RESTRICTED 內部文件

香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八六年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1986

附加數學（試卷二）
ADDITIONAL MATHEMATICS II

評卷參考
MARKING SCHEME

請在學校任教之閱卷員特別留意

本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應嚴予拒絕。閱卷員在任何情況下洩露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

Special Note for Teacher Markers

It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

Markers should therefore resist pleas from their students to have access to this document. Making it available would constitute misconduct on the part of the marker and is, moreover, in breach of the 1977 Hong Kong Examinations Authority Ordinace.

RESTRICTED 內部文件

SOLUTIONS	MARKS	REMARKS
1. When $n = 1$, L.S. $= \frac{1}{(1)(2)} = \frac{1}{2}$		
R.S. $= \frac{1}{1+1} = \frac{1}{2}$		
∴ the equality is true for $n = 1$.	1	
Assume that the equality holds for some positive integer k, then for $n = k + 1$,	1	
L.S. $= \frac{1}{(1)(2)} + \frac{1}{(2)(3)} + \dots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}$		
$= \frac{k}{(k+1)} + \frac{1}{(k+1)(k+2)}$	1A	
$= \frac{k(k+2)+1}{(k+1)(k+2)}$		
$= \frac{(k+1)^2}{(k+1)(k+2)}$		
$= \frac{k+1}{k+2}$	1A	
By mathematical induction, the equality is true for any positive integer n.	1 —— 5	Awarded only if above correct
2. Coefficient of 3rd term $= {}_nC_2 \cdot 2^2$ or $\frac{n(n-1)}{2} \cdot 2^2$	1A	
$\frac{n(n-1)}{2} \cdot 4 = 40$	1M	
$n^2 - n - 20 = 0$	1A	
$(n-5)(n+4) = 0$		
$n = 5$	1A	
Coefficient of $x^4 = {}_5C_3 \cdot 2^3$		
$= 80$	1A —— 5	
3. For equal roots, $(-4\cos\theta)^2 - 4(3)(2)\sin\theta = 0$	2A	
$16\cos^2\theta - 24\sin\theta = 0$		
$2(1-\sin^2\theta) - 3\sin\theta = 0$		
$2\sin^2\theta + 3\sin\theta - 2 = 0$	1A	
$(2\sin\theta - 1)(\sin\theta + 2) = 0$		
$\sin\theta = \frac{1}{2}$ or -2		This may be omitted.
Rejecting $\sin\theta = -2$		
$\sin\theta = \frac{1}{2}$	1A	
θ is obtuse		
∴ $\theta = 150°$ $\left(\text{or } \frac{5\pi}{6}\right)$	1A —— 5	

SOLUTIONS	MARKS	REMARKS
4. $\sin 2\theta + \sin 4\theta = \cos\theta$		For answers with mixed units, pp-1
$2\sin 3\theta\cos\theta = \cos\theta$	1A	
$\cos\theta = 0$ or $\sin 3\theta = \frac{1}{2}$	1A+1A	
$\theta = (2n+1)\frac{\pi}{2}$ or $3\theta = n\pi + (-1)^n\frac{\pi}{6}$	1A+1A	
[or $\theta = 2n\pi \pm \frac{\pi}{2}$]		
$\theta = \frac{n\pi}{3} + (-1)^n\frac{\pi}{18}$	1A 6	
(n is an integer)		
5. (a) $\frac{t+2}{s+1} = \frac{6-(-2)}{3-(-1)}$	1A	Alt. Solution: Equation of AB : $\frac{y+2}{x+1} = \frac{6-(-2)}{3-(-1)}$ 1A $y = 2x$ Sub. P(s, t), $t = 2s$ 1A
$t = 2s$	1A	
Alt. Solution: Area of $\Delta ABP = 2(2s-t)$ 1A $= 0$ $t = 2s$ 1A		
Alt. Solution: Let $\frac{AP}{PB} = r$ $s = \frac{3-r}{1+r}$, $t = \frac{6-2r}{1+r}$ $t = 2s$	1A 1A	
(b) Area of $\Delta APC = \frac{1}{2}\begin{vmatrix} 3 & 6 \\ s & 2s \\ 5 & -3 \\ 3 & 6 \end{vmatrix}$		
$= \frac{1}{2}(-13s+39)$	1A	
$\frac{1}{2}(-13s+39) = \pm\frac{13}{2}$	1M	Accept no '±'
$s = 2$ or 4	1A+1A 6	
Alt. Solution: Height of ΔAPC = distance of C from AB $= \frac{10+3}{\sqrt{5}}$ $= \frac{13}{\sqrt{5}}$ $AP = \sqrt{(s-3)^2 + (2s-6)^2}$ $= \sqrt{5}\,\lvert s-3\rvert$,		
Area of $\Delta APC = \frac{1}{2}\cdot\frac{13}{\sqrt{5}}\cdot\sqrt{5}\,\lvert s-3\rvert$	1A	Accept (s-3) or (3-s)
$\frac{1}{2}\cdot\frac{13}{\sqrt{5}}\cdot\sqrt{5}\,(s-3) = \pm\frac{13}{2}$	1M	
$s = 2$ or 4	1A+1A	

SOLUTIONS	MARKS	REMARKS
6. AB : $\frac{y-2}{x-3} = m$	1A	–
$y = mx + (2 - 3m)$		
Sub. in $y = (x-2)^2$		
$mx + (2 - 3m) = (x-2)^2$	1M	
$x^2 - (m+4)x + (3m+2) = 0$	1A	Alt. Solution:
$x_1 + x_2 = m + 4$		$x_1, x_2 = \frac{(m+4) \pm \sqrt{D}}{2}$
C is the mid-point, $\frac{m+4}{2} = 3$	1M+1A	$x_1 + x_2 = m + 4$
$m = 2$	1A	$\frac{m+4}{2} = 3$ 1M+1
		$m = 2$ 1
	6	
7 $\frac{dy}{d\theta} = \tan^2\theta\sec^2\theta - \sec^2\theta$	1A	
$= \tan^2\theta(1 + \tan^2\theta) - (1 + \tan^2\theta)$		
$= \tan^4\theta - 1$	1A	
$\tan^4\theta = \frac{dy}{d\theta} + 1$		
Integrating both sides		
$\int \tan^4\theta \, d\theta = \int (\frac{dy}{d\theta} + 1) \, d\theta$ or $\int \frac{dy}{d\theta} d\theta = \int(\tan^4\theta - 1)d\theta$	1A	
$= \int \frac{dy}{d\theta} d\theta + \int d\theta$		
$= y + \theta + C$	1M	For $\int \frac{dy}{d\theta} d\theta = y$
$= \frac{\tan^3\theta}{3} - \tan\theta + \theta + C$	2A	-1 if C omitted.
	6	
Alt. Solution:		
$\int \tan^4\theta d\theta$		
$= \int \tan^2\theta(\sec^2\theta - 1)d\theta$	1A	
$= \int \tan^2\theta\sec^2\theta d\theta - \int \tan^2\theta d\theta$		
$= \int \tan^2\theta d(\tan\theta) - \int (\sec^2\theta - 1)d\theta$	1M	For putting $u = \tan\theta$
$= \frac{\tan^3\theta}{3} - \tan\theta + \theta + c$	2A	-1 if c omitted.

SOLUTIONS	MARKS	REMARKS
8.(a) Putting $a - x = t$, ...	1A	
$dx = -dt$	1A	
When $x = 0$, $t = a$) ; When $x = a$, $t = 0$)	1A	
$\int_0^a f(x)\,dx$		
$= \int_a^0 -f(a-t)\,dt$		
$= \int_0^a f(a-t)\,dt$	1	
$= \int_0^a f(a-x)\,dx$		
	4	
(b)(i) $\int_0^\pi \cos^{2n+1} x\,dx$		
$= \int_0^\pi \cos^{2n+1}(\pi - x)\,dx$	1A	
$= \int_0^\pi (-\cos x)^{2n+1}\,dx$	1A	
$= -\int_0^\pi \cos^{2n+1} x\,dx$	1A	
$\therefore 2\int_0^\pi \cos^{2n+1} x\,dx = 0$		
$\int_0^\pi \cos^{2n+1} x\,dx = 0$	1A	
(ii) $\int_0^\pi x\sin^2 x\,dx$		
$= \int_0^\pi (\pi - x)\sin^2(\pi - x)\,dx$	1A	
$= \int_0^\pi (\pi - x)\sin^2 x\,dx$		
$= \int_0^\pi \pi\sin^2 x\,dx - \int_0^\pi x\sin^2 x\,dx$	1M	
$\int_0^\pi x\sin^2 x\,dx = \frac{\pi}{2}\int_0^\pi \sin^2 x\,dx$	1A	
$= \frac{\pi}{2}\int_0^\pi \frac{1-\cos 2x}{2}\,dx$	1M	For $\sin^2 x = \frac{1-\cos 2x}{2}$
$= \frac{\pi}{4}\left[x - \frac{1}{2}\sin 2x\right]_0^\pi$	1A	
$= \frac{\pi^2}{4}$	1A	

SOLUTIONS	MARKS	REMARKS
3.(b)(ii)		
Alt. Solution:		
$\int_0^{\pi} x\sin^2 x\,dx = \int_0^{\pi} x\,\frac{1-\cos 2x}{2}\,dx$	1M	
$= \frac{1}{2}\int_0^{\pi} x\,dx - \frac{1}{2}\int_0^{\pi} x\cos 2x\,dx$	1M	
$= \frac{1}{2}\left[\frac{x^2}{2}\right]_0^{\pi} - \frac{1}{2}\int_0^{\pi} x\cos 2x\,dx$		
$\int_0^{\pi} x\cos 2x\,dx = \int_0^{\pi} (\pi - x)\cos 2(\pi - x)\,dx$	1A	
$= \int_0^{\pi} (\pi - x)\cos 2x\,dx$		
$= \pi\int_0^{\pi} \cos 2x\,dx - \int_0^{\pi} x\cos 2x\,dx$		
$\int_0^{\pi} x\cos 2x\,dx = \frac{\pi}{2}\int_0^{\pi} \cos 2x\,dx$	1A	
$= \frac{\pi}{2}\left[\frac{1}{2}\sin 2x\right]_0^{\pi}$		
$= 0$	1A	
$\therefore \int_0^{\pi} x\sin^2 x\,dx = \frac{\pi^2}{4}$	1A	
(iii) $\int_0^{\frac{\pi}{2}} \frac{\sin x\,dx}{\sin x + \cos x} = \int_0^{\frac{\pi}{2}} \frac{\sin(\frac{\pi}{2} - x)\,dx}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)}$	1A	
$= \int_0^{\frac{\pi}{2}} \frac{\cos x\,dx}{\cos x + \sin x}$	1A	
$\int_0^{\frac{\pi}{2}} \frac{\sin x\,dx}{\sin x + \cos x} = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx$	2A	
$= \frac{1}{2}\int_0^{\frac{\pi}{2}} dx$	1A	
$= \frac{\pi}{4}$	1A	
	16	

SOLUTIONS	MARKS	REMARKS
9.(a)(i) slope of $L_1 = \frac{1}{2}$		**Alt. Solution:**
slope of reqd. line $= \frac{3k+2}{2k-1}$	1A	slope of required line $= \frac{3k+2}{2k-1}$ 1A $= m$
$\frac{\frac{3k+2}{2k-1} - \frac{1}{2}}{1 + \frac{1}{2}\cdot\frac{3k+2}{2k-1}} = \pm \tan 45°$ (Accept no "±") $= \pm 1$	1M	$\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m} = \pm\tan 45°$ 1M
$\frac{6k+4-2k+1}{4k-2+3k+2} = \pm 1$		$m = 3$ or $-\frac{1}{3}$
$4k + 5 = \pm(7k)$		$\frac{3k+2}{2k-1} = 3$ or $-\frac{1}{3}$
$k = \frac{5}{3}$ or $-\frac{5}{11}$	1A+1A	$k = \frac{5}{3}$ or $-\frac{5}{11}$ 1A+1A
Equations of lines: $3x - y - 4 = 0$, $x + 3y - 18 = 0$	1A	$3x - y - 4 = 0$, $x + 3y - 18 = 0$ 1A
Alt. Solution: The family of lines pass through (3, 5).	2A	
Let slope of required line be m. $\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m} = \pm\tan 45°$	1M	
$m = 3$ or $-\frac{1}{3}$ Equations of lines: $\frac{y-5}{x-3} = 3$ or $-\frac{1}{3}$	1A	
$3x - y - 4 = 0$, $x + 3y - 18 = 0$	1A	
(ii) $\frac{3k+2}{2k-1} = \frac{1}{2}$	1M	
$6k + 4 = 2k - 1$		
$k = -\frac{5}{4}$		
$L: x - 2y + 7 = 0$	1A	
L_2 is of the form $x - 2y + c = 0$	1M	
Take $(-7, 0)$ on L		
Distance from $(-7, 0)$ to $L_1 = \left\lvert\frac{-7+4}{\sqrt{1^2+2^2}}\right\rvert$	1M	Accept expression with no absolute sign.
Distance from $(-7, 0)$ to $L_2 = \left\lvert\frac{-7+c}{\sqrt{1^2+2^2}}\right\rvert$		
$-7 + c = \pm 3$	1M	Accept no '±'
$c = 10$ or 4 (rejected)		
$L_2: x - 2y + 10 = 0$	1A	
	11	

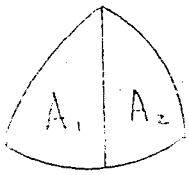
SOLUTIONS	MARKS	REMARKS
(b) x - intercept $= \dfrac{11-k}{3k+2}$	1A	
y - intercept $= \dfrac{k-11}{2k-1}$	1A	
Area $S = -\dfrac{1}{2}\,\dfrac{(k-11)^2}{(3k+2)(2k-1)}$	1M	For area $= \frac{1}{2}$(x-intercept)(y-intercept)
$\dfrac{dS}{dk} = \dfrac{(3k+2)(2k-1)(-2)(k-11)+(k-11)^2(12k+1)}{4(3k+2)^2(2k-1)^2}$		
$= \dfrac{(k-11)(-133k-7)}{4(3k+2)^2(2k-1)^2}$		
$= 0$	1M	
$k = 11$ or $-\dfrac{1}{19}$	1A	
x-intercept and y-intercept are positive, reject $k = 11$		
$k = -\dfrac{1}{19}$ (or -0.0526)	1A	
Testing for minimum	1M	
	7	
(c) $3x - 2y + 1 = 0$	2A	

SOLUTIONS	MARKS	REMARKS
10.(a)(i) $C_1 - C_2$	1M	
$6x + 6y - 18 = 0$		
$x + y - 3 = 0$	1A	
(ii) $x^2 + y^2 - 4x + 2y + 1 + k(x + y - 3) = 0$	1M+1A	
$x^2 + y^2 + (k - 4)x + (k + 2)y + (1 - 3k) = 0$		
$r^2 = [\frac{1}{2}(k - 4)]^2 + [\frac{1}{2}(k + 2)]^2 + 3k - 1$	1M+1A	
$= \frac{1}{2}k^2 + 2k + 4$		
Area $S = \pi(\frac{1}{2}k^2 + 2k + 4)$		
$\frac{dS}{dk} = \pi(k + 2)$ or $\frac{d(r^2)}{dk} = (k + 2)$		
$= 0$	1M	
$k = -2$	1A	
$x^2 + y^2 - 6x + 7 = 0$	1A	
	9	
Alt. Solution (1):		
$x^2 + y^2 - 10x - 4y + 19 + k(x + y - 3) = 0$	1M+1A	
$x^2 + y^2 + (k - 10)x + (k - 4)y + (19 - 3k) = 0$		
$r^2 = [\frac{1}{2}(k - 10)]^2 + [\frac{1}{2}(k - 4)]^2 + 3k - 19$	1M+1A	
$= \frac{1}{2}k^2 - 4k + 10$		
Area $S = \pi(\frac{1}{2}k^2 - 4k + 10)$		
$\frac{dS}{dk} = \pi(k - 4)$		
$= 0$	1M	
$k = 4$	1A	
$x^2 + y^2 - 6x + 7 = 0$	1A	
Alt. Solution (2):		
$x^2 + y^2 - 4x + 2y + 1 + k(x^2+y^2-10x-4y+19) = 0$	1M+1A	
$(1+k)x^2 + (1+k)y^2 + (-4-10k)x + (2-4k)y + 19k + 1 = 0$		
$r^2 = (\frac{2 + 5k}{1+k})^2 + (\frac{2k - 1}{1+k})^2 - \frac{19k + 1}{1 + k}$	1M+1A	
$= \frac{2(5k^2 - 2k + 2)}{(1 + k)^2}$		
$\frac{d(r^2)}{dk} = \frac{2(1 + k)(12k - 6)}{(1 + k)^4}$		
$= 0$	1M	
$k = 1/2$	1A	
$\frac{3}{2}x^2 + \frac{3}{2}y^2 - 9x + \frac{21}{2} = 0$		
$x^2 + y^2 - 6x + 7 = 0$	1A	

SOLUTIONS	MARKS	REMARKS
10.(a)(ii) Alt. Solution (3):		
Solving equation of AB with equation of C_1 or C_2	1M	
Points of intersection: (2, 1) and (4, -1)	1A+1A	
For circle of minimum area, (2, 1) and (4, -1) are ends of a diameter.	2M	
$(x-2)(x-4)+(y-1)(y+1)=0$	1A	or centre: (3, 0), radius = $\sqrt{2}$ } 1A
$x^2+y^2-6x+7=0$	1A	
10.(b) Centre of C_3 : (2, -1)	1A	
distance from (2, -1) to AB		
$= \left\lvert \frac{2-1-3}{\sqrt{1^2+1^2}} \right\rvert$ (Accept no absolute sign)	1M	
$=\sqrt{2}$	1A	
C_3 : $(x-2)^2+(y+1)^2=2$ .... or $x^2+y^2-4x+2y+3=0$	1A	
	4	
Alt. Solution:		
Centre of C_3: (2, -1)	1A	
C_3: $(x-2)^2+(y+1)^2=R^2$		
Sub. $x+y-3=0$ in equation of C_3		
$2x^2-12x+(20-R^2)=0$	1A	
$(-12)^2-4(2)(20-R^2)=0$	1M	
$R^2=2$		
$(x-2)^2+(y+1)^2=2$	1A	
(c) Centre of C_1 = (2, -1), centre of C_2 = (5, 2)		
$\frac{\sqrt{(x-2)^2+(y+1)^2}}{\sqrt{(x-5)^2+(y-2)^2}}=\frac{1}{k}$	1M+1A	
$(k^2-1)x^2+(k^2-1)y^2+(10-4k^2)x+(4+2k^2)y+(5k^2-29)=0$	1	
(i) When k = 2,		
$3x^2+3y^2-6x+12y-9=0$		
$x^2+y^2-2x+4y-3=0$	1A	
a circle (with centre at (1, -2) and radius $2\sqrt{2}$).	1A	
(ii) The locus represents a straight line.		
$k^2-1=0$	1M	
$k=1$	1A	
	7	

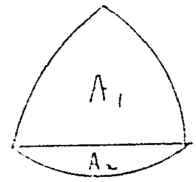
SOLUTIONS	MARKS	REMARKS
11.(a)(i) Putting $x = 2\sin\theta$, $dx = 2\cos\theta\, d\theta$	1A	
When $x = 1$, $\theta = \frac{\pi}{6}$) When $x = 2$, $\theta = \frac{\pi}{2}$)	1A	
$\int_1^2 \sqrt{4 - x^2}\, dx$		
$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 4\cos^2\theta\, d\theta$	1A	For integrand
$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 4 \cdot \frac{1}{2}(1 + \cos 2\theta)\, d\theta$	1M	For $\cos^2\theta = \frac{1 + \cos 2\theta}{2}$
$= 2[\theta + \frac{1}{2}\sin 2\theta]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$	1A	
$= 2[\frac{\pi}{3} - \frac{\sqrt{3}}{4}]$ or 1.23	1A	(1.228)
(ii) $3 + 2x - x^2$		
$= 2^2 - (x - 1)^2$	1A	
$\int_0^1 \sqrt{3 + 2x - x^2}\, dx$		
$= \int_0^1 \sqrt{4 - (x - 1)^2}\, dx$		
Putting $x - 1 = 2\sin\theta$, $dx = 2\cos\theta\, d\theta$	1A	
When $x = 0$, $\theta = -\frac{\pi}{6}$) When $x = 1$, $\theta = 0$)	1A	
$= \int_{-\frac{\pi}{6}}^{0} 4\cos^2\theta\, d\theta$	1A	
$= 2[\theta + \frac{1}{2}\sin 2\theta]_{-\frac{\pi}{6}}^{0}$		
$= \frac{\pi}{3} + \frac{\sqrt{3}}{2}$ or 1.91	1A 11	(1.913)

SOLUTIONS	MARKS	REMARKS
11. (b)(i) $y = -\sqrt{4-(x-1)^2} + \sqrt{3}$	1A	
(ii) Required area $= A_1 + A_2$	1M	



SOLUTIONS	MARKS	REMARKS
$A_1 = \int_0^1 [\sqrt{3x} - (-\sqrt{4-(x-1)^2} + \sqrt{3})]\,dx$	1M+1A	
$= [\frac{\sqrt{3}\cdot 2x^{3/2}}{3}]_0^1 + (\frac{\pi}{3} + \frac{\sqrt{3}}{2}) - \sqrt{3}$		
$= (\frac{\sqrt{3}}{6} + \frac{\pi}{3})$ or 1.336 .... (Accept 1.34)	1A	
$A_2 = \int_1^2 [\sqrt{4-x^2} - (-\sqrt{4-(x-1)^2} + \sqrt{3})]\,dx$	1M+1A	
$= 2[\frac{\pi}{3} - \frac{\sqrt{3}}{4}] + \int_1^2 \sqrt{4-(x-1)^2}\,dx - \sqrt{3}$		
$= \frac{2\pi}{3} - \frac{3\sqrt{3}}{2} + \int_0^1 \sqrt{4-(x-1)^2}\,dx$		
$= \frac{2\pi}{3} - \frac{3\sqrt{3}}{2} + (\frac{\pi}{3} + \frac{\sqrt{3}}{2})$		
$= (\pi - \sqrt{3})$ or 1.410	1A	Accept 1.41
Area $= (\frac{4\pi}{3} - \frac{5\sqrt{3}}{6})$ or 2.75	1A	
	9	

Alt. Solution (2):



Area $= A_1 + A_2$ 1M

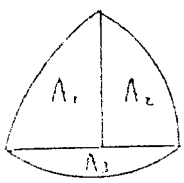
$A_1 = \int_0^{\sqrt{3}} (\sqrt{4-y^2} - \frac{y^2}{3})\,dy$ 1M+1A

$= \frac{2\pi}{3} + \frac{\sqrt{3}}{6}$ 1A

(or 2.383)

A_2 same as A_3 in Alt. solution (1).

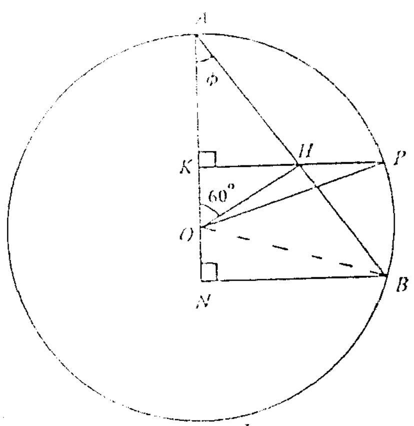
Alt. Solution (1)



SOLUTIONS	MARKS	REMARKS
Area $= A_1 + A_2 + A_3$	1M	
$A_1 = \int_0^1 \sqrt{3x}\,dx$	1A	
$= \frac{2\sqrt{3}}{3}$ or 1.155	1A	Accept 1.15
$A_2 = \int_1^2 \sqrt{4-x^2}\,dx$	1A	
$= (\frac{2\pi}{3} - \frac{\sqrt{3}}{2})$ or 1.228		Accept 1.23
$A_3 = \left\| \int_0^2 (-\sqrt{4-(x-1)^2} + \sqrt{3})\,dx \right\|$		
or $2\left\| \int_0^1 (-\sqrt{4-(x-1)^2} + \sqrt{3})\,dx \right\|$	1A	Accept no absolute sign
$= \left\| -2(\frac{\pi}{3} + \frac{\sqrt{3}}{2}) + 2\sqrt{3} \right\|$		
$= (\frac{2}{3}\pi - \sqrt{3})$ or 0.3623	1A	Accept 0.362
$A = (\frac{4\pi}{3} - \frac{5\sqrt{3}}{6})$ or 2.75	1A	

SOLUTIONS	MARKS	REMARKS
12.(a)(i) $\sin 108^\circ = \sin(3 \times 36^\circ)$		$\sin 108^\circ$
$= 3\sin 36^\circ - 4\sin^3 36^\circ$	1A	$= \sin 72^\circ$
$\sin 72^\circ = 2\sin 36^\circ \cos 36^\circ$	1A	$= 2\sin 36^\circ \cos 36^\circ$ 1A
$3\sin 36^\circ - 4\sin^3 36^\circ = 2\sin 36^\circ \cos 36^\circ$		$= 2\sin 36^\circ \sqrt{1-\sin^2 36^\circ}$ 1A
$3 - 4\sin^2 36^\circ = 2\cos 36^\circ$		
$3 - 4(1 - \cos^2 36^\circ) = 2\cos 36^\circ$		
$4\cos^2 36^\circ - 2\cos 36^\circ - 1 = 0$	1A	
$\cos 36^\circ = \frac{1 \pm \sqrt{5}}{4}$	1A	
$\cos 36^\circ > 0$		
$\therefore \cos 36^\circ = \frac{1+\sqrt{5}}{4}$	1	
(ii) $\cos 72^\circ = 2\cos^2 36^\circ - 1$	1A	
$= 2\left(\frac{1+\sqrt{5}}{4}\right)^2 - 1$		
$= \frac{\sqrt{5}-1}{4}$	1A	
	7	

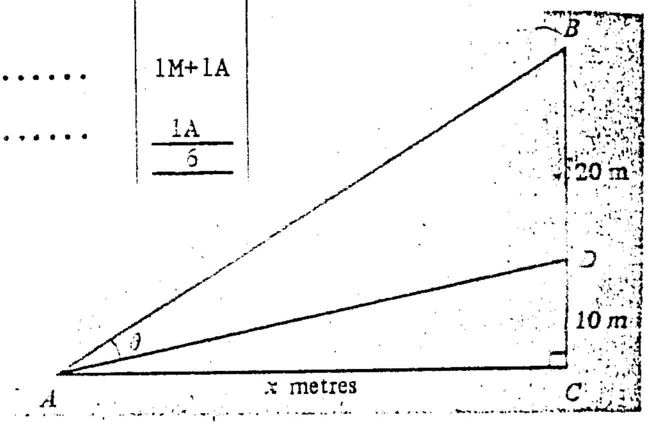
(b)(i)

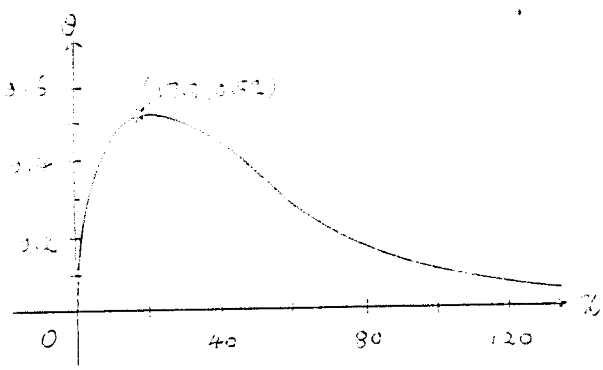


SOLUTIONS	MARKS	REMARKS
In $\triangle AOH$,		
$\frac{OH}{\sin \emptyset} = \frac{1}{\sin(180^\circ - 60^\circ - \emptyset)}$	1M	
$OH = \frac{\sin \emptyset}{\sin(60^\circ + \emptyset)}$ or $\frac{\sin \emptyset}{\sin(120^\circ - \emptyset)}$	1A	
$= \frac{\sin \emptyset}{\sin 60^\circ \cos \emptyset + \cos 60^\circ \sin \emptyset}$		
or $\frac{\sin \emptyset}{\sin 120^\circ \cos \emptyset - \cos 120^\circ \sin \emptyset}$		
$= \frac{\tan \emptyset}{\frac{\sqrt{3}}{2} + \frac{1}{2}\tan \emptyset}$	2A	
$= \frac{2\tan \emptyset}{\sqrt{3} + \tan \emptyset}$		

SOLUTIONS	MARKS	REMARKS
12.(b)(i) $\cos\angle POK$		
$= \frac{OK}{OP}$	1A	
$= OK$		
$= OH\cos 60°$	1M	
$= \frac{2\tan\phi}{\sqrt{3}+\tan\phi}\cdot\frac{1}{2}$		
$= \frac{\tan\phi}{\sqrt{3}+\tan\phi}$	1A	
(ii)(1) $ON = \frac{1}{4}$		
$BN = \sqrt{OB^2 - ON^2}$	1	
$= \frac{\sqrt{15}}{4}$	1A	
$\tan\phi = \frac{BN}{AN}$		
$= \frac{\frac{\sqrt{15}}{4}}{\frac{5}{4}}$		
$= \frac{\sqrt{15}}{5}$	1A	
(2) $\cos\angle POK = \frac{\frac{\sqrt{15}}{5}}{\sqrt{3}+\frac{\sqrt{15}}{5}}$	1M	For substitution
$= \frac{(\sqrt{5})(\sqrt{3})}{5\sqrt{3}+(\sqrt{5})(\sqrt{3})}$		
$= \frac{\sqrt{5}}{5+\sqrt{5}}$		
$= \frac{1}{1+\sqrt{5}}$		
$= \frac{\sqrt{5}-1}{4}$	1A	
Compared with (a)(ii)		
$\angle POK = 72°$	1A	Do not award this mark if a candidate had not completed (a)(ii).
	13	

SOLUTIONS	MARKS	REMARKS
11. (c) If $x = 50$, $\frac{d\theta}{dx} = \frac{20(300 - 50^2)}{50^4 + 1000(50)^2 + 90\ 000}$	1M	
$= \frac{-44\ 000}{8\ 840\ 000}$		
$= -0.0050$ (correct to 4 d.p.)	1A	Follow through for -0.005
$1° = 0.0175$ radians		
Since $\Delta x \doteqdot \Delta\theta \frac{1}{\frac{d\theta}{dx}}$ (or $\Delta\theta \doteqdot \frac{d\theta}{dx}\Delta x$), ...	1M	
at $x = 50$,		
$\Delta x \doteqdot \frac{-0.0175}{-0.005}$	1M+1A	
$= 3.5$ (correct to the nearest $\frac{1}{10}$ m)	1A	
	6	



SOLUTIONS	MARKS	REMARKS
(d) At $x = 0$, $\theta = 0$.	1A	May be indicated in diag.
At $x = \sqrt{300}$,		
$\tan\theta = 0.577$		
$\theta = 0.524$ (or 30°)	1A	
As $x \to \infty$, $\theta \to 0$	1A	
	2	1 shape, 1 tail
	5	