HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1986

ADDITIONAL MATHEMATICS PAPER II

Time allowed: Two Hours

SECTION A (39 marks)
Answer All questions in this section.

1. Prove, by mathematical induction, that for any positive integer n,

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}.$$

(5 marks)

2. In the expansion of $(x^2+2)^n$ in descending powers of x, where n is a positive integer, the coefficient of the third term is 40. Find the value of n and the coefficient of x^4.

(5 marks)

3. If θ is an obtuse angle and the equation in x

$$3x^2-(4\cos\theta)x+2\sin\theta=0$$

has equal roots, find the value of θ.

(5 marks)

4. Using the identity $\sin A+\sin B=2\sin\frac{A+B}{2}\cos\frac{A-B}{2}$, find the general solution of $\sin 2\theta+\sin 4\theta=\cos\theta$.

(6 marks)

5. $A(3,6), B(-1,-2)$ and $C(5,-3)$ are three points. $P(s,t)$ is a point on the line AB.

(a) Find t in terms of s.

(b) If the area of $\triangle APC$ is $\frac{13}{2}$, find the two values of s.

(6 marks)

6. A straight line through $C(3,2)$ with slope m cuts the curve $y=(x-2)^2$ at the points A and B. If C is the mid-point of AB, find the value of m.

(6 marks)

7. Let $y=\frac{\tan^3\theta}{3}-\tan\theta$.
Find $\frac{\mathrm{d}y}{\mathrm{d}\theta}$ in terms of $\tan\theta$.
Hence, or otherwise, find $\int\tan^4\theta\,\mathrm{d}\theta$.

(6 marks)

SECTION B (60 marks)
Answer any THREE questions from this section. Each question carries 20 marks.

8. (a) Show that $\int_0^a f(x)\,\mathrm{d}x=\int_0^a f(a-x)\,\mathrm{d}x$.

(4 marks)

(b) Using the result in (a), or otherwise, evaluate the following integrals:

(i) $\int_0^{\pi}\cos^{2n+1}x\,\mathrm{d}x$, where n is a positive integer,

(ii) $\int_0^{\pi}x\sin^2x\,\mathrm{d}x$,

(iii) $\int_0^{\frac{\pi}{2}}\frac{\sin x\,\mathrm{d}x}{\sin x+\cos x}$.

(16 marks)

9. A family of straight lines is given by the equation

$$(3k+2)x-(2k-1)y+(k-11)=0,$$

where k is any constant.

(a) L_1 is the line $x-2y+4=0$.

(i) There are two lines in the family each making an angle of 45° with L_1. Find the equations of these lines.

(ii) Find the equation of the line L in the family which is parallel to L_1.
The line L_1 and another line L_2 are equidistant from L. Find the equation of L_2.

(11 marks)

(b) For what value of k does the line in the family form a triangle of minimum area with the two positive coordinate axes?

(7 marks)

(c) The straight lines in the family pass through a fixed point Q. Write down the equation of the line which passes through Q but which does not belong to the family.

(2 marks)

10. The circles $C_1 : x^2 + y^2 - 4x + 2y + 1 = 0$ and $C_2 : x^2 + y^2 - 10x - 4y + 19 = 0$ have a common chord AB.

(a) (i) Find the equation of the line AB.

(ii) Find the equation of the circle with AB as a chord such that the area of the circle is a minimum.

(9 marks)

(b) The circle C_1 and another circle C_3 are concentric. If AB is a tangent to C_3, find the equation of C_3.

(4 marks)

(c) $P(x, y)$ is a variable point such that

$$\frac{\text{distance from } P \text{ to the center of } C_1}{\text{distance from } P \text{ to the center of } C_2} = \frac{1}{k} \qquad (k > 0).$$

Find the equation of the locus of P.

(i) When $k = 2$, write down the equation of the locus of P and name the locus.

(ii) For what value of k is the locus of P a straight line?

(7 marks)

11. (a) (i) Using the substitution $x = 2\sin\theta$, evaluate

$$\int_1^2 \sqrt{4 - x^2}\,\mathrm{d}x.$$

(ii) Express $3 + 2x - x^2$ in the form $a^2 - (x - b)^2$ where a and b are constants.
Using the substitution $x - b = a\sin\theta$, evaluate

$$\int_0^1 \sqrt{3 + 2x - x^2}\,\mathrm{d}x.$$

(11 marks)

(b) In Figure 1, the shaded region is bounded by the two circles $C_1 : x^2 + y^2 = 4$, $C_2 : (x - 1)^2 + (y - \sqrt{3})^2 = 4$ and the parabola $S : y^2 = 3x$.

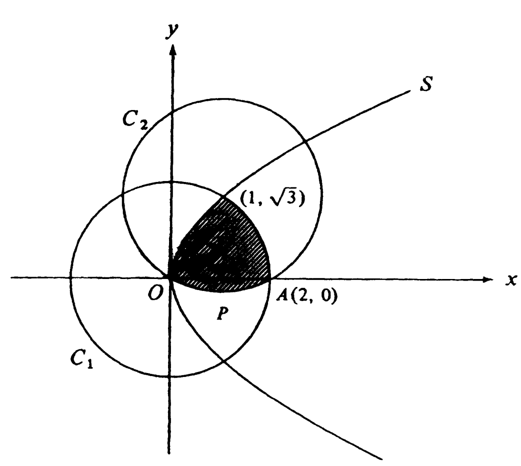

Figure 1

(i) $P(x, y)$ is a point on the minor arc OA of C_2. Express y in terms of x.

(ii) Find the area of the shaded region.

(9 marks)

12. (a) (i) Express $\sin 108°$ in terms of $\sin 36°$.

Using this result and the relation $\sin 108° = \sin 72°$, show that $\cos 36° = \dfrac{1+\sqrt{5}}{4}$.

(ii) Find $\cos 72°$ in surd form.

(7 marks)

(b) In Figure 2, O is the center of the circle APB of radius 1 unit. $AKON, AHB$ and KHP are straight lines. PK and BN are both perpendicular to AN. $\angle AOH = 60°$. Let $\angle OAH = \phi$.

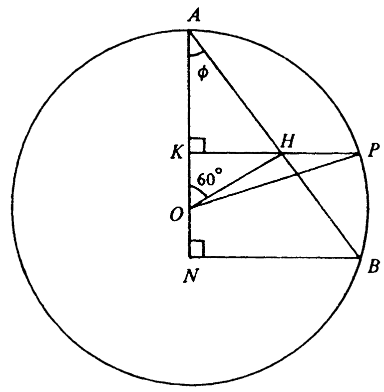

Figure 2

(i) By considering $\triangle AOH$, express OH in terms of $\tan\phi$.

Hence show that $\cos \angle POK = \dfrac{\tan\phi}{\sqrt{3}+\tan\phi}$.

(ii) It is given that $ON = \frac{1}{4}$.

(1) Find BN and hence the value of $\tan\phi$. Give your answers in surd form.

(2) Find the value of $\cos \angle POK$ in surd form.
Hence find $\angle POK$ without using calculators.

(13 marks)

END OF PAPER