HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1987

附加數學　試卷一
ADDITIONAL MATHEMATICS　PAPER I

8.30 am–10.30 am　(2 hours)

This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.

SECTION A (39 marks)

Answer ALL questions in this section.

1. Let $f(x) = \operatorname{cosec}^2 3x$. Find $f'(\frac{\pi}{12})$. (4 marks)

2. Let $x = y + \sin y$.

   Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of y. (5 marks)

3. For any complex number z, let $\bar{z}$, $|z|$ and $\operatorname{Re}(z)$ be its conjugate, modulus and real part respectively.

   Show that $z + \bar{z} = 2\operatorname{Re}(z)$ and $|z| \geqslant \operatorname{Re}(z)$.

   Hence, or otherwise, show that for any complex numbers z_1 and z_2,

   $$z_1 z_2 + \bar{z}_1 \bar{z}_2 \leqslant 2|z_1||z_2| .$$

   (5 marks)

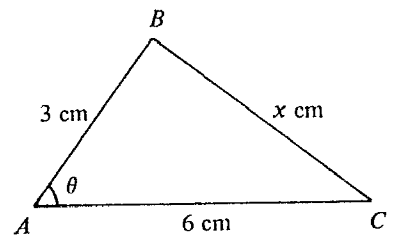
4. 

   Figure 1

   In Figure 1, $AB = 3$ cm, $AC = 6$ cm, $BC = x$ cm and $\angle A = \theta$.

   (a) Express x^2 in terms of θ.

   (b) If θ increases at the rate of $\frac{1}{3}$ radian per second, find the rate of change of x with respect to time when $\theta = \frac{\pi}{3}$. (6 marks)

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5. The equation $x^2 + 4x + p = 0$, where p is a real constant, has distinct real roots α and β.

(a) Find the range of values of p.

(b) If $\alpha^2 + \beta^2 + \alpha^2\beta^2 + 3(\alpha + \beta) - 19 = 0$, find the value of p.

(6 marks)

6.

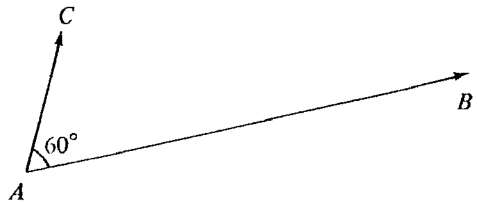

Figure 2

In Figure 2, $|\overrightarrow{AB}| = 3$, $|\overrightarrow{AC}| = 1$ and $\angle CAB = 60°$.

Find (a) $\overrightarrow{AB} \cdot \overrightarrow{AC}$,

(b) $|\overrightarrow{AB} + 2\overrightarrow{AC}|$.

(6 marks)

7. Solve the inequality $(x + 2)|x - 2| < -5$.

(7 marks)

SECTION B (60 marks)

Answer any THREE questions from this section.
Each question carries 20 marks.

8.

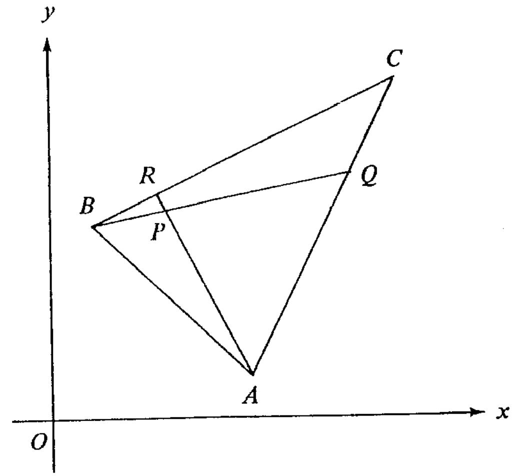

Figure 3

In Figure 3, R is a point on BC such that $BR : RC = m : 1$. Q is a point on AC. BQ intersects AR at P. $\overrightarrow{OA} = 4\mathbf{i} + \mathbf{j}$, $\overrightarrow{OB} = \mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OC} = 7\mathbf{i} + 7\mathbf{j}$ and $\overrightarrow{BQ} = 5\mathbf{i} + \mathbf{j}$.

(a) (i) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(ii) Express $\overrightarrow{AR}$ in terms of m, $\mathbf{i}$ and $\mathbf{j}$.

(4 marks)

(b) Suppose AR is perpendicular to BC.

(i) Show that $m = \frac{1}{4}$.

(ii) Find $\angle QPR$.

(iii) If $\overrightarrow{BQ} = \lambda\overrightarrow{BA} + \mu\overrightarrow{BC}$, find the values of λ and μ.

(iv) If $AP : PR = n : 1$, express $\overrightarrow{BP}$ in terms of n, $\mathbf{i}$ and $\mathbf{j}$.

Hence find the value of n.

(16 marks)

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9. (a)

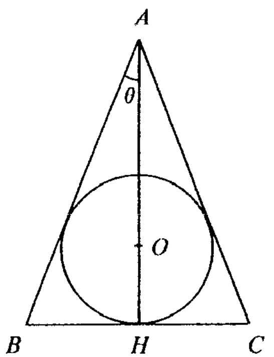

Figure 4(a)

Figure 4(a) shows a circle of centre O and radius a inscribed in an isosceles triangle ABC with $AB = AC$. Let $\angle OAB = \theta$.

(i) Find, in terms of a and θ, the height AH of $\triangle ABC$.

Hence show that the area of $\triangle ABC$ is

$$\frac{a^2(1 + \sin\theta)^2}{\sin\theta\cos\theta}.$$

(ii) For what value of θ is the area of $\triangle ABC$ a minimum? (Testing for maximum/minimum is not required.) (10 marks)

(b)

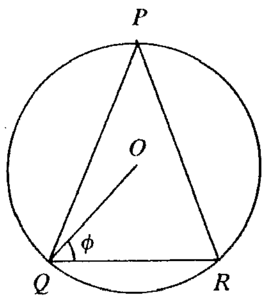

Figure 4(b)

Figure 4(b) shows a circle of centre O and radius b circumscribing an isosceles triangle PQR with $PQ = PR$. Let $\angle OQR = \phi$.

(i) Show that the area of $\triangle PQR$ is

$$b^2\cos\phi(1 + \sin\phi).$$

(ii) When $\triangle PQR$ is equilateral, show that its area is a maximum. (10 marks)

10. (a) Let $z = \cos\theta + i\sin\theta$, where θ is not a multiple of π.

If $z^2 - 2\bar{z} + \frac{1}{z}$ is real, find the two values of z. (9 marks)

(b) Let z_1 and z_2 be the two values of z obtained in (a).

(i) Show that $z_1^2 = z_2$ and $z_2^2 = z_1$.

(ii) Find the values of z_1^3 and z_2^3.

(iii) Find the values of $z_1^k + z_2^k$ when

(1) $k = 3n$,
(2) $k = 3n + 1$,
(3) $k = 3n + 2$,

where n is a positive integer.

(iv) For any positive integer k, show that

$$z_1^{2k} + z_2^{2k} = \begin{cases} 2 & \text{when } k \text{ is a multiple of } 3, \\ -1 & \text{when } k \text{ is not a multiple of } 3. \end{cases}$$

(11 marks)

11. It is given that the equation

$z^2 - 2z + k = 0$ (k is real) ....(*)

has no real roots.

(a) Find the range of values of k. (2 marks)

(b) Find the quadratic equation whose roots are the cubes of the roots of (*) and show that the discriminant of this equation is $4(1 - k)(4 - k)^2$.

If this equation has real roots, deduce the value of k. (11 marks)

(c) Find, in terms of k, the squares of the roots of (*), expressing the answers in the form $x + iy$ where x and y are real.

As k varies, find the equation of the locus of the points in the Argand plane representing the squares of the roots of (*). (7 marks)

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12.

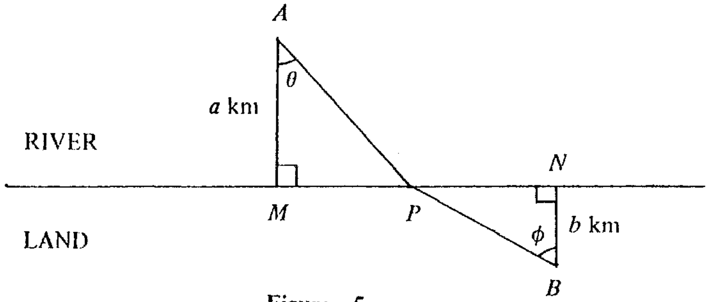

Figure 5

In Figure 5, A is a fixed point in water a km from a straight river bank. B is a fixed point on land b km from the river. M and N are the points on the bank nearest to A and B respectively. P is a point between M and N. Let $\angle MAP = \theta$ and $\angle NBP = \phi$. A man can swim at a speed of u km/h and run at a speed of v km/h, where $u < v$.

(a) The man swims from A to P and then runs to B.

(i) Express MN in terms of a, b, θ and ϕ.

Hence show that $\frac{d\phi}{d\theta} = -\frac{a\sec^2\theta}{b\sec^2\phi}$.

(ii) Let t hours be the time taken to travel from A to B via P.

Show that $t = \frac{a}{u}\sec\theta + \frac{b}{v}\sec\phi$.

If t is a minimum, show that $\frac{u}{v} = \frac{\sin\theta}{\sin\phi}$.

(Testing for maximum/minimum is not required.) (12 marks)

(b) Let $MN = h$ km. Suppose the man swims from A to P and then runs to N.

(i) Express the time taken in terms of a, h, u, v and θ.

(ii) Using the result in (b)(i), find MP in terms of a, u and v when the time taken is a minimum.
(Testing for maximum/minimum is not required.) (5 marks)

(c) Suppose C is a point in water c km from N and $CN \perp MN$. If the man swims from A to C via P in the minimum time, find $MP : PN$. (3 marks)

END OF PAPER

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附加數學 試卷二
ADDITIONAL MATHEMATICS PAPER II

11.15 am–1.15 pm (2 hours)
This paper must be answered in English

Answer **ALL** questions in Section A and any **THREE** questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.

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