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香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八六年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1986

附加數學（試卷一）
ADDITIONAL MATHEMATICS I

評卷參考
MARKING SCHEME

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本評卷參考並非標準答案，故極不宜落於學生手中，以免引起誤會。

遇有學生求取此文件時，閱卷員應斷予拒絕。閱卷員在任何情況下披露本評卷參考內容，均有違閱卷員守則及「一九七七年香港考試局法例」。

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It is highly undesirable that this marking scheme should fall into the hands of students. They are likely to regard it as a set of model answers, which it certainly is not.

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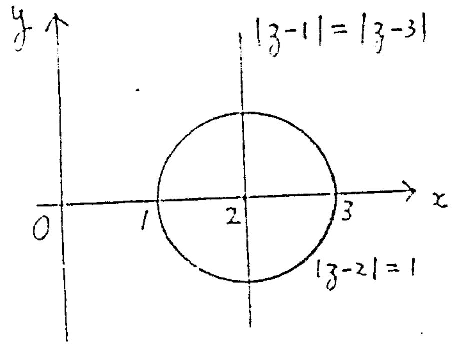
SOLUTIONS	MARKS	REMARKS
1. $\frac{d}{dx}(x^3) = \lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 - x^3}{\Delta x}$	1	
$= \lim_{\Delta x \to 0} \frac{x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3}{\Delta x}$	1A	For expanding $(x+\Delta x)^3$.
$= \lim_{\Delta x \to 0} [3x^2 + 3x(\Delta x) + (\Delta x)^2]$	1A	
$= 3x^2$	1A	
	4	
2. The discriminant $= (\log b)^2 - 4(\log a)(\log b)$	1A	Alt. Solution:
For equal roots, $(\log b)^2 - 4(\log a)(\log b) = 0$	1M	Differentiating, $2x\log a + \log b = 0$ 1M
The roots are non-zero, $\log b \neq 0$.	1	Solving with given eqt.
(Accept rejecting $\log b = 0$)		$(x+2)\log b = 0$ 1A; $x = -2$ 1A
$\therefore \log b = 4 \log a$	1A	$-4\log a + \log b = 0$ 1A
$= \log a^4$		$b = a^4$ 1A
$b = a^4$	1A	
	5	
3. $f'(x) = 18 - 2kx$		Alt. Solution:
$= 0$	1M	$4k + 18x - kx^2 = 45$ 1
$x = \frac{9}{k}$	1A	$kx^2 - 18x + 45 - 4k = 0$
Alt. Solution:		For equal roots,
$f(x)$ is quadratic, its maximum occurs when $x = \frac{-18}{2(-k)}$	1A	$(-18)^2 - 4k(45-4k) = 0$ 1M+1A
$= \frac{9}{k}$	1A	$4k^2 - 45k + 81 = 0$ 1A
		$k = 9$ or $\frac{9}{4}$ 1A
Alt. Solution:		
$f(x) = -kx^2 + 18x + 4k$		
$= -k[\ (x - \frac{9}{k})^2 - 4 - \frac{81}{k^2}\ ]$	1M+1A	
$f(\frac{9}{k}) = 45$ or $4k + \frac{81}{k} = 45$	1M	
$4k^2 - 45k + 81 = 0$	1A	
$(k-9)(4k-9) = 0$		
$k = 9$ or $\frac{9}{4}$	1A	For both answers
	5	

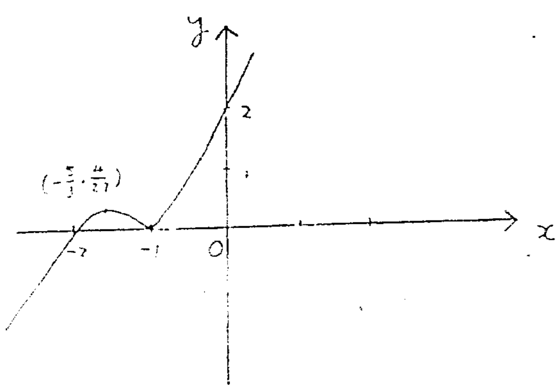
SOLUTIONS	MARKS	REMARKS
4. Differentiating with respect to x	1M	
$2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$	2A	
Substituting (2, 1),		
$4 + 2\frac{dy}{dx} + 1 + 2\frac{dy}{dx} = 0$		
$\therefore \frac{dy}{dx} = -\frac{5}{4}$	1A	
Equation of tangent: $\frac{y-1}{x-2} = -\frac{5}{4}$	1M	For point-slope form
$5x + 4y - 14 = 0$	1A	
	6	
5. $(\underline{i}+\underline{j}) \cdot [(c+4)\underline{i} + (c-4)\underline{j}] = \lvert \underline{i}+\underline{j} \rvert \lvert (c+4)\underline{i} + (c-4)\underline{j} \rvert \cos\theta$	1M	Dot must be shown
$(c+4) + (c-4) = \sqrt{2}\sqrt{2}\sqrt{c^2+16}\,(-\frac{3}{5})$	1A+1A	1A for L.S. 1A for R.S.
$c = -\frac{3}{5}\sqrt{c^2+16}$		
$c^2 = 9$	1A	
$c = \pm 3$		
After checking,	1M	
$c = -3$	1A	
	6	
Alt. Solution:		
$\tan\alpha = 1$, $\tan\beta = \frac{c-4}{c+4}$	1A	
$\tan(\alpha - \beta) = \dfrac{1 - \frac{c-4}{c+4}}{1 + \frac{c-4}{c+4}}$	1M	
$\tan\theta = \pm\frac{4}{c}$		
$\cos\theta = -\frac{3}{5}$		
$\tan\theta = -\frac{4}{3}$	1A	
$\therefore \frac{4}{c} = \pm\frac{4}{3}$		
$c = \pm 3$	1A	
After checking,	1M	
$c = -3$	1A	

SOLUTIONS	MARKS	REMARKS
6. 		
Circle	1	
Radius & centre	1A	
Line ⊥ x-axis	1A	
Line passes through (2,0)	1A	
$2 + i$ and $2 - i$	1A+1A	
	6	
		Alt.Sol. for last part:
		$\lvert z - 2\rvert = 1$
		$(x - 2)^2 + y^2 = 1$
		$\lvert z - 1\rvert = \lvert z - 3\rvert$
		$x = 2$
		$y = \pm 1$
		$2+i$ and $2-i$ 1A+1A
7. (a) $x > 0$,		
$x > \frac{3}{x} + 2$		
$x^2 > 3 + 2x$	1A	
$x^2 - 2x - 3 > 0$		
$(x - 3)(x + 1) > 0$		
$x > 3$ or $x < -1$	1M	
but $x > 0$	1A	
$\therefore$ $x > 3$		
(b) $x < 0$,		
$x > \frac{3}{x} + 2$		
$x^2 < 3 + 2x$	2A	
$x^2 - 2x - 3 < 0$		
$(x - 3)(x + 1) < 0$		
$3 > x > -1$	1M	
but $x < 0$	1A	
$\therefore$ $-1 < x < 0$	7	

SOLUTIONS	MARKS	REMARKS
3.(a) $\overrightarrow{OC} = \underline{a} + 2\underline{b}$	1A	If vector sign omitted, or division of vectors, pp-1.
$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$	1A	
$= \underline{a} + 2\underline{b} - \underline{b}$		
$= \underline{a} + \underline{b}$	1A	
$\overrightarrow{OQ} = \overrightarrow{OB} + \overrightarrow{BQ} = \overrightarrow{OB} + \frac{1}{3}\overrightarrow{BC}$		
$= \underline{b} + \frac{1}{3}(\underline{a} + \underline{b})$	1M	
$= \frac{1}{3}\underline{a} + \frac{4}{3}\underline{b}$	1A	
	5	
(b) $\overrightarrow{OR} = h\overrightarrow{OQ} + (1-h)\overrightarrow{OP}$	1	Alt. Solution: $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$
$= h(\frac{1}{3}\underline{a} + \frac{4}{3}\underline{b}) + (1-h)\frac{1}{2}\underline{a}$	1M	$= \overrightarrow{OP} + h\overrightarrow{PQ}$
$= (\frac{1}{2} - \frac{h}{6})\underline{a} + \frac{4h}{3}\underline{b}$	1A	$= \overrightarrow{OP} + h(\overrightarrow{OQ} - \overrightarrow{OP})$ 1
		$= \frac{1}{2}\underline{a} + h[(\frac{1}{3}\underline{a} + \frac{4}{3}\underline{b}) - \frac{1}{2}\underline{a}]$ 1M
		$= (\frac{1}{2} - \frac{h}{6})\underline{a} + \frac{4h}{3}\underline{b}$ 1A
		$\overrightarrow{OC} = \underline{a} + 2\underline{b}$
$\overrightarrow{OR} = k\overrightarrow{OC}$		
$= k\underline{a} + 2k\underline{b}$	1A	OR // OC
$\left.\begin{array}{l}\frac{1}{2} - \frac{h}{6} = k \\ \frac{4h}{3} = 2k\end{array}\right\}$	2M+1A	$\frac{4h/3}{2} = \frac{\frac{1}{2} - h/6}{1}$ 2M+1A
		$h = 3/5$ 1A
Solving, $h = \frac{3}{5}$	1A	$\overrightarrow{OR} = 2/5\,\underline{a} + 4/5\,\underline{b}$
		$= 2/5(\underline{a} + 2\underline{b})$ 1A
$k = \frac{2}{5}$	1A	$= 2/5\,\overrightarrow{OC}$
	9	$\therefore k = 2/5$ 1A
(c) $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$		
$= \frac{4}{3}\underline{b} - \frac{1}{6}\underline{a}$	1A	
$\overrightarrow{PT} = \overrightarrow{OT} - \overrightarrow{OP}$		
$= \lambda\underline{b} - \frac{1}{2}\underline{a}$	1A	
PQ // PT		Alt. Solution:
$\frac{\frac{4}{3}}{\lambda} = \frac{\frac{1}{6}}{\frac{1}{2}}$	2M+1A	Let $\overrightarrow{PT} = \mu\overrightarrow{PQ}$
$\lambda = 4$	1A	$= \frac{4}{3}\mu\underline{b} - \frac{\mu}{6}\underline{a}$
	5	$\left.\begin{array}{l}\frac{1}{2} = \frac{\mu}{6} \\ \lambda = \frac{4}{3}\mu\end{array}\right\}$ 2M+1A
		Solving $\mu = 3$, $\lambda = 4$ 1A

SOLUTIONS	MARKS	REMARKS
(a) $\cos x = \frac{1}{\sqrt{2}}$		If mixed units, pp-1.
$x = 2n\pi \pm \frac{\pi}{4}$ or $(n)(360°) \pm 45°$	2A	
(n is an integer)	1A	
	3	
(b)(i) $z = r(\cos\theta + i\sin\theta)$		
$z^m = r^m(\cos m\theta + i\sin m\theta)$	1A	
$\bar{z} = r[\cos(-\theta) + i\sin(-\theta)]$	1A	
$(\bar{z})^m = r^m[\cos(-m\theta) + i\sin(-m\theta)]$		
$= r^m(\cos m\theta - i\sin m\theta)$	1A	
$z^m + (\bar{z})^m = 2r^m\cos m\theta$	1	
(ii) $z = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$		
$= (\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$	1A	
$(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i)^m + (\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i)^m = \sqrt{2}$		
$2\cos\frac{m\pi}{4} = \sqrt{2}$	1M	
$\cos\frac{m\pi}{4} = \frac{1}{\sqrt{2}}$		
$\frac{m\pi}{4} = 2n\pi \pm \frac{\pi}{4}$	1M	
$m = 8n \pm 1$	1A	
$m = 1$ or $m = 8n\pm1$ where n is a positive integer.	1A	
	9	
(c) $(1+i)^p - (1-i)^p = 0$		Alt. Solution:
$(\frac{1+i}{1-i})^p = 1$	1A	$(\frac{1-i}{1+i})^p = 1$ 1A
$[\frac{(1+i)^2}{2}]^p = 1$	1A	$[\frac{(1-i)^2}{2}]^p = 1$ 1A
$i^p = 1$	1A	$(-i)^p = 1$ 1A
$p = 4n$, n is a +ve integer	1A	$p = 4n$,
(Accept $p = 4, 8, 12, \ldots$)		n is a +ve integer 1A

SOLUTIONS	MARKS	REMARKS
9.(c)(i)		
Alt. Solution:		
$(1+i)^p - (1-i)^p = 0$		
$1 + i = \sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$	1A	
$(1+i)^p - (1-i)^p$		
$= 2(\sqrt{2})^p\, i\sin\frac{p\pi}{4}$	1A	
$= 0$		
$\sin\frac{p\pi}{4} = 0$	1A	
$\frac{p\pi}{4} = n\pi$		
$p = 4n$, n is a +ve integer	1A	
(ii) $\frac{(1+i)^{4k+1}}{(1-i)^{4k-1}} = \left(\frac{1+i}{1-i}\right)^{4k}(1+i)(1-i)$	1M+1A	
$= 1 - i^2$	1A	
$= 2$	1A	
	3	
Alt. Solution		
$\frac{(1+i)^{4k+1}}{(1-i)^{4k-1}} = \frac{(1+i)^{8k}}{2^{4k-1}}$	1M+1A	
$= \frac{(\sqrt{2})^{8k}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})^{8k}}{2^{4k-1}}$		
$= \frac{2^{4k}}{2^{4k-1}}(\cos 2k\pi + i\sin 2k\pi)$	1A	
$= 2$	1A	

SOLUTIONS	MARKS	REMARKS
10. (a) $f(x) = x^3 + hx^2 + kx + 2$		
$f'(x) = 3x^2 + 2hx + k$	1A	
$= 0$	1M	
For 2 distinct turning points,		
$(2h)^2 - 4(3)(k) > 0$	1M	
$h^2 > 3k$	1	
Put $y = 2$ in $y = x^3 + hx^2 + kx + 2$	1	
$x^3 + hx^2 + kx = 0$		
$x(x^2 + hx + k) = 0$	1A	
$x^2 + hx + k = 0$ has no real roots	1M	
$\therefore h^2 < 4k$	1	
	8	
(b)(i) Sub. $(-2, 0)$ in $f(x) = x^3 + hx^2 + kx + 2$	1	
$-8 + 4h - 2k + 2 = 0$		
$k = 2h - 3$	1A	
(ii) $4k > h^2 > 3k$		
$8h - 12 > h^2 > 6h - 9$	1M	
$h^2 - 8h + 12 < 0$ and $h^2 - 6h + 9 > 0$	1A+1A	1A For 2 ineq. 1A For 'and' (Accept omitting 'and'. Do not accept 'or'.)
$(h - 2)(h - 6) < 0$ and $(h - 3)^2 > 0$		
$6 > h > 2$ and $h \neq 3$	1A+1A	
h is an integer		
$\therefore h = 4$ or 5		
(iii) For $h = 4$,		
$f'(x) = 3x^2 + 8x + 5 = 0$		
$(3x + 5)(x + 1) = 0$		
$x = -\frac{5}{3}$ or -1	1A	
$f''(x) = 6x + 8$		
$x = -\frac{5}{3}$, $f''(x) < 0$, $\therefore (-\frac{5}{3}, \frac{4}{27})$ is a maximum point	1A	(1.67, 0.148)
$x = -1$, $f''(x) > 0$, $\therefore (-1, 0)$ is a minimum point	1A	
	1A	For shape
	1A	For 3 points out of the 4 points $(-2, 0)$, $(-5/3, 4/27)$, $(-1, 0)$, $(0, 2)$
	12	

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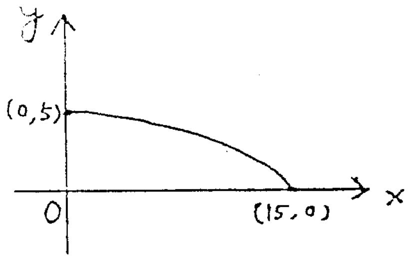
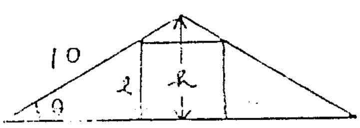
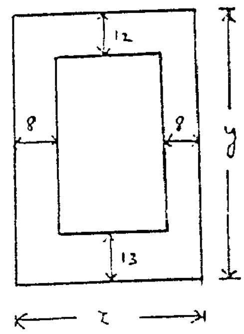
SOLUTIONS	MARKS	REMARKS
11.(a) $s = 20\cos\theta$	1A	
$\frac{ds}{dt} = \frac{ds}{d\theta}\cdot\frac{d\theta}{dt}$	1	
$\frac{ds}{d\theta} = -20\sin\theta$	1A	
$\frac{ds}{dt} = 10$		
$\therefore\ \frac{d\theta}{dt} = \frac{10}{-20\sin\theta}$		
$= \frac{-1}{2\sin\theta}$	1A	
When $s = 10$, $\theta = \frac{\pi}{3}$		
$\frac{d\theta}{dt} = -\frac{1}{\sqrt{3}}\ (s^{-1})$ (or $-0.577\ s^{-1}$)	1A / 5	Unit optional
(b) $x = 15\cos\theta$	1A	
$y = 5\sin\theta$	1A	
$\frac{x^2}{15^2} + \frac{y^2}{5^2} = 1$	1A	
$(x, y \geq 0)$		
	1A	Shape
	1A / 5	Labelling the two end-points
(c) 		
$h = 10\sin\theta$	1A	
$\frac{h-\ell}{h} = \frac{\ell}{20\cos\theta}$	1M+1A	1M For similar Δ s.
$1 - \frac{\ell}{h} = \frac{\ell}{20\cos\theta}$		Alt. Solution: Equating lengths 1M; Correct equation 2A; Final answer 1
$\ell\left(\frac{1}{h} + \frac{1}{20\cos\theta}\right) = 1$		
$\ell = \frac{1}{\left(\frac{1}{10\sin\theta} + \frac{1}{20\cos\theta}\right)}$		
$= \frac{20\sin\theta\cos\theta}{\sin\theta + 2\cos\theta}$	1	

SOLUTIONS	MARKS	REMARKS
11.(c) A = Area of square = ℓ^2		
$\frac{dA}{d\theta} = 2\ell\frac{d\ell}{d\theta}$	1M	
$= 2\ell\frac{(\sin\theta + 2\cos\theta)20(-\sin^2\theta+\cos^2\theta) - 20\sin\theta\cos\theta(\cos\theta-2\sin\theta)}{(\sin\theta + 2\cos\theta)^2}$		
$= 0$	1M	
$-\sin^3\theta-2\cos\theta\sin^2\theta+\sin\theta\cos^2\theta+2\cos^3\theta-\sin\theta\cos^2\theta+2\sin^2\theta\cos\theta = 0$		
Alt. Solution (1):		
$\frac{dA}{d\theta} = 2\ell\frac{d\ell}{d\theta}$	1M	
$= 2\ell\cdot 20\frac{d}{d\theta}\left[\frac{1}{\frac{1}{\cos\theta}+\frac{2}{\sin\theta}}\right]$		
$= 40\ell\frac{-1}{\left(\frac{1}{\cos\theta}+\frac{2}{\sin\theta}\right)^2}\left[\frac{\sin\theta}{\cos^2\theta}-\frac{2\cos\theta}{\sin^2\theta}\right]$		
$= 0$	1M	
Alt. Solution (2):		
$A = \ell^2$		
$= \frac{400\sin^2\theta\cos^2\theta}{(\sin\theta + 2\cos\theta)^2}$		
$\frac{dA}{d\theta} = 400\cdot\frac{(\sin\theta+2\cos\theta)^2(2\sin\theta\cos^3\theta-2\cos\theta\sin^3\theta)-\sin^2\theta\cos^2\theta\cdot 2(\sin\theta+2\cos\theta)(\cos\theta-2\sin\theta)}{(\sin\theta + 2\cos\theta)^4}$	1M	For quotient rule
$= 0$	1M	
$(\sin\theta + 2\cos\theta)(\cos^2\theta - \sin^2\theta) - \sin\theta\cos\theta(\cos\theta - 2\sin\theta) = 0$		
$-\sin^3\theta + 2\cos^3\theta = 0$	2A	
$\tan^3\theta = 2$	1A	
$\theta = 51.6°$	1A	51.561°
	10	

SOLUTIONS	MARKS	REMARKS
12.(a) $A = (x-16)(y-25)$	1A	
$= (x-16)(\frac{3600}{x} - 25)$	1A / 2	
$= 4000 - 25x - \frac{16(3600)}{x}$		
Alt. Solution:		
$A = 3600 - 2(8y) - 12(x-16) - 13(x-16)$	1A	
$= 4000 - 25x - \frac{16(3600)}{x}$	1A	
(b) $\frac{dA}{dx} = -25 + \frac{16(3600)}{x^2}$	1A	
$= 0$	1M	
$x^2 = \frac{16(3600)}{25}$		
$x = \pm 48$		
Rejecting $x = -48$,		
$x = 48$	1A	
Maximum $A = 1600$		
Testing for maximum		
$\frac{d^2A}{dx^2} = -\frac{2(16)(3600)}{x^3}$	1A	
< 0		
Explaining why A is largest when $x = 48$	1M / 5	
c)(i) A decreases as x increases		
$\frac{dA}{dx} < 0$		
$-25 + \frac{16(3600)}{x^2} < 0$	1M	
$x > 48$ or $x < -48$ (rejected)		
$\therefore (144 >) x > 48$	1A	Accept $x \geq 48$
(ii) If $x \geq 50$,		
$\because$ A is decreasing	2	
$\therefore$ Largest value of A occurs when $x = 50$	1A	
Largest value of $A = 4000 - (25)(50) - \frac{16(3600)}{50}$		
$= 1598$	1A / 6	

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SOLUTIONS	MARKS	REMARKS
12.(d) $\frac{4}{9} \leq \frac{x}{y} \leq \frac{9}{16}$		
$\frac{4}{9} \leq \frac{x}{\frac{3600}{x}} \leq \frac{9}{16}$	1A	
$\frac{4}{9} \leq \frac{x^2}{3600} \leq \frac{9}{16}$		
$1600 \leq x^2 \leq 2025$	1A	
$40 \leq x \leq 45$	1A	
For $x < 48$, $\frac{dA}{dx} > 0$		
A is increasing	2	
Largest value of A occurs when $x = 45$	1A	
Largest value of $A = 4000 - (25)(45) - \frac{16(3600)}{45}$		
$= 1595$	1A	
	7	