2 - 4(2)(k^2 - 7) = 0$	1M	
$k^2 + 2k - 15 = 0$		
$k = 3$ or -5	2A	
For $k = 3$, $2x^2 - 4x + 2 = 0$		
$x^2 - 2x + 1 = 0$		
$x = 1$, $y = 2$	1A	
The point is $(1, 2)$.		
For $k = -5$, $2x^2 + 12x + 18 = 0$		
$x^2 + 6x + 9 = 0$		
$x = -3$, $y = 2$	1A	
The point is $(-3, 2)$.		
	6	

SOLUTIONS	MARKS	REMARKS
10.(c) $f(x) > g(x)$		
$x^2 + 2x - 1 > -x^2 + 2kx - k^2 + 6$		
$2x^2 + (-2k + 2)x + k^2 - 7 > 0$	1A	
This is true for any real value of x,		
$(2 - 2k)^2 - 4(2)(k^2 - 7) < 0$	2M	
$k^2 + 2k - 15 > 0$		
$k > 3$ or $k < -5$	2A	
	5	

Alt. Solution:

$f(x) > g(x)$

$x^2 + 2x - 1 > -x^2 + 2kx - k^2 + 6$

$2x^2 + (-2k + 2)x + k^2 - 7 > 0$ 1A

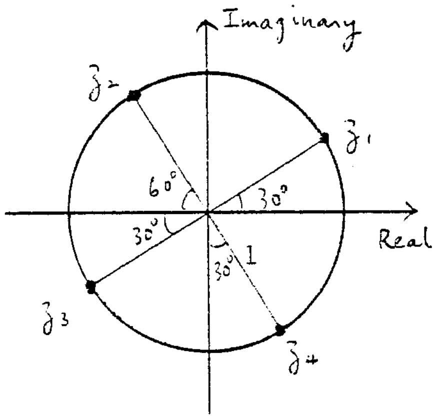
$x^2 + (1 - k)x + \frac{1}{2}(k^2 - 7) > 0$

$(x + \frac{1 - k}{2})^2 + \frac{1}{2}(k^2 - 7) - (\frac{1 - k}{2})^2 > 0$ 1M

$\frac{1}{2}(k^2 - 7) - (\frac{1 - k}{2})^2 > 0$ 1M

$k^2 + 2k - 15 > 0$

$k > 3$ or $k < -5$ 2A

SOLUTIONS	MARKS	REMARKS
11. (a) (i) $z = \cos\theta + i\sin\theta$		
$z^n = \cos n\theta + i\sin n\theta$	1A	
$\frac{1}{z^n} = \cos n\theta - i\sin n\theta$	1A	Accept $\frac{1}{z^n} = z^{-n}$ $= \cos(-n\theta) + i\sin(-n\theta)$
$z^n + \frac{1}{z^n} = 2\cos n\theta$) $z^n - \frac{1}{z^n} = 2i\sin n\theta$)	1	
(ii) $\frac{(z^2 - \frac{1}{z^2})i}{z^2 + \frac{1}{z^2}} = \frac{2i^2\sin 2\theta}{2\cos 2\theta}$	1A	
$= -\tan 2\theta$	1A	
$\tan 2\theta = \sqrt{3}$	1M	
$2\theta = n\pi + \frac{\pi}{3}$		
$\theta = \frac{n\pi}{2} + \frac{\pi}{6}$		
$z = \cos(\frac{n\pi}{2} + \frac{\pi}{6}) + i\sin(\frac{n\pi}{2} + \frac{\pi}{6})$) where $n = 0, 1, 2, 3$) ..	2A	NOTE: If one or more roots missing, award 1 mar[k]
[Accept $n = 0, 1, 2, 3, \ldots$/n is an integer.]		
OR $z = \frac{\sqrt{3}}{2} + \frac{i}{2}, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{\sqrt{3}}{2} - \frac{i}{2}, \frac{1}{2} - \frac{\sqrt{3}}{2}i$.		
 Positions should be either specified by coordinates or unit modulus and angles.	1A+1A	1A For two points. 2A For four points. NOTE: If positions not specified, deduct 1 mark.
	10	
(b) (i) $x = \frac{1 \pm \sqrt{1-4}}{2}$		
$= \frac{1 \pm \sqrt{3}i}{2}$		
$= \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$	1A	
or $\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3})$ [Accept $\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}$]	1A	Accept $\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$

SOLUTIONS	MARKS	REMARKS
11.(b) (ii) Product of roots $= \left(\frac{\alpha}{\beta}\right)^k \left(\frac{\beta}{\alpha}\right)^k$		
$= 1$	1A	
$\frac{\alpha}{\beta} = \dfrac{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}{\cos(\frac{-\pi}{3}) + i\sin(\frac{-\pi}{3})}$ or $\dfrac{\cos(\frac{-\pi}{3}) + i\sin(\frac{-\pi}{3})}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$		
$= \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$ or $\cos(\frac{-2\pi}{3}) + i\sin(\frac{-2\pi}{3})$	1A	
Sum of roots $= \left(\frac{\alpha}{\beta}\right)^k + \left(\frac{\beta}{\alpha}\right)^k$		
$= z^k + \left(\frac{1}{z}\right)^k$		
$= 2\cos\frac{2k\pi}{3}$	2A	
Required equation is:		
$x^2 - 2\cos\frac{2k\pi}{3} \cdot x + 1 = 0$	1A	This can be omitted.
(1) When $k = 3n$,		
$\frac{2k\pi}{3}$ is a multiple of 2π,		
equation becomes $x^2 - 2x + 1 = 0$.	1A	
(2) When $k = 3n + 1$,		
$\cos\frac{2k\pi}{3} = \cos(2n\pi + \frac{2\pi}{3}) = -\frac{1}{2}$		
equation becomes $x^2 + x + 1 = 0$	1A	
(3) When $k = 3n + 2$,		
$\cos\frac{2k\pi}{3} = \cos(2n\pi + \frac{4\pi}{3}) = -\frac{1}{2}$		
equation becomes $x^2 + x + 1 = 0$	1A	
	10	

Alt. Solution:

(b) (ii) (1) $k = 3n$,

product of roots $= 1$ 1A

sum of roots $= \left(\frac{\alpha}{\beta}\right)^{3n} + \left(\frac{\beta}{\alpha}\right)^{3n}$

$= 2\cos\frac{2(3n\pi)}{3}$ 1A

$= 2$ 1A

Equation: $x^2 - 2x + 1 = 0$ 1A

(2) $k = 3n + 1$,

sum of roots $= -1$ 1A

Equation: $x^2 + x + 1 = 0$ 1A

(3) $k = 3n + 2$,

sum of roots $= -1$ 1A

Equation: $x^2 + x + 1 = 0$ 1A

SOLUTIONS	MARKS	REMARKS
12.(a) (i) $2\pi r = \ell\phi$		
$r = \frac{\ell\phi}{2\pi}$	1A	
(ii) Let h be the height of the cone.		
$h^2 = \ell^2 - r^2$		
$= \ell^2 - \frac{\ell^2\phi^2}{4\pi^2}$	1M	For Pythagoras' Theorem.
Volume of the cone $= \frac{1}{3}\pi r^2 h$		
$= \frac{1}{3}\pi \cdot \frac{\ell^2\phi^2}{4\pi^2} \cdot \sqrt{\ell^2 - \frac{\ell^2\phi^2}{4\pi^2}}$	1M	For substitution.
$V^2 = \frac{\ell^6}{576\pi^4}[4\pi^2\phi^4 - \phi^6]$		
$= k(4\pi^2\phi^4 - \phi^6)$	1	
(iii) $\frac{d(V^2)}{d\phi} = k(16\pi^2\phi^3 - 60^5)$	1A	
$= 0$	1M	
$\phi \neq 0$, $\phi^2 = \frac{8\pi^2}{3}$		
$\phi > 0$, $\phi = \frac{2\sqrt{6}}{3}\pi$ (or 5.13) (or 1.63π)	2A	Accept $\phi = 0$ or $\pm\frac{2\sqrt{6}}{3}\pi$
$\frac{d^2(V^2)}{d\phi^2} = k(48\pi^2\phi^2 - 300^4)$		
$= 6k\phi^2(8\pi^2 - 5\phi^2)$		
$\frac{d^2(V^2)}{d\phi^2}\Big\vert_{\phi^2=\frac{8\pi^2}{3}} = 6k \cdot \frac{8\pi^2}{3}(8\pi^2 - 5\cdot\frac{8\pi^2}{3})$		
< 0	1M	
V^2 is a maximum when $\phi = \frac{2\sqrt{6}}{3}\pi$)		
)	1A	
V is a maximum when $\phi = \frac{2\sqrt{6}}{3}\pi$)	10	
(b) (i) $\ell = r - r\cos\theta$	1A	
(ii) $A = \frac{1}{2}(r^2)(2\theta) - \frac{1}{2}r^2\sin 2\theta$		
$= r^2\theta - \frac{1}{2}r^2\sin 2\theta$	2A	or $r^2\theta - r^2\sin\theta\cos\theta$
(iii) $\frac{dA}{d\theta} = r^2 - r^2\cos 2\theta$	1A	or $2r^2\sin^2\theta$
$\frac{d\theta}{d\ell} = \frac{1}{r\sin\theta}$	1A	
$\frac{dA}{dt} = \frac{dA}{d\theta}\cdot\frac{d\theta}{d\ell}\cdot\frac{d\ell}{dt}$	1M	
$= (r^2 - r^2\cos 2\theta)\cdot\frac{1}{r\sin\theta}\cdot u$	1M	
$= \frac{ru(1-\cos 2\theta)}{\sin\theta}$	1A	Accept $\frac{u(r - r\cos 2\theta)}{\sin\theta}$
$= 2ru\sin\theta$	1A	
When $\theta = \frac{\pi}{6}$, $\frac{dA}{dt} = 2ru\sin\frac{\pi}{6}$		
$= ru$	1	
	10	

RESTRICTED 内部文件