HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1985

# 附加數學　試卷一

# ADDITIONAL MATHEMATICS　PAPER I

8.30 am–10.30 am　(2 hours)

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

Unless otherwise specified in a question, it is sufficient for numerical answers to be given correct to three significant figures.



## SECTION A (39 marks)

**Answer ALL questions in this section.**

1. Let $f(x) = x\sqrt{1-x^2}$ . Find the value of $f'(\frac{1}{2})$ .

(5 marks)

2. Express $1 - i$ in polar form.

Hence find the cube roots of $1 - i$ (give your answers in polar form).

(6 marks)

3. Solve the inequality $x^2 - ax - 4 \leqslant 0$ , where $a$ is real. If among the possible values of $x$ satisfying the above inequality, the greatest is 4 , find the least.

(6 marks)

4. In Figure 1 , $\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j}$ , $\overrightarrow{OB} = 4\mathbf{i} - 3\mathbf{j}$ . $C$ is a point on $AB$ such that $\frac{AC}{CB} = r$ .

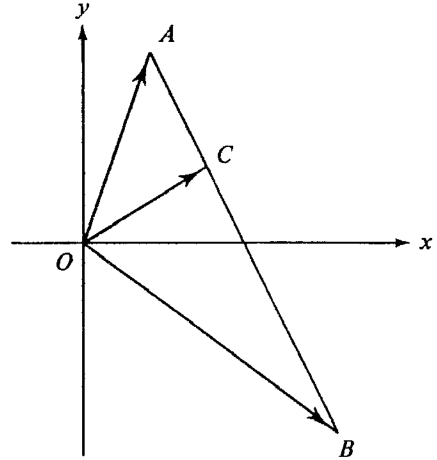


Figure 1

(a) Express $\overrightarrow{OC}$ in terms of $r$ .

(b) Find the value of $r$ if $OC$ is perpendicular to $AB$ . Hence find the coordinates of $C$ .

(6 marks)



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5.

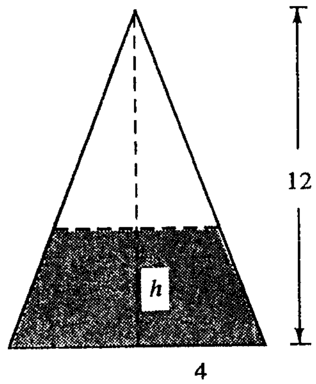


Figure 2

Figure 2 shows a vessel in the shape of a right circular cone with base radius 4 cm and height 12 cm. Water is poured into the vessel through the apex. Find the volume of the water in the vessel when the depth of the water is $h$ centimetres. If water is poured into the vessel at a rate of $\pi$ cm$^3$/s, how fast is the water level rising when the depth of the water is 6 cm ?

(8 marks)

6. Find the two real values of $p$ for which the equation

$$\log_{10} |x^2 + 2px| = 0$$

has a double root.

(8 marks)



## SECTION B (60 marks)

**Answer any THREE questions from this section.**
**Each question carries 20 marks.**

7. (a) In the equation $ax^2 + bx + c = 0$, $a, b$ and $c$ are complex numbers and $a \neq 0$.

(i) By the method of completing the square, factorize the expression $ax^2 + bx + c$.

(ii) Show that if $a, b$ and $c$ are real numbers such that $b^2 - 4ac < 0$, then the given equation has imaginary roots.

(iii) Show that if $a = 3i$, $b = -2$ and $c = 5i$, then $b^2 - 4ac > 0$, but the equation still has imaginary roots.

(9 marks)

(b) The equation

$$x^2 - 2\lambda x + (2\lambda^2 - 2\lambda\mu + \mu^2) = 0$$

has real roots. If $\lambda$ and $\mu$ are real, find the relation between them.

(4 marks)

(c) The coefficients of the following equations are real :

$x^2 + ax + b = 0$ ..................................(1)
$x^2 + cx + d = 0$ ..................................(2)
$2x^2 + (a + c)x + (b + d) = 0$ ............(3) .

Prove that if the roots of (1) and (2) are imaginary, so are the roots of (3) .

(7 marks)



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8. In Figure 3, $\Delta OBA$ is right-angled at $B$. $OB$ is produced to $C$ such that $OB = BC$. $CD$ is drawn in the direction of $OA$ such that $CD = kOA$. $P$ is a point on $AD$ such that $CP \parallel BA$. Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{DP} = \lambda \overrightarrow{DA}$.

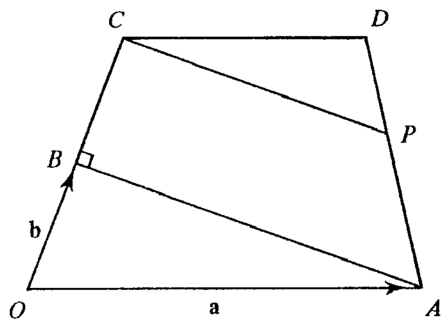



Figure 3

(a) (i) Express $\overrightarrow{OD}$ and $\overrightarrow{DA}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $k$.

(ii) Find $\overrightarrow{BA}$ in terms of $\mathbf{a}$ and $\mathbf{b}$ and express $\overrightarrow{CP}$ in terms of $\mathbf{a}$, $\mathbf{b}$, $\lambda$ and $k$.

Hence find $\lambda$ in terms of $k$.

(9 marks)

(b) (i) Show that $\mathbf{a} \cdot \mathbf{b} = OB^2$.

(ii) If $OB = \frac{1}{4}OA$, show that $\overrightarrow{OD} \cdot \overrightarrow{DA} = (-16k^2 + 12k - 2)OB^2$.

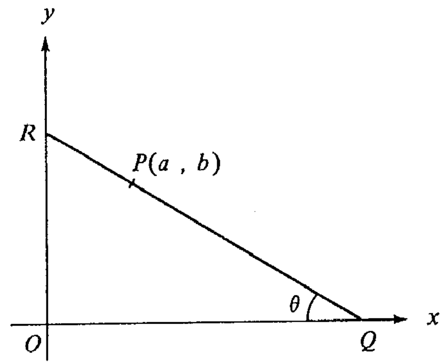
Hence find the values of $k$ and $\lambda$ if $OD \perp DA$.

(11 marks)



9. (a) In Figure 4, $P(a, b)$ is a point in the first quadrant. A variable line segment $QR$ passes through $P$ with the end $Q$ on the $x$-axis and $R$ on the $y$-axis. Let $\angle RQO = \theta$ and $QR = s$.

Figure 4

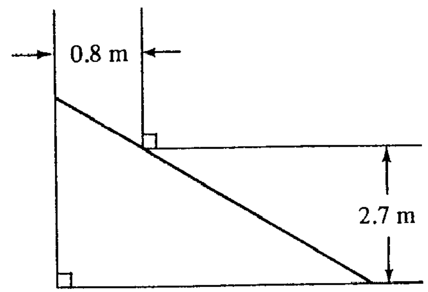



(i) Express $s$ in terms of $a$, $b$ and $\theta$.

(ii) Show that $s$ will be least when $\tan\theta = \sqrt[3]{\frac{b}{a}}$. (11 marks)

(b) Figure 5 shows two corridors meeting at right angles. The width of one corridor is 0.8 m and that of the other is 2.7 m. A pipe is to be moved from one corridor into the other.

Figure 5




(i) If the pipe is to lie completely on the horizontal floor when it is being moved round the corner, what is the greatest possible length of the pipe?

(ii) If the height of the ceiling of each corridor is 3 m, find the length of the longest pipe that can be carried round the corner.

(9 marks)



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10. Let $z$ be a complex number not equal to $-1$ and $w = \frac{z-1}{z+1}$ .

(a) Let $w = p + qi$, where $p$ and $q$ are real.

Show that $p = \frac{1}{2}(w + \bar{w})$

and $q = \frac{1}{2i}(w - \bar{w})$ .

Hence show that $p = \frac{z\bar{z} - 1}{z\bar{z} + z + \bar{z} + 1}$

and $q = \frac{i(\bar{z} - z)}{z\bar{z} + z + \bar{z} + 1}$ .

(7 marks)

(b) In each of the following cases, find the locus of $z$ and interpret the result geometrically :

(i) $w$ is real,

(ii) $w$ is purely imaginary,

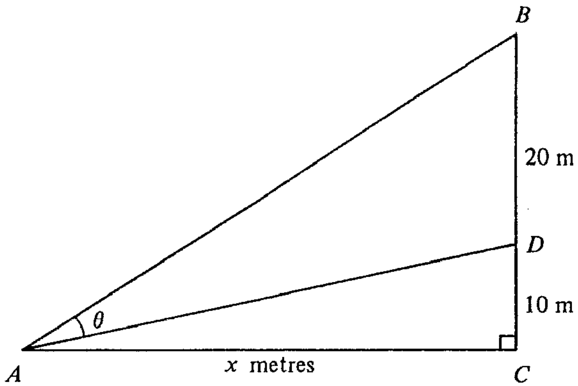
(iii) $|w| = 1$ . [Hint : You may use $|w|^2 = w\bar{w}$ .]

(13 marks)



11. In Figure 6 , $BD$ is an advertisement painted on a vertical wall $BDC$ of a building. $BD = 20$ m , $DC = 10$ m . An observer at $A$ , $x$ metres from the wall, finds the angle subtended by the advertisement at his eye to be $\theta$ .

Figure 6




(a) Show that $\tan\theta = \frac{20x}{x^2 + 300}$ .

(3 marks)

(b) By differentiating both sides of the result in (a) with respect to $x$ , show that $\frac{d\theta}{dx} = \frac{20(300 - x^2)}{x^4 + 1000x^2 + 90\,000}$ .

Hence find the value of $x$ for which $\theta$ is a maximum.

(6 marks)

(c) Find the value of $\frac{d\theta}{dx}$ at $x = 50$ , correct to 4 decimal places.

Hence estimate the increase in the distance between the observer and the wall if the angle subtended is to be decreased by $1^\circ$ from that observed at $x = 50$ (your answer should be correct to the nearest $\frac{1}{10}$ m) .

(6 marks)

(d) Sketch the graph of $\theta$ against $x$ for $x \geqslant 0$ .

(5 marks)

END OF PAPER

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