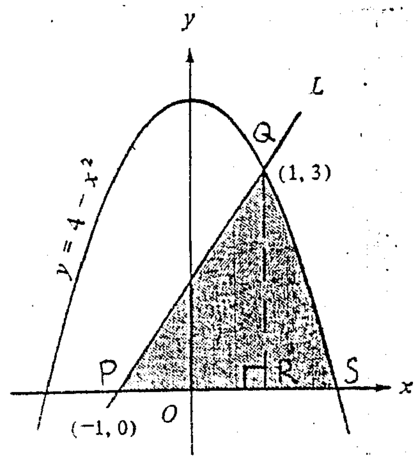
1985. PAPER II

SOLUTIONS	MARKS	REMARKS
1. General term $= C^n_r (ax)^{n-r} \frac{1}{x^{2r}}$.		If other terms given, disregard wrong but irrelevant terms.
The 4th term of the expansion		
$= C^n_3 (ax)^{n-3} \frac{1}{x^6}$		
$= C^n_3 a^{n-3} x^{n-9}$	2A	
If this term is independent of x, $n - 9 = 0$		
$n = 9$	1A	
$C^9_3 a^6 = \frac{21}{2}$	1M	
$a^6 = \frac{21}{2} \cdot \frac{3 \cdot 2}{9 \cdot 8 \cdot 7}$		
$= \frac{1}{8}$		
$a = \frac{1}{\sqrt{2}}$ (as $a > 0$) (or $\frac{\sqrt{2}}{2}$ or 0.707)	1A / 5	
2. For $n = 1$, L.S. $= \frac{1 \times (1+2)}{(1+1)^2} = \frac{3}{4}$		
R.S. $= \frac{1+2}{2(1+1)} =$ L.S.	1	
Assume that the equality holds for some positive integer k, ...	1	
then for $n = k + 1$,		
L.S. $= T_1 \times T_2 \times \ldots \times T_{k+1}$		
$= (T_1 \times T_2 \times \ldots \times T_k) \times T_{k+1}$		
$= \frac{k+2}{2(k+1)} \times \frac{(k+1)(k+3)}{(k+2)^2}$	1A	
$= \frac{k+3}{2(k+2)}$	1A	
= R.S.		
$\therefore$ the equality also holds for $n = k + 1$.		
By mathematical induction, the equality holds for all positive integers n.	1 / 5	Awarded only if above correct.

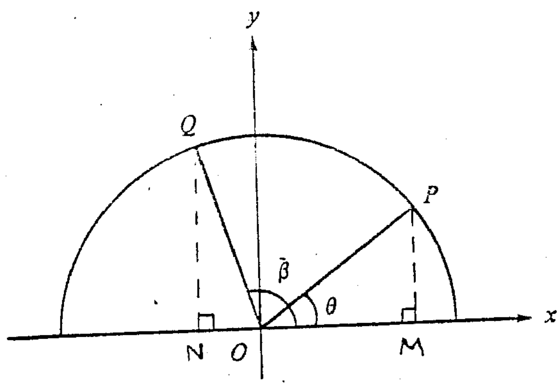
SOLUTIONS	MARKS	REMARKS
3. Let $u = 25 - x^2$, $du = -2xdx$.	1A	
When $x = 3, u = 16$; $x = 4, u = 9$	1A	
$\int_3^4 \frac{x}{\sqrt{25 - x^2}} dx = \int_{16}^{9} -\frac{1}{2\sqrt{u}} du$	1M+1A	1M for limits, 1A for $-\frac{1}{2\sqrt{u}} du$
$= \frac{1}{2} [2u^{\frac{1}{2}}]_9^{16}$		
$= 1$	1A	
	5	
Alt. Solution :		
Let $u = 25 - x^2$, $du = -2xdx$.	1A	
$\int \frac{x}{\sqrt{25 - x^2}} dx = -\frac{1}{2} \int \frac{1}{\sqrt{u}} du$	1A	
$= -\sqrt{u} + c$	1A	
$= -\sqrt{25 - x^2} + c$	1M	
$\therefore \int_3^4 \frac{x}{\sqrt{25 - x^2}} dx = [-\sqrt{25 - x^2}]_3^4$		
$= 1$	1A	
4. Area of the shaded part = area of PQR + area of RQS	1M	
Area of PQR $= \frac{1}{2} \times 3 \times 2 = 3$	1A	
Area of RQS $= \int_1^2 (4 - x^2) dx$	1A	
$= [4x - \frac{x^3}{3}]_1^2$		
$= (8 - \frac{8}{3}) - (4 - \frac{1}{3}) = 1\frac{2}{3}$ (or 1.67)	1A	
$\therefore$ total area $= 3 + 1\frac{2}{3} = 4\frac{2}{3}$ (or 4.67)	1A	
	5	
Alt. Solution :		
Equation of L : $y - 3 = \frac{3}{2}(x - 1)$		
$x = \frac{2}{3}y - 1$	1A	
Area $= \int_0^3 [\sqrt{4 - y} - (\frac{2}{3}y - 1)] dy$	1A+1A+1M	1A for limits; 1A for integrand; 1M for ' − '
$= [-\frac{2}{3}(4 - y)^{\frac{3}{2}} - \frac{1}{3}y^2 + y]_0^3$		
$= 4\frac{2}{3}$	1A	



SOLUTIONS	MARKS	REMARKS
5. The equation of the family of circles passing through A and B is		
$x^2 + y^2 - 2y + k(x - y) = 0$	1A	
[or $x - y + k(x^2 + y^2 - 2y) = 0$, $(k \neq 0)$]		
The equation can be written as		
$x^2 + y^2 + kx - (2 + k)y = 0$		
Radius of the circle = $\sqrt{(\frac{k}{2})^2 + (\frac{2+k}{2})^2}$	1M	
$\sqrt{(\frac{k}{2})^2 + (\frac{2+k}{2})^2} = \sqrt{5}$	1M	
$k^2 + 2k - 8 = 0$	1A	
$\therefore$ $k = 2$ or -4		If k is wrong, no marks below
The two circles are $x^2 + y^2 + 2x - 4y = 0$	1A	
and $x^2 + y^2 - 4x + 2y = 0$	1A	
(or $(x+1)^2 + (y-2)^2 = 5$	6	
$(x-1)^2 + (y+1) = 5$)		
6. Let the equation of the line through (-1, 0) be		Alt. Solution:
$y = m(x + 1)$	1A	Eqn. of the tangent at (x_1, y_1) is
Substituting in the equation of the parabola	1M	$y_1 y = 2(x_1 + x)$ 1A
$m^2(x + 1)^2 = 4x$		If the tangent passes through (-1, 0),
$m^2x^2 + (2m^2 - 4)x + m^2 = 0$	1A	
$(2m^2 - 4)^2 - 4m^4 = 0$	1M	$0 = 2(x_1 - 1)$ 1M
For the line to be a tangent,		$x_1 = 1$ 1A
$m^2 = 1$		Putting $x=1$ in $y^2=4x$ 1M
$m = \pm 1$		$y_1 = \pm 2$
		Equations of tangents are
$\therefore$ the equations of the tangents are		$y = \pm(x + 1)$.......1A+1A
$y = \pm(x + 1)$	1A+1A	i.e. $x - y + 1 = 0$
i.e. $x - y + 1 = 0$		and $x + y + 1 = 0$
$x + y + 1 = 0$	6	

SOLUTIONS	MARKS	REMARKS
7. (a) Since $A + B + C = \pi$		
$\sin C = \sin(\pi - (A + B))$	1A	
$= \sin(A + B)$		
$= \sin A \cos B + \cos A \sin B$	1A	
Since A, B, C are acute		
$\sin A = \frac{5}{13} \Rightarrow \cos A = \frac{12}{13}$) $\sin B = \frac{3}{5} \Rightarrow \cos B = \frac{4}{5}$)	1A	
$\therefore \sin C = \frac{5}{13}\cdot\frac{4}{5} + \frac{12}{13}\cdot\frac{3}{5}$		
$= \frac{56}{65}$	1A	
(b) The 3 sides a, b, c satisfy		
$a : b : c = \sin A : \sin B : \sin C$	1M	for sine rule
$= \frac{5}{13} : \frac{3}{5} : \frac{56}{65}$		
$= 25 : 39 : 56$		
If the perimeter is 12 cm,		
the longest side $c = \frac{56}{120} \times 12 = 5.6$ cm	2A 7	
8. (a) $\int_0^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt = \int_0^{\frac{\pi}{2}} \sin t \cos^4 t (1 - \cos^2 t) \, dt$	1M	For $\sin^3 t = \sin t(1-\cos^2 t)$
$= \int_0^{\frac{\pi}{2}} \sin t \cos^4 t \, dt - \int_0^{\frac{\pi}{2}} \sin t \cos^6 t \, dt$		
$= \left[-\frac{1}{5}\cos^5 t + \frac{1}{7}\cos^7 t \right]_0^{\frac{\pi}{2}}$	1A+1A	
$= \frac{2}{35}$	1A 4	
(b) Putting $t = \frac{\pi}{2} - u$, $dt = -du$		
When $t = 0$, $u = \frac{\pi}{2}$;		
$t = \frac{\pi}{2}$, $u = 0$.	1A	
$\int_0^{\frac{\pi}{2}} \cos^3 t \sin^4 t \, dt = -\int_{\frac{\pi}{2}}^{0} \cos^3(\frac{\pi}{2} - u) \sin^4(\frac{\pi}{2} - u) \, du$	1A	
$= \int_0^{\frac{\pi}{2}} \sin^3 u \cos^4 u \, du$	1A	
$= \int_0^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt$	1A 4	

SOLUTIONS	MARKS	REMARKS
3. (c) Putting $t = -u$, $dt = -du$	1A	
When $t = -\frac{\pi}{2}$, $u = \frac{\pi}{2}$;		
$t = 0$, $u = 0$.	1A	
$\int_{-\frac{\pi}{2}}^{0} \cos^3 t \sin^4 t \, dt = -\int_{\frac{\pi}{2}}^{0} \cos^3(-u) \sin^4(-u) du$	1A	
$= \int_{0}^{\frac{\pi}{2}} \cos^3 u \sin^4 u \, du$		
$= \int_{0}^{\frac{\pi}{2}} \cos^3 t \sin^4 t \, dt$	1A	
$\int_{-\frac{\pi}{2}}^{0} \sin^3 t \cos^4 t \, dt = -\int_{\frac{\pi}{2}}^{0} \sin^3(-u) \cos^4(-u) \, du$	1A	
$= \int_{\frac{\pi}{2}}^{0} \sin^3 u \cos^4 u \, du$		
$= -\int_{0}^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt$	1A / 6	
(d) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{8} \sin^3 2t \, (\sin t + \cos t) \, dt$		
$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 t \cos^3 t \, (\sin t + \cos t) \, dt$	1M	for $\sin 2t = 2\sin t \cos t$
$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3 t \sin^4 t \, dt + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt$	1A	
$= [\int_{-\frac{\pi}{2}}^{0} \cos^3 t \sin^4 t \, dt + \int_{0}^{\frac{\pi}{2}} \cos^3 t \sin^4 t \, dt]$		
$+ [\int_{-\frac{\pi}{2}}^{0} \sin^3 t \cos^4 t \, dt + \int_{0}^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt]$	1M	
$= 2\int_{0}^{\frac{\pi}{2}} \cos^3 t \sin^4 t \, dt$		
$+ [-\int_{0}^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt + \int_{0}^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt]$		
$= 2\int_{0}^{\frac{\pi}{2}} \sin^3 t \cos^4 t \, dt$	2A	
$= 2 \times \frac{2}{35} = \frac{4}{35}$ (or 0.114)	1A / 6	

SOLUTIONS	MARKS	REMARKS
9.		
		
(a) The radius of the semi-circle is 1		
$\therefore$ $P = (\cos\theta, \sin\theta)$		
$Q = (\cos\beta, \sin\beta)$		
Volume generated by rotating PONM about the x-axis.		
$= \int_{\cos\beta}^{\cos\theta} \pi(1 - x^2)\,dx$	1M+1M + 1A	1M for vol. 1M for limits, accept -cos 1A for integrand
$= \pi\left[x - \frac{x^3}{3}\right]_{\cos\beta}^{\cos\theta}$		
$= \pi[(\cos\theta - \cos\beta) - \frac{1}{3}(\cos^3\theta - \cos^3\beta)]$	1A	
Volume of the two cones generated by rotating POM and QON are $\lvert\frac{1}{3}\pi\sin^2\theta\cos\theta\rvert$, $\lvert\frac{1}{3}\pi\sin^2\beta\cos\beta\rvert$	1A+1A	Accept vol. without absolute value signs
Volume V of the solid		
$= \pi[(\cos\theta - \cos\beta) - \frac{1}{3}(\cos^3\theta - \cos^3\beta)]$ $- \frac{1}{3}\pi\sin^2\theta\cos\theta + \frac{1}{3}\pi\sin^2\beta\cos\beta$	1M+2A	
$= \frac{\pi}{3}[3(\cos\theta-\cos\beta)-\cos^3\theta+\cos^3\beta-\sin^2\theta\cos\theta+\sin^2\beta\cos\beta]$		
$= \frac{\pi}{3}[3(\cos\theta-\cos\beta)-\cos\theta(\cos^2\theta+\sin^2\theta)+\cos\beta(\cos^2\beta+\sin^2\beta)]$		
$= \frac{2\pi}{3}(\cos\theta - \cos\beta)$	1A 10	

SOLUTIONS	MARKS	REMARKS

9. (b) If $\beta = 2\theta$, $V = \frac{2}{3}\pi(\cos\theta - \cos 2\theta)$,

$\frac{dV}{d\theta} = \frac{2}{3}\pi(-\sin\theta + 2\sin 2\theta)$ 1A

Putting $\frac{dV}{d\theta} = 0$, $-\sin\theta + 2\sin 2\theta = 0$ 1M

$4\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(4\cos\theta - 1) = 0$

$\therefore$ $\sin\theta = 0$ or $\cos\theta = \frac{1}{4}$ $(\theta = 0 \text{ or } 1.318)$ .. 1A

Obviously the volume is minimum if $\sin\theta = 0$.

$\frac{d^2V}{d\theta^2} = \frac{2\pi}{3}(-\cos\theta + 4\cos 2\theta)$

$\frac{d^2V}{d\theta^2} < 0$ if $\cos\theta = \frac{1}{4}$ 1A

V is maximum at $\cos\theta = \frac{1}{4}$

Its max. value is

$\frac{2\pi}{3}(\frac{1}{4} - 2(\frac{1}{4})^2 + 1) = \frac{3}{4}\pi$ (or 2.36) (cu. units) 1A

5

Alt. Solution:

If $\beta = 2\theta$,

$V = \frac{2}{3}\pi(\cos\theta - \cos 2\theta)$

$= \frac{2}{3}\pi(1 + \cos\theta - 2\cos^2\theta)$ ...1A

$= \frac{4}{3}\pi(\frac{9}{16} - (\frac{1}{4} - \cos\theta)^2)$ 1M+1A

$\therefore$ V is a max. when $\cos\theta = \frac{1}{4}$ & the max. value is $\frac{3}{4}\pi$ (cu. units) 2A

(c) If $\beta - \theta = \frac{\pi}{3}$, $V = \frac{2\pi}{3}(\cos\theta - \cos(\frac{\pi}{3} + \theta))$

$\frac{dV}{d\theta} = \frac{2\pi}{3}(-\sin\theta + \sin(\frac{\pi}{3} + \theta))$ 1A

$= \frac{2\pi}{3}(-\sin\theta + \sin\frac{\pi}{3}\cos\theta + \cos\frac{\pi}{3}\sin\theta)$

$= \frac{2\pi}{3}(-\frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta)$

Putting $\frac{dV}{d\theta} = 0$, $\tan\theta = \sqrt{3}$ 1M

$\theta = \frac{\pi}{3}$ 1A

$\frac{d^2V}{d\theta^2} = \frac{2\pi}{3}(-\cos\theta + \cos(\frac{\pi}{3} + \theta)) < 0$ if $\theta = \frac{\pi}{3}$ 1A

$\therefore$ V is max. at $\theta = \frac{\pi}{3}$ and its value is

$\frac{2\pi}{3}(\frac{1}{2} + \frac{1}{2}) = \frac{2\pi}{3}$ (or 2.09) (cu. units) 1A

5

Alt. Solution:

If $\beta - \theta = \frac{\pi}{3}$,

$V = \frac{2\pi}{3}(\cos\theta - \cos(\frac{\pi}{3} + \theta))$

$= \frac{2}{3}\pi[2\sin\frac{1}{2}(\frac{\pi}{3} + 2\theta)\sin\frac{\pi}{6}]$ ..1A

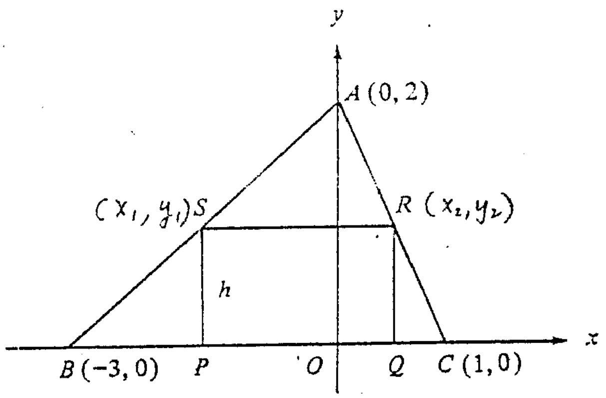
$= \frac{2\pi}{3}\sin(\frac{\pi}{6} + \theta)$ 1A

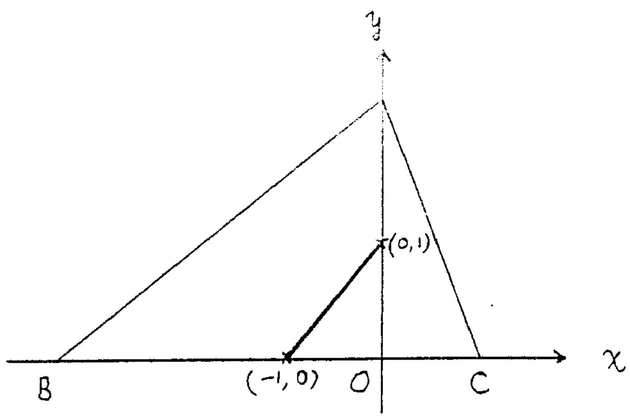
$\therefore$ V is a max. if $\theta = \frac{\pi}{3}$ 1M

[1M for $\sin(\ ) \leqslant 1$]

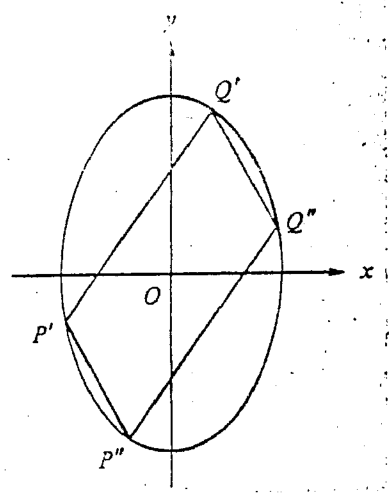
and the max. value is

$\frac{2}{3}\pi$ (cu. units) 2A

SOLUTIONS	MARKS	REMARKS
10.		
		
(a) Let $S = (x_1, y_1)$, $R = (x_2, y_2)$		**Alt. Solution :**
$y_1\ (= y_2) = h$	1A	$y_1\ (= y_2) = h$, 1A
By similar triangles		Equation of AB is
$\frac{-3 - x_1}{-3} = \frac{h}{2}$	1A	$y = \frac{2}{3}x + 2$ 1A
$\therefore x_1 = \frac{3h}{2} - 3$	1A	Substituting $y = h$
$\frac{1 - x_2}{1} = \frac{h}{2}$	1A	$x_1 = \frac{3}{2}(h - 2)$ 1A
$\therefore x_2 = 1 - \frac{h}{2}$	1A	Equation of AC is $y = 2 - 2x$ 1A
		Substituting $y = h$
		$x_2 = 1 - \frac{h}{2}$ 1A
	5	
(b) If PQRS is a square $x_2 - x_1 = h$	1M	
$4 - 2h = h$		
$h = \frac{4}{3}$	1A	
$\therefore A_1 = h^2\ (= \frac{16}{9})$	1A	
Area of rectangle $= h(4 - 2h)$	1A	
$= -2(h^2 - 2h + 1) + 2$		
$= 2 - 2(h - 1)^2$	1M	or $\frac{dA}{dh} = 0$
$\therefore A_2 = 2$	1A	
$A_3 = \frac{1}{2} \times 2 \times (1 - (-3)) = 4$	1A	
$\therefore A_1 : A_2 : A_3 = \frac{16}{9} : 2 : 4$ (or $8 : 9 : 18$)	1A	
	8	

SOLUTIONS	MARKS	REMARKS
10. (c) The coordinates of the centre $M(x, y)$ of PQRS are given by		
$x = \frac{x_2 + x_1}{2}$		
$= \frac{1}{2}(h - 2)$	1A	
$y = \frac{h}{2}$	1A	
Eliminating h, ...	1M	Attempt to eliminate
$x - y = \frac{1}{2}(h - 2) - \frac{h}{2}$		
$= -1$	1A	
Since $0 \leqslant h \leqslant 2$ (or $0 < h < 2$), the locus of M is the part of the straight line $x - y = -1$ lying between $(-1, 0)$ and $(0, 1)$ (end-points included/excluded)		
 Locus of M	3A	Line segment with end-point on axes ....2 End points correct ......1 (only awarded if equation correct)
	7	
11. (a) (i) $PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$		
$= \sqrt{(x_1 - x_2)^2 + [(2x_1 + c) - (2x_2 + c)]^2}$	1M+1A	1M for sub. y
$= \sqrt{5}\,\lvert x_1 - x_2 \rvert$	1A	
(ii) Putting $y = 2x + c$, $x^2 + \frac{(2x + c)^2}{16} = 1$	1M	
$16x^2 + (4x^2 + 4cx + c^2) = 16$		
$20x^2 + 4cx + (c^2 - 16) = 0$ ....(*) ...	1A	
Since (x_1, y_1) (x_2, y_2) satisfy the equations		
$y = 2x + c$ and $x^2 + \frac{y^2}{16} = 1$,		
x_1, x_2 are the roots of (*)		

SOLUTIONS	MARKS	REMARKS
11. (a) (iii) If $PQ = 2\sqrt{2}$, since x_1, x_2 are roots of (*),		
$\sqrt{5}\,\lvert x_1 - x_2\rvert = \sqrt{5}\sqrt{(x_1+x_2)^2 - 4x_1x_2}$	1A	
$= \sqrt{5}\sqrt{\left(\frac{-4c}{20}\right)^2 - \frac{4(c^2-16)}{20}}$	1M+1M	Sub. $x_1+x_2 = \frac{-4C}{20}$, $x_1x_2 = \frac{c^2-16}{20}$
$= \sqrt{\frac{80-4c^2}{5}}$	1A	
$= 2\sqrt{2}$ ————————	1M	
$\Rightarrow 80 - 4c^2 = 40$, $c^2 = 10$		
$c = \pm\sqrt{10}$	1A	
	11	
(b) Let the equations of P'Q' and P"Q" be $y = 2x + \sqrt{10}$ and $y = 2x - \sqrt{10}$ respectively.		
(i) $(0, \sqrt{10})$ is a point on P'Q'.	1A	
Distance between P'Q' and P"Q" is		
$\frac{2 \times 0 - \sqrt{10} - \sqrt{10}}{\pm\sqrt{2^2+1^2}}$	1M	
$= 2\sqrt{2}$ ————————	1A	
Area of parallelogram $= 2\sqrt{2} \times 2\sqrt{2}$	1M	
$= 8$ (sq. units)	1A	
(ii) If $P' = (x_1, y_1)$, $Q' = (x_2, y_2)$		
by symmetry $P'' = (-x_2, -y_2)$	1M	
$\therefore P'P'' = \sqrt{(x_1+x_2)^2 + (y_1+y_2)^2}$		
$= \sqrt{(x_1+x_2)^2 + 4(x_1+x_2+c)^2}$	1M	
$= \sqrt{\left(\frac{-c}{5}\right)^2 + 4\left(\frac{4c}{5}\right)^2}$		
$= \sqrt{\frac{65}{25}c^2}$	1A	
$= \sqrt{\frac{130}{5}}$		
$= \sqrt{26}$	1A	
	9	



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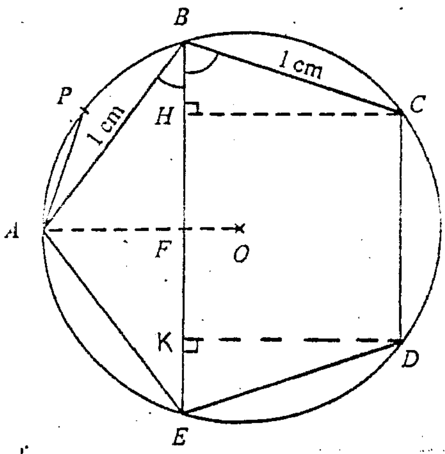
SOLUTIONS	MARKS	REMARKS
Alt. Solution:		
11. (a)		
(iii) $x = \frac{-4c \pm \sqrt{16c^2 - 80(c^2 - 16)}}{40} = \frac{-c \pm \sqrt{80 - 4c^2}}{10}$	1A	
$y = 2x + c = \frac{-c \pm \sqrt{80 - 4c^2}}{5} + c$		
$= \frac{4c \pm \sqrt{80 - 4c^2}}{5}$	1A	
Let $P = \left(\frac{-c - \sqrt{80 - 4c^2}}{10}, \frac{4c - \sqrt{80 - 4c^2}}{5} \right)$		
$Q = \left(\frac{-c + \sqrt{80 - 4c^2}}{10}, \frac{4c + \sqrt{80 - 4c^2}}{5} \right)$		
$PQ^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$	1M	
$= \left(\frac{\sqrt{80 - 4c^2}}{5} \right)^2 + \left(\frac{2\sqrt{80 - 4c^2}}{5} \right)^2$		
$= \frac{80 - 4c^2}{5}$	1A	
$PQ = 2\sqrt{2} \Rightarrow \frac{80 - 4c^2}{5} = (2\sqrt{2})^2$	1M	
i.e. $c = \pm\sqrt{10}$	1A	
(b) (i) $\sqrt{80 - 4c^2} = \sqrt{40} = 2\sqrt{10}$	1A	
$P' = \left(\frac{-\sqrt{10} - 2\sqrt{10}}{10}, \frac{4\sqrt{10} - 2\sqrt{10}}{5} \right)$		
$= \left(\frac{-3\sqrt{10}}{10}, \frac{2\sqrt{10}}{5} \right)$	1A	
$Q' = \left(\frac{-\sqrt{10} + 2\sqrt{10}}{10}, \frac{4\sqrt{10} + 2\sqrt{10}}{5} \right)$		
$= \left(\frac{\sqrt{10}}{10}, \frac{6\sqrt{10}}{5} \right)$	1A	
$P'' = \left(\frac{\sqrt{10} - 2\sqrt{10}}{10}, \frac{-4\sqrt{10} - 2\sqrt{10}}{5} \right)$		
$= \left(\frac{-\sqrt{10}}{10}, \frac{-6\sqrt{10}}{5} \right)$	1A	
Area of parallelogram P'Q'Q"P" = 2 Δ P'Q'P"	1M	
$= \left\| \frac{-3\sqrt{10}}{10}\left(\frac{6\sqrt{10}}{5} - \frac{-6\sqrt{10}}{5} \right) + \frac{\sqrt{10}}{10}\left(\frac{-6\sqrt{10}}{5} - \frac{2\sqrt{10}}{5} \right) - \frac{\sqrt{10}}{10}\left(\frac{2\sqrt{10}}{5} - \frac{6\sqrt{10}}{5} \right) \right\|$		
$= \left\| -\frac{36}{5} - \frac{8}{5} + \frac{4}{5} \right\|$		
$= 8$	2A	
(ii) $(P'P'')^2 = \left(\frac{2\sqrt{10}}{10} \right)^2 + \left(\frac{8\sqrt{10}}{5} \right)^2$	1M	
$= \frac{2}{5} + \frac{128}{5}$		
$= 26$		
$\therefore$ P'P" $= \sqrt{26}$	1A	

SOLUTIONS | MARKS | REMARKS



12. (a) $\angle ABC = \frac{2 \times 5 - 4}{5} \times 90°$

$= 108°$ 1A

$\angle ABE = \frac{(180 - 108)°}{2}$

$= 36°$ 1A

$\angle CBE = 108° - 36° = 72°$ 1A

$BE = BH + HK + KE$ 1M

$= \cos 72° + 1 + \cos 72°$

$= 2 \cos 72° + 1$ 1A

Also, $BE = 2BF = 2 \cos 36°$..................... 1A — see Alt. Solution

$\therefore 2 \cos 72° + 1 = 2 \cos 36°$

i.e. $\cos 36° - \cos 72° = \frac{1}{2}$ 1A

$\cos 36° - (2 \cos^2 36° - 1) = \frac{1}{2}$ 1A

$4 \cos^2 36° - 2 \cos 36° - 1 = 0$

$\cos 36° = \frac{2 + \sqrt{4 + 16}}{8}$ (-ve root rejected as $\cos 36° > 0$)

$= \frac{1 + \sqrt{5}}{4}$ 1A

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(b) $\frac{\frac{1}{2}AB}{OA} = \cos 54°$ 1A

$= \sin 36°$ 1A

$\therefore OA = \frac{1}{2 \sin 36°}$

$= \frac{1}{2\sqrt{1 - \cos^2 36°}}$ 1A

$= \frac{1}{2\sqrt{1 - \frac{(1 + \sqrt{5})^2}{16}}}$ 1M

$= \frac{2}{\sqrt{16 - (1 + 5 + 2\sqrt{5})}}$

$= \frac{2}{\sqrt{10 - 2\sqrt{5}}}$ cm 1A

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Alt. Solution:

$OA^2 + OB^2 - AB^2$
$= 2OA \cdot OB \cos AOB$ 1A

$2OA^2 - 1 = 2OA^2 \cos 72°$

$OA^2 = \frac{1}{2(1 - \cos 72°)}$ ..1A

$= \frac{1}{2(1 - \cos 36° + \frac{1}{2})}$ ...1M

$= \frac{1}{3 - \frac{1 + \sqrt{5}}{2}}$

$= \frac{2}{5 - \sqrt{5}}$ 1A

$\therefore OA = \sqrt{\frac{2}{5 - \sqrt{5}}}$

$= \frac{2}{\sqrt{10 - 2\sqrt{5}}}$ ...1A

SOLUTIONS	MARKS	REMARKS
12. (c) Each angle of a regular decagon		
$= \dfrac{2 \times 10 - 4}{10} \times 90° = 144°$	1A	or $\angle AOP = 36°$
$\therefore \angle PAO = 72°$	1A	
$\dfrac{\frac{1}{2}AP}{AO} = \cos 72°$	1A	
$AP = 2\cos 72° \times AO$		
$= 2(\cos 36° - \frac{1}{2}) \times AO$	1A	
$= 2\left(\dfrac{\sqrt{5}-1}{4}\right)\dfrac{2}{\sqrt{10-2\sqrt{5}}}$	1M	see Alt. Solution
$= \dfrac{(\sqrt{5}-1)(\sqrt{5}+1)}{\sqrt{10-2\sqrt{5}}\,(\sqrt{5}+1)}$		
$= \dfrac{4}{\sqrt{(10-2\sqrt{5})(6+2\sqrt{5})}}$		
$= \dfrac{4}{\sqrt{40+8\sqrt{5}}}$		
$= \dfrac{2}{\sqrt{10+2\sqrt{5}}}$ cm	1A	
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<u>Alt. Solution</u>

SOLUTIONS	MARKS	REMARKS
12. (a) In $\triangle ABE$,		
$BE = \sqrt{1+1-2\cos 108°}$		
$= \sqrt{2+2\cos 72°}$	1A	
In $\triangle BCE$, $BE = EC$,		
$1^2 = BE^2 + BE^2 - 2BE^2 \cos 36°$		
$BE = \dfrac{1}{\sqrt{2-2\cos 36°}}$	1A	
$2 + 2\cos 72° = \dfrac{1}{2 - 2\cos 36°}$		
$\cos 36° - \cos 72° = \dfrac{3}{4} - \cos 72° \cos 36°$		
$= \dfrac{3}{4} - \dfrac{\cos 36° \cos 72° \cdot \sin 36°}{\sin 36°}$	1	
$= \dfrac{3}{4} - \dfrac{1}{2}\,\dfrac{\sin 72° \cos 72°}{\sin 36°}$		
$= \dfrac{3}{4} - \dfrac{1}{4}\,\dfrac{\sin 144°}{\sin 36°}$		
$= \dfrac{1}{2}$	1A	

SOLUTIONS	MARKS	REMARKS
Alt. Solution (1)		
12. (c) $\angle PAB = 72° - 54°$		
$= 18°$	1A	
$\cos 18° = \frac{\frac{1}{2}}{AP}$		
$AP = \frac{1}{2\cos 18°}$		
$= \frac{1}{2\sqrt{\frac{1+\cos 36°}{2}}}$	1A	
$= \frac{1}{2\sqrt{\frac{1+\frac{1+\sqrt{5}}{4}}{2}}}$	1M	
$= \frac{\sqrt{2}}{\sqrt{5}+\sqrt{5}}$		
$= \frac{2}{\sqrt{10+2\sqrt{5}}}$	1A	
Alt. Solution (2)		
In $\triangle PAO$, $AP = AO$,		
$AP^2 = AO^2 + AO^2 - 2(AO)^2 \cos 36°$	2A	
$= 2\left(\frac{2}{\sqrt{10-2\sqrt{5}}}\right)^2 (1 - \cos 36°)$		
$= \frac{8}{10-2\sqrt{5}}\left(1 - \frac{1+\sqrt{5}}{4}\right)$	1M	
$= \frac{2(3-\sqrt{5})(3+\sqrt{5})}{(10-2\sqrt{5})(3+\sqrt{5})}$		
$= \frac{4}{10+2\sqrt{5}}$		
$\Rightarrow AP = \frac{2}{\sqrt{10+2\sqrt{5}}}$	1A	