12.

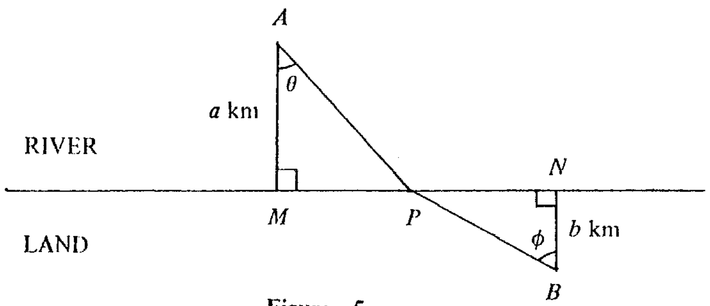

Figure 5

In Figure 5, A is a fixed point in water a km from a straight river bank. B is a fixed point on land b km from the river. M and N are the points on the bank nearest to A and B respectively. P is a point between M and N. Let $\angle MAP = \theta$ and $\angle NBP = \phi$. A man can swim at a speed of u km/h and run at a speed of v km/h, where $u < v$.

(a) The man swims from A to P and then runs to B.

(i) Express MN in terms of a, b, θ and ϕ.

Hence show that $\dfrac{d\phi}{d\theta} = -\dfrac{a\sec^2\theta}{b\sec^2\phi}$.

(ii) Let t hours be the time taken to travel from A to B via P.

Show that $t = \dfrac{a}{u}\sec\theta + \dfrac{b}{v}\sec\phi$.

If t is a minimum, show that $\dfrac{u}{v} = \dfrac{\sin\theta}{\sin\phi}$.

(Testing for maximum/minimum is not required.) (12 marks)

(b) Let $MN = h$ km. Suppose the man swims from A to P and then runs to N.

(i) Express the time taken in terms of a, h, u, v and θ.

(ii) Using the result in (b)(i), find MP in terms of a, u and v when the time taken is a minimum.
(Testing for maximum/minimum is not required.) (5 marks)

(c) Suppose C is a point in water c km from N and $CN \perp MN$. If the man swims from A to C via P in the minimum time, find $MP : PN$. (3 marks)

END OF PAPER

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1987

附加數學 試卷二
ADDITIONAL MATHEMATICS PAPER II

11.15 am–1.15 pm (2 hours)
This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.

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SECTION A (39 marks)

Answer ALL questions in this section.

1. If the coefficient of x^2 in the expansion of $(1 + x + x^2)^n$ is 21 and n is a positive integer, find the value of n.

(5 marks)

2. Prove, by mathematical induction, that

$$\frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1},$$

for all positive integers n.

(5 marks)

3. Find the equations of the two lines through $(1, 2)$, each making an angle of θ (where $\tan\theta = \frac{1}{2}$) with the line $3x - y = 0$.

(5 marks)

4. Using the substitution $x = \sin\theta$, evaluate $\int_0^{\frac{1}{2}} \frac{2x^2}{\sqrt{1-x^2}}\,dx$.

(6 marks)

5. The slope at any point (x, y) of a curve is given by

$$\frac{dy}{dx} = (3x^2 - 2)(x^3 - 2x + 1)^{\frac{1}{3}}$$

If the curve passes through the origin, find the equation of the curve.
[Hint: Put $x^3 - 2x + 1 = u$.]

(6 marks)

6. Express $\sin 3\theta$ in terms of $\sin\theta$. Hence find the three roots of the equation $8x^3 - 6x + 1 = 0$ to 2 significant figures.

(6 marks)

7. Find the equations of the two tangents to the curve $x^2 - y^2 = 3$ which are parallel to the line $y = 2x$.

(6 marks)

SECTION B (60 marks)

Answer any THREE questions from this section.
Each question carries 20 marks.

8. (a) Using the substitution $u = \tan x$, find

$$\int \tan^{n-2} x \sec^2 x\,dx,$$

where n is an integer and $n \geqslant 2$.

(4 marks)

(b) (i) By writing $\tan^n x$ as $\tan^{n-2} x \tan^2 x$, show that

$$\int_0^{\frac{\pi}{4}} \tan^n x\,dx = \frac{1}{n-1} - \int_0^{\frac{\pi}{4}} \tan^{n-2} x\,dx,$$

where n is an integer and $n \geqslant 2$.

(ii) Evaluate $\int_0^{\frac{\pi}{4}} \tan^6 x\,dx$.

(9 marks)

(c) Show that $\int_{-\frac{\pi}{4}}^{0} \tan^6 x\,dx = \int_0^{\frac{\pi}{4}} \tan^6 x\,dx$.

Hence evaluate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^6 x\,dx$.

(7 marks)

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9.

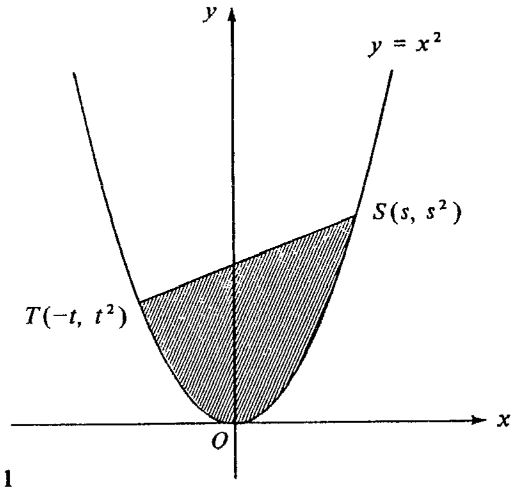

Figure 1

In Figure 1, $S(s, s^2)$ and $T(-t, t^2)$ are two points on the curve $y = x^2$, where s and t are positive real numbers.

(a) Find the area of the region bounded by the curve, the x-axis and the line $x = s$.

Hence, or otherwise, show that the area of the region bounded by the curve and the chord ST (the shaded part in the figure) is $\frac{1}{6}(s + t)^3$. (7 marks)

(b) Suppose the chord ST passes through $H(0, 1)$.

(i) Show that $t = \frac{1}{s}$.

(ii) Using the results in (a) and (b)(i), find the value of s such that the area of the region bounded by the curve and the chord ST is a minimum. (7 marks)

(c) If the region of minimum area in (b)(ii) is revolved about the x-axis, find the volume of the solid generated. (6 marks)

10.

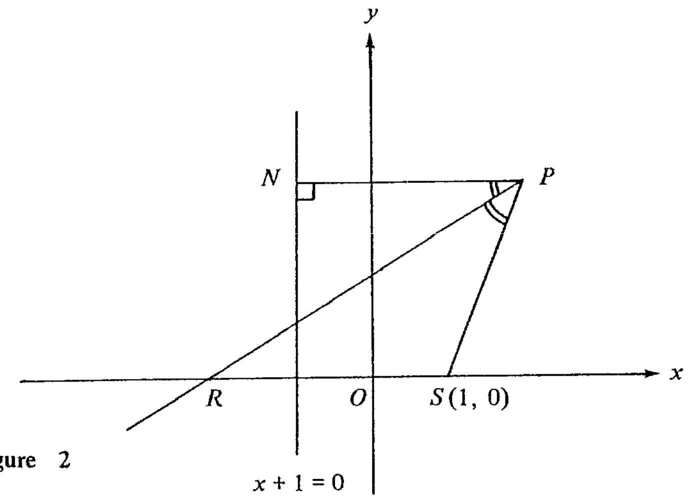

Figure 2

In Figure 2, $P(x, y)$ is a point equidistant from the point $S(1, 0)$ and the line $x + 1 = 0$.

(a) Show that the equation of the locus of P is $y^2 = 4x$. (4 marks)

(b) Let the y-coordinate of P be $2t$.

(i) Find the x-coordinate of P in terms of t.

(ii) N is the foot of the perpendicular from P to the line $x + 1 = 0$. The bisector of $\angle SPN$ intersects the x-axis at R.

(1) Show that the equation of PR is $x - ty + t^2 = 0$.

(2) Show that PR touches $y^2 = 4x$ at P.

(3) Find the equation of the locus of the mid-point of PR. (16 marks)

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11.

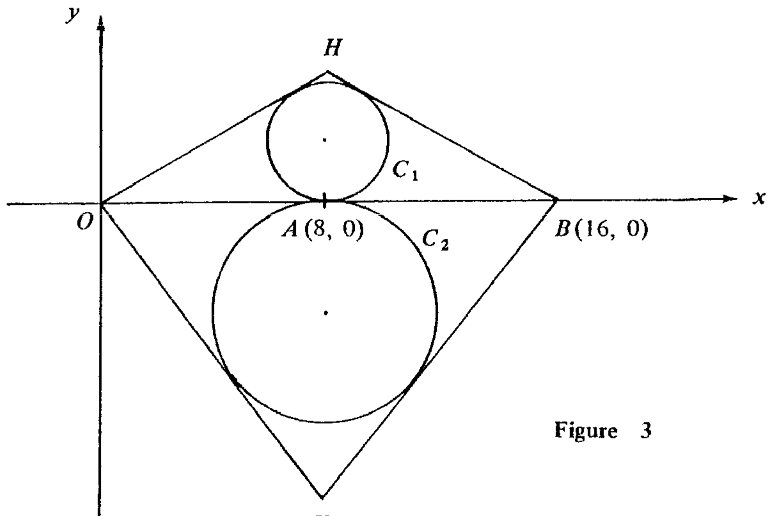

Figure 3

In Figure 3, A and B are the points (8, 0) and (16, 0) respectively. The equation of the circle C_1 is $x^2 + y^2 - 16x - 4y + 64 = 0$. OH and BH are tangents to C_1.

(a) (i) Show that C_1 touches the x-axis at A.

(ii) Find the equation of OH.

(iii) Find the equation of BH.

(12 marks)

(b) In the figure, the equation of OK is $4x + 3y = 0$. The circle C_2 : $x^2 + y^2 - 16x + 2fy + c = 0$ is the inscribed circle of ΔOBK and touches the x-axis at A.

(i) Find the values of the constants c and f.

(ii) Find area of ΔOBH : area of ΔOBK.

(8 marks)

12. (a) (i) If $7 \sin\theta - 24\cos\theta$ is expressed in the form $r\sin(\theta - A)$ where $r > 0$ and $0^\circ < A < 90^\circ$, find r and A.

(ii) Let $y = 14\sin\theta - 48\cos\theta + 14$.

Using the result in (i), find the maximum and minimum values of y.

Find also the general values of θ at which y attains its maximum.

(12 marks)

(b) α and β are two acute angles satisfying the equation

$$6\cos^2\theta - 5\cos\theta + 1 = 0 .$$

Without solving the equation, show that

$$\cos\frac{\alpha+\beta}{2} + \cos\frac{\alpha-\beta}{2} = \sqrt{2} .$$

[Hint: $\cos^2\frac{x}{2} = \frac{1}{2}(1 + \cos x)$.]

(8 marks)

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Additional Mathematics Paper I

1. -12

2. $\frac{dy}{dx} = \frac{1}{1 + \cos y}$

   $\frac{d^2y}{dx^2} = \frac{\sin y}{(1 + \cos y)^3}$

4. (a) $45 - 36\cos\theta$

   (b) $1\ (s^{-1})$

5. (a) $p < 4$

   (b) -3

6. (a) $\frac{3}{2}$

   (b) $\sqrt{19}$

7. $x < -3$

8. (a) (i) $\overrightarrow{AB} = -3\mathbf{i} + 3\mathbf{j}$

   $\overrightarrow{AC} = 3\mathbf{i} + 6\mathbf{j}$

   (ii) $\frac{1}{1 + m}[(-3 + 3m)\mathbf{i} + (3 + 6m)\mathbf{j}]$

   (b) (ii) $105°$

   (iii) $\lambda = \frac{1}{3}$, $\mu = \frac{2}{3}$

   (iv) 10

9. (a) (i) $a + \frac{a}{\sin\theta}$

   (ii) $30°$

10. (a) $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$,

    $\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}$

    (b) (ii) $z_1^3 = 1$, $z_2^3 = 1$

    (iii) (1) 2

    (2) -1

    (3) -1

11. (a) $k > 1$

    (b) $z^2 + (6k - 8)z + k^3 = 0$

    $k = 4$

    (c) $(2 - k) \pm 2\sqrt{k - 1}\, i$

    $y^2 = 4(1 - x)$

12. (a) (i) $a\tan\theta + b\tan\phi$

    (b) (i) $\frac{a\sec\theta}{u} + \frac{h - a\tan\theta}{v}$

    (ii) $\frac{au}{\sqrt{v^2 - u^2}}$

    (c) $a : c$

1987

Additional Mathematics Paper II

1. 6

3. $7x + y - 9 = 0$,

   $x - y + 1 = 0$

4. $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$

5. $y = \frac{3}{4}(x^3 - 2x + 1)^{\frac{4}{3}} - \frac{3}{4}$

6. 0.17, 0.77, -0.94

7. $2x - y - 3 = 0$,

   $2x - y + 3 = 0$

8. (a) $\frac{\tan^{n-1} x}{n - 1} + c$

   (b) (ii) $\frac{13}{15} - \frac{\pi}{4}$

   (c) $2(\frac{13}{15} - \frac{\pi}{4})$

9. (a) $\frac{s^3}{3}$

   (b) (ii) 1

   (c) $\frac{8\pi}{5}$

10. (b) (i) t^2

    (ii) (3) $x = 0$

11. (a) (ii) $8x - 15y = 0$

    (iii) $8x + 15y - 128 = 0$

    (b) (i) $c = 64$, $f = 4$

    (ii) $2 : 5$

12. (a) (i) $r = 25$, $A = 73.7°$

    (ii) Maximum value of $y = 64$

    Minimum value of $y = -36$

    $180n° + (-1)^n 90° + 73.7°$

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