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香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八七年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1987

附加數學（卷一）

ADDITIONAL MATHEMATICS (Paper I)

評卷參考

MARKING SCHEME

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遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員在任何情況下披露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

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RESTRICTED 內部文件

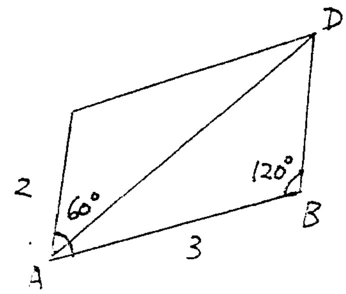
SOLUTIONS	MARKS	REMARKS
1. $f(x) = \operatorname{cosec}^2 3x.$		Alt. Solution:
$f'(x) = 2 \operatorname{cosec} 3x(-\operatorname{cosec} 3x \cot 3x) \cdot 3$	2A	$f'(x) = -2(\sin 3x)^{-3} \cdot \cos 3x (3$
$= -6 \operatorname{cosec}^2 3x \cot 3x$ or $\frac{-6\cos 3x}{\sin^3 3x}$		
$\therefore f'(\frac{\pi}{12}) = -6 \operatorname{cosec}^2 \frac{\pi}{4} \cot \frac{\pi}{4}$	1M	$= -2(\sin \frac{\pi}{4})^{-3} \cos \frac{\pi}{4} \cdot 3$
$= -12$	1A	$= -12$ ....
	4	
2. $x = y + \sin y$		Alternative solution
$\frac{dx}{dy} = 1 + \cos y$	1M+1A	Diff. both sides
$\therefore \frac{dy}{dx} = \frac{1}{1 + \cos y}$	1A	$1 = y' + (\cos y)(y')$ 1M+
		$= y'(1 + \cos y)$
		$y' = \frac{1}{1 + \cos y}$
$\frac{d^2y}{dx^2} = \frac{-(-\sin y)}{(1 + \cos y)^2} \cdot \frac{dy}{dx}$	1A	$0 = y'' - \sin y \cdot (y')^2 + \cos y \cdot y''$
$= \frac{\sin y}{(1 + \cos y)^3}$	1A	$y'' = \frac{\sin y}{(1 + \cos y)^3}$ 1
	5	
3. let $z = x + iy$		
$z + \bar{z} = x + yi + x - yi$		
$= 2x$		
$= 2\mathrm{Re}(z)$	1	
$\lvert z \rvert = \sqrt{x^2 + y^2}$		
$\geqslant \sqrt{x^2} = \lvert x \rvert \geqslant x$		
$= \mathrm{Re}(z)$	1	
$z_1z_2 + \bar{z}_1\bar{z}_2 = z_1z_2 + \overline{z_1z_2}$		
$= 2\mathrm{Re}(z_1z_2)$	1A	
$\leqslant 2\lvert z_1z_2 \rvert$	1A	
$= 2\lvert z_1 \rvert \lvert z_2 \rvert$	1	
	5	
Alternatively		
$z = \lvert z \rvert(\cos\theta + i\sin\theta)$		
$\bar{z} = \lvert z \rvert(\cos\theta - i\sin\theta)$		
$z + \bar{z} = 2\lvert z \rvert\cos\theta = 2\mathrm{Re}(z)$	1	
$\mathrm{Re}(z) = \lvert z \rvert\cos\theta \leqslant \lvert z \rvert$	1	
$z_1z_2 + \bar{z}_1\bar{z}_2$		
$= \lvert z_1 \rvert \lvert z_2 \rvert \operatorname{cis}(\theta_1 + \theta_2) + \lvert z_1 \rvert \lvert z_2 \rvert \operatorname{cis}(-\theta_1 - \theta_2)$	1A	
$= 2\lvert z_1 \rvert \lvert z_2 \rvert \cos(\theta_1 + \theta_2)$	1A	
$\leqslant 2\lvert z_1 \rvert \lvert z_2 \rvert$	1	

SOLUTIONS	MARKS	REMARKS
3. Alternative Solution:		
$z_1z_2 + \bar{z}_1\bar{z}_2$		
$= (x_1 + iy_1)(x_2 + iy_2) + (x_1 - iy_1)(x_2 - iy_2)$		
$= 2(x_1x_2 - y_1y_2)$	1A	
$2\|z_1\|\|z_2\|$		
$= 2\|z_1z_2\|$		
$= 2\sqrt{(x_1x_2 - y_1y_2)^2 + (x_1y_2 + x_2y_1)^2}$		
$\geq 2\sqrt{(x_1x_2 - y_1y_2)^2}$	1A	
$= 2\|x_1x_2 - y_1y_2\|$		
$\geq 2(x_1x_2 - y_1y_2)$		
$= z_1z_2 + \bar{z}_1\bar{z}_2$	1	
4. (a) $x^2 = 3^2 + 6^2 - (2)(3)(6)\cos\theta$		
$= 45 - 36\cos\theta$	1A	$x^2 = 45 - 36\cos\theta$ 1A
(b) Differentiating with respect to time,	1M	$x = \sqrt{45 - 36\cos\theta}$
$2x\frac{dx}{dt} = 36\sin\theta\frac{d\theta}{dt}$	1A+1A	Differentiating 1M
When $\theta = \frac{\pi}{3}$, $x^2 = 45 - 36(\frac{1}{2})$		$\frac{dx}{dt} = \frac{36\sin\theta}{2\sqrt{45 - 36\cos\theta}} \cdot \frac{d\theta}{dt}$ 2A
$= 27$		$= \frac{1}{2\sqrt{45 - 18}} \cdot 36 \cdot \frac{\sqrt{3}}{2} \cdot \frac{1}{3}$
$x = \sqrt{27}$		$= 1(s^{-1})$ 2A
$\frac{dx}{dt} = (36)(\frac{\sqrt{3}}{2})(\frac{1}{3})(\frac{1}{2\sqrt{27}})$		
$= 1\ (s^{-1})$	2A	Do not deduct marks for wrong units or no units.
	6	
5. (a) $4^2 - 4p > 0$	1A	
$p < 4$	1A	
(b) $\alpha + \beta = -4$, $\alpha\beta = p$	1A	
$(\alpha^2 + \beta^2) + \alpha^2\beta^2 + 3(\alpha + \beta) - 19 = 0$		
$16 - 2p + p^2 - 12 - 19 = 0$		
$p^2 - 2p - 15 = 0$	1A	
$(p - 5)(p + 3) = 0$		
$p = 5$ or -3	1A	
but $p < 4$, $\therefore p = -3$	1A	
	6	

SOLUTIONS	MARKS	REMARKS
6. (a) $\vec{AB}\cdot\vec{AC} = 3\cdot 1\cdot\cos 60°$	1A	if vector sign omitted, pp-
$= \frac{3}{2}$	1A	
(b) $\lvert\vec{AB} + 2\vec{AC}\rvert^2$		**Alternative Solution:**
$= (\vec{AB} + 2\vec{AC})\cdot(\vec{AB} + 2\vec{AC})$	1A	$\vec{AB} = 3\underline{i}$ 1
$= \lvert\vec{AB}\rvert^2 + 4\lvert\vec{AC}\rvert^2 + 4\vec{AB}\cdot\vec{AC}$	1A	$\vec{AC} = \frac{1}{2}\underline{i} + \frac{\sqrt{3}}{2}\underline{j}$ 1
$= 9 + 4 + 6$		$\vec{AB} + 2\vec{AC} = 4\underline{i} + \sqrt{3}\underline{j}$ 1
$= 19$	1A	$\lvert\vec{AB} + 2\vec{AC}\rvert = \sqrt{19}$ 1
$\therefore \lvert\vec{AB} + 2\vec{AC}\rvert = \sqrt{19}$	1A	
	6	

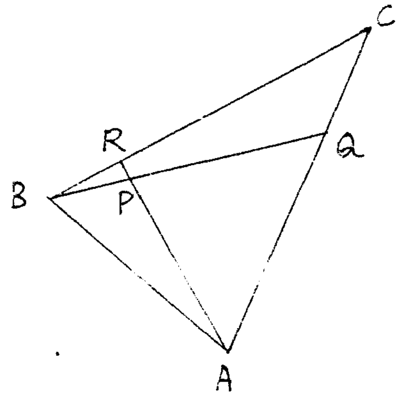
Alternative Solution:

$\lvert\vec{AB} + 2\vec{AC}\rvert = AD$	(1A)
$AD^2 = 3^2 + 2^2 - 2(2)(3)\cos 120°$	(1A)
$= 19$	(1A)
$\lvert\vec{AB} + 2\vec{AC}\rvert = \sqrt{19}$	(1A)



SOLUTIONS	MARKS	REMARKS
7. 2 cases $[x-2 \geq 0$ or $x-2 < 0]$ or $[x-2 > 0$ or $x-2 \leq 0]$	1	**Alternative Solution:**
Case (i) $x - 2 \geq 0$ i.e. $x \geq 2$		L.S. < 0
$(x + 2)\lvert x - 2\rvert < -5$		$\therefore x + 2 < 0$ 1
$(x + 2)(x - 2) < -5$	1A	$x < -2$ 1
$x^2 - 4 < -5$		
$x^2 < -1$ impossible	1A	L.S. $= (x+2)(2-x)$
Case (ii) $x - 2 < 0$ i.e. $x < 2$		$(x+2)(2-x) < -5$ 1
$(x + 2)\lvert x - 2\rvert < -5$		$x^2 > 9$
$(x + 2)(-x + 2) < -5$	1A	$x > 3$ or $x < -3$ 1
$-x^2 + 4 < -5$		$x < -3$ 2
$x^2 > 9$		**Alternative Solution:**
$x > 3$ or $x < -3$	1A	$(x+2)^2(x-2)^2 > 25$ 2
$x < -3$	1A	$(x^2+1)(x^2-9) > 0$ 1
Combining (i) and (ii)		$x > 3$ or $x < -3$ 2
$\therefore x < -3$	1A	After checking, $x < -3$ 2
	7	

SOLUTIONS	MARKS	REMARKS
8. (a) (i) $\overrightarrow{AB} = -3\underline{i} + 3\underline{j}$	1A	if vector sign omitted, or division of vectors, pp-1
$\overrightarrow{AC} = 3\underline{i} + 6\underline{j}$	1A	
(ii) $\overrightarrow{AR} = \frac{1}{1+m}[\overrightarrow{AB} + m\overrightarrow{AC}]$	1M	
$= \frac{1}{1+m}[(-3+3m)\underline{i} + (3+6m)\underline{j}]$	1A	
	4	
(b) (i) $\overrightarrow{BC} = 6\underline{i} + 3\underline{j}$		
$AR \perp BC$, $\therefore \overrightarrow{AR} \cdot \overrightarrow{BC} = 0$		
$\frac{1}{1+m}[6(-3+3m) + 3(3+6m)] = 0$	1M+1A	
$m = \frac{1}{4}$	1	



Alternative Solution:

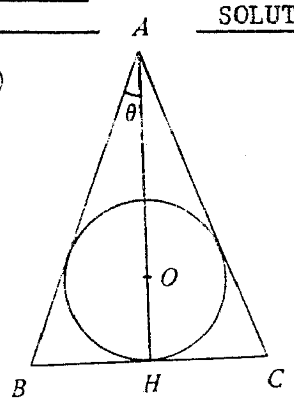
	MARKS
slope of BC · slope of AR = −1	
$\frac{1}{2} \cdot \left[\frac{\frac{7m+4}{m+1} - 1}{\frac{7m+1}{m+1} - 4}\right] = -1$	1M+1A
or $\frac{1}{2} \cdot \frac{3+6m}{-3+3m} = -1$	
$m = \frac{1}{4}$	1A

SOLUTIONS	MARKS	REMARKS
(ii) $\overrightarrow{AR} = -\frac{9}{5}\underline{i} + \frac{18}{5}\underline{j}$		**Alternative Solution:**
Let $\angle QPR = \theta$		$\angle CBQ = \phi$
$\overrightarrow{BQ} \cdot \overrightarrow{AR} = \lvert\overrightarrow{BQ}\rvert\lvert\overrightarrow{AR}\rvert \cos\theta$	1M	$\overrightarrow{BQ} \cdot \overrightarrow{BC} = \lvert\overrightarrow{BQ}\rvert\lvert\overrightarrow{BC}\rvert \cos\phi$ 1M
$\overrightarrow{BQ} \cdot \overrightarrow{AR} = -\frac{27}{5}$	1A	$\overrightarrow{BQ} \cdot \overrightarrow{BC} = 33$ 1A
$\lvert\overrightarrow{BQ}\rvert\lvert\overrightarrow{AR}\rvert \cos\theta = \sqrt{26}\sqrt{(-\frac{9}{5})^2 + (\frac{18}{5})^2}\cos\theta$	1A	$\lvert\overrightarrow{BQ}\rvert\lvert\overrightarrow{BC}\rvert \cos\phi = \sqrt{26} \cdot \sqrt{45} \cos\phi$ 1A
$-\frac{27}{5} = \frac{9}{5} \cdot \sqrt{5} \cdot \sqrt{26} \cos\theta$		$\angle QBC = 15.25°$
$\theta = 105°$ (Accept answers roundable to 105°)	1A	$\theta = 105°$ 1A

Alternative Solution:

	MARKS
$\tan \angle CBQ = \frac{m_1 - m_2}{1 + m_1 m_2}$	
$= \frac{\frac{3}{6} - \frac{1}{5}}{1 + \frac{1}{5} \cdot \frac{3}{6}}$	1M+2A
(Accept $\frac{1}{5} - \frac{3}{6}$ in the numerator)	
$\angle CBQ = 15.25°$	
$\theta = 105°$	1A

SOLUTIONS	MARKS	REMARKS
8. (b) (iii) $\overrightarrow{BQ} = \lambda\overrightarrow{BA} + \mu\overrightarrow{BC}$		
$5\underline{i} + \underline{j} = \lambda(3\underline{i} - 3\underline{j}) + \mu(6\underline{i} + 3\underline{j})$		
$= (3\lambda + 6\mu)\underline{i} + (-3\lambda + 3\mu)\underline{j}$		
$3\lambda + 6\mu = 5$) $-3\lambda + 3\mu = 1$)	2M+1A	
$\lambda = \frac{1}{3}$, $\mu = \frac{2}{3}$	1A	
(iv) $\overrightarrow{BP} = \frac{1}{1+n}[\overrightarrow{BA} + n\overrightarrow{BR}]$		
$= \frac{1}{1+n}[\overrightarrow{BA} + \frac{n}{5}\overrightarrow{BC}]$	1A	
$= \frac{1}{1+n}[(3 + \frac{6}{5}n)\underline{i} + (-3 + \frac{3}{5}n)\underline{j}]$	1A	
$\overrightarrow{BQ} = 5\underline{i} + \underline{j}$		
$\overrightarrow{BP} \mathbin{//} \overrightarrow{BQ}$		
$\therefore \frac{3 + \frac{6n}{5}}{-3 + \frac{3n}{5}} = \frac{5}{1}$	1M+1A	
$3 + \frac{6}{5}n = -15 + \frac{15}{5}n$		
$n = 10$	1A	
	16	

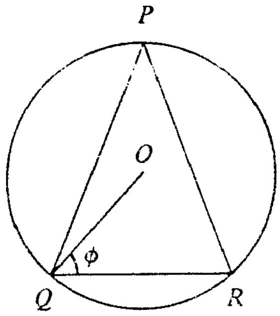
SOLUTIONS	MARKS	REMARKS
9. (a)(i) 		
$OA = \frac{a}{\sin\theta}$		
$AH = a + \frac{a}{\sin\theta}$	1A	
base $BC = (2)(AH\tan\theta)$	1M	
$= 2a[1 + \frac{1}{\sin\theta}]\tan\theta$	1A	
$= 2a\frac{(1+\sin\theta)}{\cos\theta}$		
Area, $S = \frac{1}{2}(a)(1 + \frac{1}{\sin\theta}) \cdot 2a\frac{(1+\sin\theta)}{\cos\theta}$		
$= \frac{a^2(1+\sin\theta)^2}{\sin\theta\cos\theta}$	1	
(ii) Writing $s = \sin\theta$, $c = \cos\theta$,		
$\frac{dS}{d\theta} = \frac{(sc)2(1+s)c - (1+s)^2(-s^2+c^2)}{s^2c^2} \cdot a^2$	1M	For differentiating S wit respect to θ .
$= \frac{a^2(1+s)[2sc^2 - c^2 + s^3 - sc^2 + s^2]}{s^2c^2}$		
$= \frac{a^2(1+s)}{s^2c^2}(s^3 + s^2 - c^2 + sc^2)$		
$= 0$	1M	
$\because 1 + s \neq 0$, $s^3 + s^2 - c^2 + sc^2 = 0$		
$s^3 + s^2 + (1 - s^2)(s - 1) = 0$		
$2s^2 + s - 1 = 0$	2A	
$(2s - 1)(s + 1) = 0$		
$s = \frac{1}{2}$	1A	
$\theta = 30°$ or $\frac{\pi}{6}$	1A	
	10	

NOTE: There are several alternative solutions in which S is expressed in different forms before differentiation,

e.g. $S = a^2(\frac{1}{sc} + \frac{s}{c} + \frac{2}{c})$,

$S = a^2\tan\theta(1 + \csc\theta)^2$

or $S = \frac{a^2(1+s)^2}{\sin 2\theta}$.

SOLUTIONS	MARKS	REMARKS
9. (b) (i) 		
$QR = 2b\cos\emptyset$	1A	
height $= b + b\sin\emptyset$	1A	
Area $A = \frac{1}{2}\cdot 2b\cos\emptyset\cdot b(1+\sin\emptyset)$		
$= b^2\cos\emptyset(1+\sin\emptyset)$	1	
(ii) When Δ is equilateral, $\emptyset = 30°$	1A	
$\frac{dA}{d\emptyset} = b^2[\cos^2\emptyset - \sin\emptyset(1+\sin\emptyset)]$	2A	
$= b^2[1 - 2\sin^2\emptyset - \sin\emptyset]$ or $b^2[\cos 2\emptyset - \sin\emptyset]$		
when $\emptyset = 30°$, $\frac{dA}{d\emptyset} = b^2[\cos^2 30° - \sin 30°(1+\sin 30°)]$	1M	1M For sub. $\emptyset = 30°$ in $\frac{dA}{d\emptyset}$
$= 0$	1A	
$\frac{d^2A}{d\emptyset^2} = b^2[-4\sin\emptyset\cos\emptyset - \cos\emptyset]$	1M	For finding 2nd derivative.
When $\emptyset = 30°$, $\frac{d^2A}{d\emptyset^2} < 0$,	1A	Do not award this mark if there is no 2nd derivative 2nd derivative is wrong.
$\therefore$ The area is a maximum.		
	10	

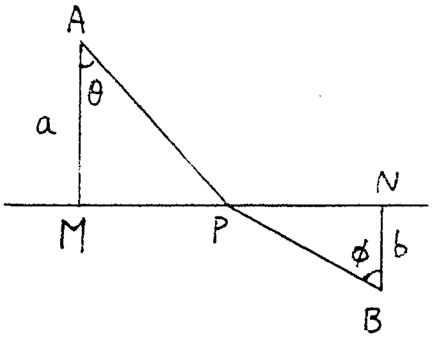
Alternatively:

$\frac{dA}{d\emptyset} = b^2[\cos^2\emptyset - \sin\emptyset(1+\sin\emptyset)]$	2A	
$= 0$	1M	
$\cos^2\emptyset - \sin\emptyset - \sin^2\emptyset = 0$		
$1 - \sin\emptyset - 2\sin^2\emptyset = 0$		
$2\sin^2\emptyset + \sin\emptyset - 1 = 0$		
$(2\sin\emptyset - 1)(\sin\emptyset + 1) = 0$		
$\sin\emptyset = \frac{1}{2}$ or -1 (rejected)		
$\emptyset = 30°$	1A	
$\angle OQR = 30°$		
$\angle PQR = 30° + 30° = 60°$		
$\therefore \Delta PQR$ is equilateral	1A	
$\frac{d^2A}{d\emptyset^2} = b^2[-2\cos\emptyset\sin\emptyset - \cos\emptyset - 2\sin\emptyset\cos\emptyset]$	1M	
$= b^2[-4\cos\emptyset\sin\emptyset - \cos\emptyset]$		
$\left.\frac{d^2A}{d\emptyset^2}\right\vert_{\emptyset = 30°} < 0$	1A	
The area is a max. when Δ is equilateral.		

SOLUTIONS	MARKS	REMARKS
10.(a) $z^2 = (\cos\theta + i\sin\theta)^2$		Alternatively:
$= \cos 2\theta + i\sin 2\theta$ or $(\cos^2\theta - \sin^2\theta) + i2\sin\theta\cos\theta$	1A	$\frac{1}{z} = \bar{z}$ 1
$\bar{z} = \cos\theta - i\sin\theta$	1A	$z^2 - 2\bar{z} + \frac{1}{z} = z^2 - \bar{z}$ 1
$\frac{1}{z} = \frac{1}{\cos\theta + i\sin\theta} = \cos\theta - i\sin\theta$	1A	$z^2 - 2\bar{z} + \frac{1}{z}$ is real
$z^2 - 2\bar{z} + \frac{1}{z} = \cos 2\theta + i\sin 2\theta - \cos\theta + i\sin\theta$		$\therefore z^2 - \bar{z} = \overline{z^2 - \bar{z}} = (\bar{z})^2 - z$ 1M+1
This is real,		$z^2 - (\bar{z})^2 = \bar{z} - z$
$\therefore \sin 2\theta + \sin\theta = 0$	1M+1A	$(z - \bar{z})(z + \bar{z}) = \bar{z} - z$
or $2\sin\theta\cos\theta + \sin\theta = 0$		$z + \bar{z} = -1$ 1
$\sin\theta \neq 0$,		$2\mathrm{Re}(z) = -1$
$\therefore \cos\theta = -\frac{1}{2}$	1A	$\mathrm{Re}(z) = -\frac{1}{2}$ 1
$\theta = 2n\pi \pm \frac{2\pi}{3}$		$\mathrm{Im}(z) = \pm\sqrt{1 - \frac{1}{4}} = \pm\frac{\sqrt{3}}{2}$
$z = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$ or $\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}$	2A+1A	$z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i,\ -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ 2A+1
$[z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ or $-\frac{1}{2} - \frac{\sqrt{3}}{2}i]$		
$[z = \mathrm{cis}120°$ or $\mathrm{cis}(-120°)]$ (Accept cis240°)	9	
(b) Take $z_1 = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$, $z_2 = \cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}$		Alternative Solution:
(i) $z_1^2 = (\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})^2$		$z_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$
$= \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$		$z_1^2 = (-\frac{1}{2} + \frac{\sqrt{3}}{2}i)(-\frac{1}{2} + \frac{\sqrt{3}}{2}i)$
$= \cos(2\pi - \frac{4\pi}{3}) - i\sin(2\pi - \frac{4\pi}{3})$		$= \frac{1}{4} - \frac{3}{4} - \frac{1}{2}(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2})i$
$= z_2$	1	$= -\frac{1}{2} - \frac{\sqrt{3}}{2}i$
$z_2^2 = (\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3})^2$		$= z_2$ 1
$= \cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}$		Similarly for $z_2^2 = z_1$ 1
$= \cos(2\pi - \frac{4\pi}{3}) + i\sin(2\pi - \frac{4\pi}{3})$		
$= z_1$	1	
(ii) $z_1^3 = (\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})^3$		$z_1^3 = z_1 z_1^2$
$= \cos 2\pi + i\sin 2\pi$		$= z_1 z_2$
$= 1$	1A	$= 1$ 1A
$z_2^3 = (\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3})^3$		$z_2^3 = z_2 z_2^2$
$= \cos 2\pi - i\sin 2\pi$		$= z_2 z_1$
$= 1$	1A	$= 1$ 1A

SOLUTIONS	MARKS	REMARKS
10.(b)(iii) $z_1^{3n} + z_2^{3n} = (z_1^3)^n + (z_2^3)^n$		
$= 1^n + 1^n$		
$= 2$	1A	
$z_1^{3n+1} + z_2^{3n+1} = z_1^{3n} \cdot z_1 + z_2^{3n} \cdot z_2$		
$= z_1 + z_2$		
$= -1$	1A	
$z_1^{3n+2} + z_2^{3n+2} = (z_1^{3n})z_1^2 + (z_2^{3n})z_2^2$		
$= z_1^2 + z_2^2$		
$= z_2 + z_1$		
$= -1$	1A	
Alternative Solution:		
$z_1^{3n} + z_2^{3n} = 2\cos 2n\pi$		
$= 2$	1A	
$z_1^{3n+1} + z_2^{3n+1} = 2\cos \frac{2(3n+1)\pi}{3}$		
$= 2\cos(2n\pi + \frac{2\pi}{3})$		
$= 2\cos \frac{2\pi}{3}$		
$= -1$	1A	
$z_1^{3n+2} + z_2^{3n+2} = 2\cos \frac{2(3n+2)\pi}{3}$		
$= 2\cos(2n\pi + \frac{4\pi}{3})$		
$= 2\cos \frac{4\pi}{3}$		
$= -1$	1A	
(iv) $z_1^{2k} + z_2^{2k} = (z_1^2)^k + (z_2^2)^k$	1A	
$= z_2^k + z_1^k$	2A	
$= \begin{cases} 2 & k \text{ is a multiple of } 3 \\ -1 & k \text{ is not a multiple of } 3 \end{cases}$	1	
	11	
Alternative Solution:		Alt. Solution:
$z_1^{2k} + z_2^{2k} = 2\cos \frac{4k\pi}{3}$	1A	$z_1^{2(3n)} + z_2^{2(3n)}$
$k = 3n$, $z_1^{2k} + z_2^{2k} = 2\cos \frac{4k\pi}{3} = 2\cos 4n\pi = 2$	1	$= \text{cis}4n\pi + \text{cis}(-4n\pi)$ 1A
$k = 3n+1$, $z_1^{2k} + z_2^{2k} = 2\cos(4n\pi + \frac{4\pi}{3}) = -1$	1	$= 2$ 1
$k = 3n+2$, $z_1^{2k} + z_2^{2k} = 2\cos(4n\pi + \frac{8\pi}{3}) = -1$	1	etc.

SOLUTIONS	MARKS	REMARKS
11.(a) $z^2 - 2z + k = 0$		
$(-2)^2 - 4k < 0$	1A	
$k > 1$	1A	
	2	
(b) Let α and β be the roots of $z^2 - 2z + k = 0$		**Alternative Solution:**
$\alpha + \beta = 2$) $\alpha\beta = k$)	1A	$z = 1 \pm \sqrt{1-k}$　1
$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha^2\beta - 3\alpha\beta^2$		$z^2 = (2-k) \pm 2\sqrt{1-k}$
$= (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$	1A	$z^3 = (4-3k) \pm (4-k)\cdot\sqrt{1-k}$　2
or $(\alpha+\beta)[(\alpha+\beta)^2 - 3\alpha\beta]$		Required equation is
$= 8 - 6k$	1A	$\{z-[(4-3k)+(4-k)\sqrt{1-k}]\}$
$\alpha^3\beta^3 = k^3$	1A	X $\{z-[(4-3k)-(4-k)\sqrt{1-k}]\} = 0$　1
$\therefore$ Required equation is $z^2 + (6k-8)z + k^3 = 0$	1A	$z^2 + (6k-8)z + k^3 = 0$　1
$\Delta = (6k-8)^2 - 4k^3$	1M	
$= -4k^3 + 36k^2 - 96k + 64$ $= -4(k^3 - 9k^2 + 24k - 16)$ $= 4(1-k)(4-k)^2$	1	
The equation has real roots,		**Alternatively:**
$4(1-k)(4-k)^2 \geq 0$	1A	$\text{Arg}(z) = \pm\tan^{-1}(\sqrt{k-1})$
but $k > 1$ 1		z^3 is real
$\therefore (k-4)^2 \leq 0$　1A		$\text{Arg}(z^3) = \pi$ 1
$k = 4$　1A		$3\text{Arg}(z) = \pi$　1
Alt. Solution: $k = 4$ or $k \leq 1$　1A but $k > 1$　1 $\therefore k = 4$　1A		$\tan^{-1}(\pm\sqrt{k-1}) = \frac{\pi}{3}$　1A $\pm\sqrt{k-1} = \sqrt{3}$ $k - 1 = 3$ $k = 4$ 1A
	11	
		Alternatively:
(c) $z = 1 \pm \sqrt{1-k}$	1A	$z = 1 \pm \sqrt{1-k}$ 1A
$z^2 = (2-k) \pm 2\sqrt{1-k}$	1A	$z^2 = 2z - k$
$= (2-k) \pm 2\sqrt{k-1}\,i$	1A	$= 2(1 \pm \sqrt{1-k}) - k$　1A
		$= (2-k) \pm \sqrt{k-1}\,i$　1A
$x = 2 - k$) $y = \pm 2\sqrt{k-1}$)	1A	
Eliminating k,	1M	
$y = \pm 2\sqrt{2-x-1}$ $= \pm 2\sqrt{1-x}$	1A	
where $x \neq 1$	1A	
i.e. $y^2 = 4(1-x)$ where $x \neq 1$.		
	7	

SOLUTIONS	MARKS	REMARKS
12.(a) (i) $MN = MP + PN$	1A	
$= a\tan\theta + b\tan\phi$	1A	
$\frac{d(MN)}{d\theta} = a\sec^2\theta + b\sec^2\phi\frac{d\phi}{d\theta}$	1M+1A	or $\frac{d(MN)}{d\phi}$
$= 0$	1M	
$\therefore \frac{d\phi}{d\theta} = -\frac{a\sec^2\theta}{b\sec^2\phi}$		
(ii) $t = \frac{AP}{u} + \frac{BP}{v}$	1A	
$= \frac{a}{u}\sec\theta + \frac{b}{v}\sec\phi$		
$\frac{dt}{d\theta} = \frac{a}{u}\sec\theta\tan\theta + \frac{b}{v}\sec\phi\tan\phi\frac{d\phi}{d\theta}$	1M+2A	
$= 0$	1M	
From (a), $\frac{a}{u}\sec\theta\tan\theta + \frac{b}{v}\sec\phi\tan\phi\left(-\frac{a\sec^2\theta}{b\sec^2\phi}\right) = 0$	1M	
$\frac{a}{u}\frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta} + \frac{b}{v}\cdot\frac{\sin\phi}{\cos^2\phi}\left(-\frac{a}{b}\cdot\frac{\cos^2\phi}{\cos^2\theta}\right) = 0$		
$\frac{u}{v} = \frac{\sin\theta}{\sin\phi}$	1 / 12	
(b) (i) $t = \frac{AP}{u} + \frac{PN}{v}$		
$= \frac{a\sec\theta}{u} + \frac{h - a\tan\theta}{v}$	1A	
(ii) When t is a minimum,		
$\frac{dt}{d\theta} = 0$	1M	
$\frac{a\sec\theta\tan\theta}{u} - \frac{a\sec^2\theta}{v} = 0$		
$\frac{\tan\theta}{u} = \frac{\sec\theta}{v}$		
$\frac{u}{v} = \sin\theta$	1A	
$MP = a\tan\theta$		
$= \frac{a\frac{u}{v}}{\sqrt{1 - \frac{u^2}{v^2}}}$		
$= \frac{au}{\sqrt{v^2 - u^2}}$	2A / 5	

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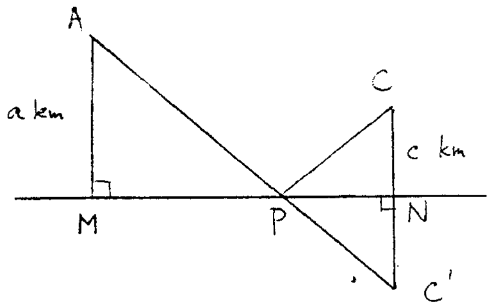
12.(c)



Time required $= \dfrac{AP + CP}{u}$

$= \dfrac{AP + C'P}{u}$

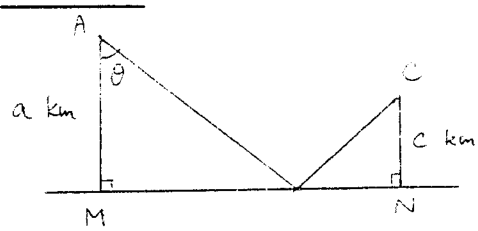
For minimum time, (AP + C'P) is a minimum.

i.e. APC' is a straight line — 2

and MP : PN = a : c .... — 1A

3

Alternative Solution:



$t = \dfrac{1}{u}\left[a\sec\theta + \sqrt{(h - a\tan\theta)^2 + c^2}\right]$

$\dfrac{dt}{d\theta} = \dfrac{1}{u}\left[a\sec\theta\tan\theta - \dfrac{2(h - a\tan\theta)\,a\sec^2\theta}{2\sqrt{(h - a\tan\theta)^2 + c^2}}\right]$........ — 1A

$= 0$

$\tan\theta\sqrt{(h - a\tan\theta)^2 + c^2} = (h - a\tan\theta)\sec\theta$

$\tan^2\theta(h - a\tan\theta)^2 + c^2\tan^2\theta = (h - a\tan\theta)^2\sec^2\theta$

$c^2\tan^2\theta = (h - a\tan\theta)^2$

$h - a\tan\theta = \pm c\tan\theta$

$(a \pm c)\tan\theta = h$

$\tan\theta = \dfrac{h}{a \pm c}$ .... — 1A

$MP = a\tan\theta = \dfrac{ah}{a \pm c}$

(Rejecting $\dfrac{ah}{a - c}$ ∵ P lies between M and N)

$MP = \dfrac{ah}{a + c}$

$PN = \dfrac{ch}{a + c}$

MP : PN = a : c .... — 1A