HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1986

ADDITIONAL MATHEMATICS PAPER I

Time allowed: Two Hours

SECTION A (39 marks)
Answer All questions in this section.

1. Find, from first principles, $\frac{\mathrm{d}}{\mathrm{d}x}(x^3)$.

(4 marks)

2. The quadratic equation

$$x^2 \log a + (x+1)\log b = 0,$$

where a and b are constants, has non-zero equal roots.
Find b in terms of a.

(5 marks)

3. The maximum value of the function $f(x) = 4k + 18x - kx^2$ (k is a positive constant) is 45. Find k.

(5 marks)

4. Find the equation of the tangent to the curve $x^2 + xy + y^2 = 7$ at the point $(2, 1)$.

(6 marks)

5. The angle between the two vectors $\mathbf{i} + \mathbf{j}$ and $(c+4)\mathbf{i} + (c-4)\mathbf{j}$ is θ, where $\cos\theta = -\frac{3}{5}$. Find the value of the constant c.

(6 marks)

6. On the same Argand diagram, sketch the locus of the point representing the complex number z in each of the following cases:

(a) $|z-2| = 1$;

(b) $|z-1| = |z-3|$.

Hence, or otherwise, find the complex numbers represented by the points of intersection of the two loci.

(6 marks)

7. Solve $x > \frac{3}{x} + 2$ for each of the following cases:

(a) $x > 0$;

(b) $x < 0$.

(7 marks)

SECTION B (60 marks)
Answer any THREE questions from this section. Each question carries 20 marks.

8. In Figure 1, $OACB$ is a trapezium with $OB \parallel AC$ and $AC = 2OB$. P and Q are points on OA and BC respectively such that $OP = \frac{1}{2}OA$ and $BQ = \frac{1}{3}BC$. Let $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

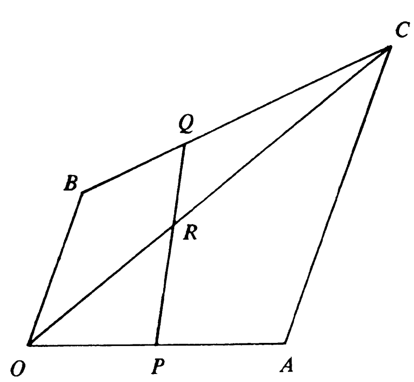

Figure 1

(a) Express $\overrightarrow{OC}$, $\overrightarrow{BC}$ and $\overrightarrow{OQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(5 marks)

(b) OC intersects PQ at the point R.
Let $PR : RQ = h : 1 - h$.

(i) Express $\overrightarrow{OR}$ in terms of $\mathbf{a}, \mathbf{b}$ and h.

(ii) If $\overrightarrow{OR} = k\overrightarrow{OC}$, find h and k.

(9 marks)

(c) OB and PQ are produced to meet at T and $\overrightarrow{OT} = \lambda\mathbf{b}$.

(i) Express $\overrightarrow{PQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
Express $\overrightarrow{PT}$ in terms of $\mathbf{a}, \mathbf{b}$ and λ.

(ii) Hence, or otherwise, find the value of λ.

(6 marks)

9. (a) Write down the general solution of the equation $\cos x = \dfrac{1}{\sqrt{2}}$.

(3 marks)

(b) Let m be a positive integer.

(i) If $z = r(\cos\theta + i\sin\theta)$, show that $z^m + \bar{z}^m = 2r^m\cos m\theta$.

(ii) By making use of (a) and (b)(i), or otherwise, find the values of m for which

$$\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)^m + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)^m = \sqrt{2}.$$

(9 marks)

(c) (i) Let p be a positive integer. Find the values of p for which

$$(1+i)^p - (1-i)^p = 0.$$

(ii) By making use of (c)(i), or otherwise, find the value of

$$\frac{(1+i)^{4k+1}}{(1-i)^{4k-1}},$$

where k is a positive integer.

(8 marks)

10. The graph of the function $f(x) = x^3 + hx^2 + kx + 2$ (h and k are constants) has 2 distinct turning points and intersects the line $y = 2$ at the point $(0, 2)$ only.

(a) Show that $3k < h^2 < 4k$.

(8 marks)

(b) It is also known that the graph of $f(x)$ passes through $(-2, 0)$.

(i) Express k in terms of h.

(ii) If h is an integer, use the results in (a) and (b)(i) to show that $h = 4$ or 5.

(iii) For $h = 4$, find the maximum and minimum points of the graph of $f(x)$ and sketch this graph.

(12 marks)

11. Figure 2 shows two rods OP and PR in the xy-plane. The rods, each 10 cm long, are hinged at P. The end O is fixed while the end R can move along the positive x-axis. $OL = 20$ cm, $OR = s$ cm and $\angle POR = \theta$, where $0 \le \theta \le \dfrac{\pi}{2}$.

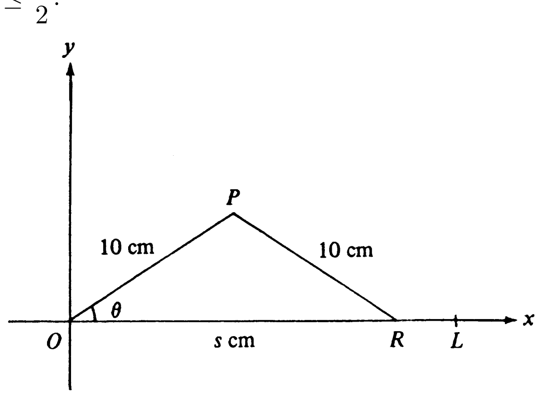

Figure 2

(a) Express s in terms of θ.
If R moves from the point O to the point L at a speed of 10 cm/s, find the rate of change of θ with respect to time when $s = 10$.

(5 marks)

(b) Find the equation of the locus of the mid-point of PR and sketch this locus.

(5 marks)

(c) A square of side ℓ cm is inscribed in $\triangle OPR$ such that one side of the square lies on OR. Show that

$$\ell = \frac{20\sin\theta\cos\theta}{\sin\theta + 2\cos\theta}.$$

Hence find θ when the area of the square is a maximum.

(10 marks)

12. Figure 3 shows a rectangular picture of area A cm^2 mounted on a rectangular piece of cardboard of area 3600 cm^2 with sides of length x cm and y cm. The top, bottom and side margins are 12 cm, 13 cm and 8 cm wide respectively.

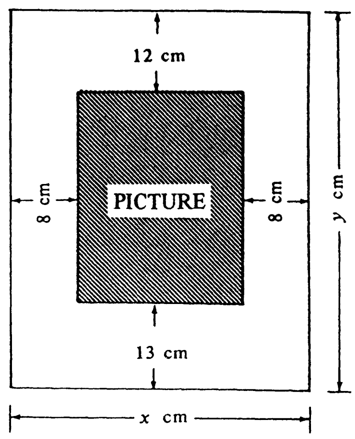

Figure 3

(a) Find A in terms of x.

(2 marks)

(b) Show that the largest value of A is 1600.

(5 marks)

(c) (i) Find the range of values of x for which A decreases as x increases.
(ii) If $x \geq 50$, find the largest value of A.

(6 marks)

(d) If $\frac{4}{9} \leq \frac{x}{y} \leq \frac{9}{16}$, find the range of values of x and the largest value of A.

(7 marks)

END OF PAPER