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一九八四年香港中學會考
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ADD. MATHEMATICS (PAPER II)
MARKING SCHEME

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SOLUTION	MARKS	REMARKS
1. $(x^2 + \frac{a}{x})^8 =$		
$= x^{16} + 8x^{14}\frac{a}{x} + 28x^{12}\frac{a^2}{x^2} + 56x^{10}\frac{a^3}{x^3} + \dots$		~~1 for B_7~~
$= x^{16} + 8ax^{13} + \underline{28a^2x^{10}} + \underline{56a^3x^7} + \dots$	3A	~~1 for B_{10}~~ 2A+1A ~~1 for the rest~~
$56a^3 = 4 \times 28a^2$	1M	Alternatively:
$\therefore a = 2$ (as $a \neq 0$)	1A	The general term $=$ ${}_8C_r\, a^r x^{16-3r}$ $16-3r = 7 \Rightarrow r = 3$ $\therefore B_7 = {}_8C_3\, a^3 = \underline{56\, a^3}$ $16-3r = 10 \Rightarrow r = 2$ $\}$ 2A+1A $\therefore B_{10} = {}_8C_2\, a^2 = \underline{28a^2}$ etc.
	5	
2. If $n = 1$, $4n^3 - n = 3$, which is divisible by 3.	1A	Lost all A marks if this part is om
Assume that 3 divides $4k^3 - k$ for some positive integer k.	1A	
Let $4k^3 - k = 3m$, where m is an integer.		
$4(k+1)^3 - (k+1) = 4(k^3+3k^2+3k+1) - (k+1)$		
$= (4k^3-k) + 3(4k^2+4k+1)$	1M+1A	1M for using assumption
$= 3m + 3(4k^2 + 4k + 1)$	1A	
$= 3(m + 4k^2 + 4k + 1)$, which is divisible by 3		
can be omitted By induction, 3 divides $4n^3 - n$ for all positive integers n.	1A	
	6	

SOLUTION	MARKS	REMARKS
3. $y = \int (4\sin^2 x + 1)dx$	1A	-1 if dx omitted
$= \int [2(1 - \cos 2x) + 1]dx$		
$= \int (3 - 2\cos 2x)dx$	1A	No penality for omitting brackets
$= 3x - \sin 2x + c$	2A	- 1 if c omitted
sub $x = \frac{\pi}{2}$, $y = 0$	1M	
$c = \sin\pi - \frac{3}{2}\pi$		
$= -\frac{3\pi}{2}$	1A	
∴ the equation of the curve is $y = 3x - \sin 2x - \frac{3\pi}{2}$		
	6	
4. The two lines $\begin{cases} x + y = 4 \\ x - y = 2p \end{cases}$ intersect at		Alternatively: The 2 lines intersect at $x = p+2$
$(2+p,\ 2-p)$. x-coordinate	1A	Area $= \int_0^{p+2} [(4-x)-(x-2p)]dx$ ..1+1A (limit, integrand)
They intersect the y-axis at $(0, 4)$ and $(0, -2p)$	1A	$= [(4+2p)x - x^2]_0^{p+2}$
Area of △ $= \frac{\text{height X base}}{2}$		$= (p+2)^2$ 1A
$= \frac{1}{2}(2+p)(4+2p)$		etc.
$= (p+2)^2$	1A	
$p^2 + 4p + 4 = 9$	1M	Accept ±9
$p^2 + 4p - 5 = 0$		
$(p+5)(p-1) = 0$		
$p = 1$ or -5	1+1A	
	6	

SOLUTION	MARKS	REMARKS
5. $\frac{d}{d\theta}\tan^3\theta = 3\tan^2\theta\sec^2\theta$	1A	
$\int \tan^2\theta\sec^2\theta\,d\theta = \frac{1}{3}\int d\tan^3\theta$		
$= \frac{1}{3}\tan^3\theta + c$	2A	-1 if c omitted still marked even if $\frac{1}{3}\tan^3\theta + C$ is not obtained by the specified method
$\int_0^{\frac{\pi}{3}} \tan^4\theta\,d\theta = \int_0^{\frac{\pi}{3}} \tan^2\theta(\sec^2\theta - 1)d\theta$	2A	**Alternatively:**
$= \int_0^{\frac{\pi}{3}} \tan^2\theta\sec^2\theta\,d\theta - \int_0^{\frac{\pi}{3}} \tan^2\theta\,d\theta$		$\int_0^{\frac{\pi}{3}} \tan^2\theta\sec^2\theta\,d\theta$
$= \int_0^{\frac{\pi}{3}} \tan^2\theta\sec^2\theta\,d\theta - \int_0^{\frac{\pi}{3}} (\sec^2\theta - 1)d\theta$		$= \int_0^{\frac{\pi}{3}} \tan^2\theta(1+\tan^2\theta)d\theta$ 2A
$= [\frac{1}{3}\tan^3\theta]_0^{\frac{\pi}{3}} - [\tan\theta - \theta]_0^{\frac{\pi}{3}}$ for sub. $\frac{1}{3}\tan^3\theta$	1M+1A	$= \int_0^{\frac{\pi}{3}} (\sec^2\theta - 1)d\theta + \int_0^{\frac{\pi}{3}} \tan^4\theta d\theta$
$= \sqrt{3} - \sqrt{3} + \frac{\pi}{3}$		$\therefore \int_0^{\frac{\pi}{3}} \tan^4\theta d\theta$
$= \frac{\pi}{3}$ (1.05)	1A	$= [\frac{\tan^3\theta}{3}]_0^{\frac{\pi}{3}} - [\tan\theta - \theta]_0^{\frac{\pi}{3}}$ ..1M+1A
	8	$= \frac{\pi}{3}$ 1A
6. $x^2 + y^2 - 2kx + 4ky + 6k^2 - 2 = 0$		$(x-k)^2 + (y+2k)^2 = 2 - k^2$
(a) radius $= \sqrt{k^2 + (2k)^2 - (6k^2-2)}$		
$= \sqrt{2 - k^2}$	1A	
$\sqrt{2 - k^2} > 1$	1M	
$\Rightarrow k^2 < 1$		
$\Rightarrow -1 < k < 1$	1A	
(b) Coordinates of the centre are $x = k$, $y = -2k$	1A	
$\therefore$ the locus of the centre lies on the line $\underline{2x + y = 0}$	2A	
Since $-1 < k < 1$, we have $-1 < x < 1$ and $2 > y > -2$	1A for	either one of the inequalities or
$\therefore$ the locus is a line segment [with end-points $(-1, 2)$ and $(1, -2)$ excluded.]	1A	
	8	

SOLUTION	MARKS	REMARKS
7. (a) $\frac{1}{x^3}+\frac{3}{(2-3x)^2} = \frac{(2-3x)^2+3x^3}{x^3(2-3x)^2}$ $= \frac{3x^3+9x^2-12x+4}{9x^5-12x^4+4x^3}$	2A	
$\int_1^2 \frac{3x^3+9x^2-12x+4}{9x^5-12x^4+4x^3}\,dx$		
$= \int_1^2 \left[\frac{1}{x^3}+\frac{3}{(2-3x)^2}\right]dx$	1M	
$= \int_1^2 \frac{1}{x^3}\,dx + \int_1^2 \frac{3}{(2-3x)^2}\,dx$		
$= -\frac{1}{2}\left[\frac{1}{x^2}\right]_1^2 + \left[\frac{1}{2-3x}\right]_1^2$	1+1A	
$= \frac{3}{8}+\frac{3}{4}$	1+1A	
$= \frac{9}{8}$		
	7	
(b) (i) Let $u=\sin\phi$, $du=\cos\phi\,d\phi$	1A	
$\int \frac{\cos\phi}{\sin^4\phi}\,d\phi = \int \frac{1}{u^4}\,du$	1A	
$= -\frac{1}{3u^3}+c$	2A	− 1 if omit c
$= -\frac{1}{3\sin^3\phi}+c$	1A	
	5	
(ii) Put $x=\tan\phi$, $dx=\sec^2\phi\,d\phi$	1A	
when $x=\frac{1}{\sqrt{3}}$, $\phi=\frac{\pi}{6}$; when $x=1$, $\phi=\frac{\pi}{4}$.	1A	
$\int_{\frac{1}{\sqrt{3}}}^{1} \frac{3\sqrt{1+x^2}}{x^4}\,dx$		
$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{3\sqrt{1+\tan^2\phi}}{\tan^4\phi}\sec^2\phi\,d\phi$	1M+1A	1M for limits, 1A for integrand
$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{3\sec^3\phi}{\tan^4\phi}\,d\phi = \boxed{\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{3\cos\phi}{\sin^4\phi}\,d\phi}$	1A	no penalty if ϕ is written as θ, etc
$= \left[-\frac{1}{\sin^3\phi}\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ by (i)	1M+1A	
$= \frac{1}{\sin^3\frac{\pi}{6}} - \frac{1}{\sin^3\frac{\pi}{4}}$		
$= \frac{1}{(\frac{1}{2})^3} - \frac{1}{(\frac{\sqrt{2}}{2})^3}$		
$= 8-2\sqrt{2}$ $(= 5.17)$	1A	any figure roundable to 5.17
	8	

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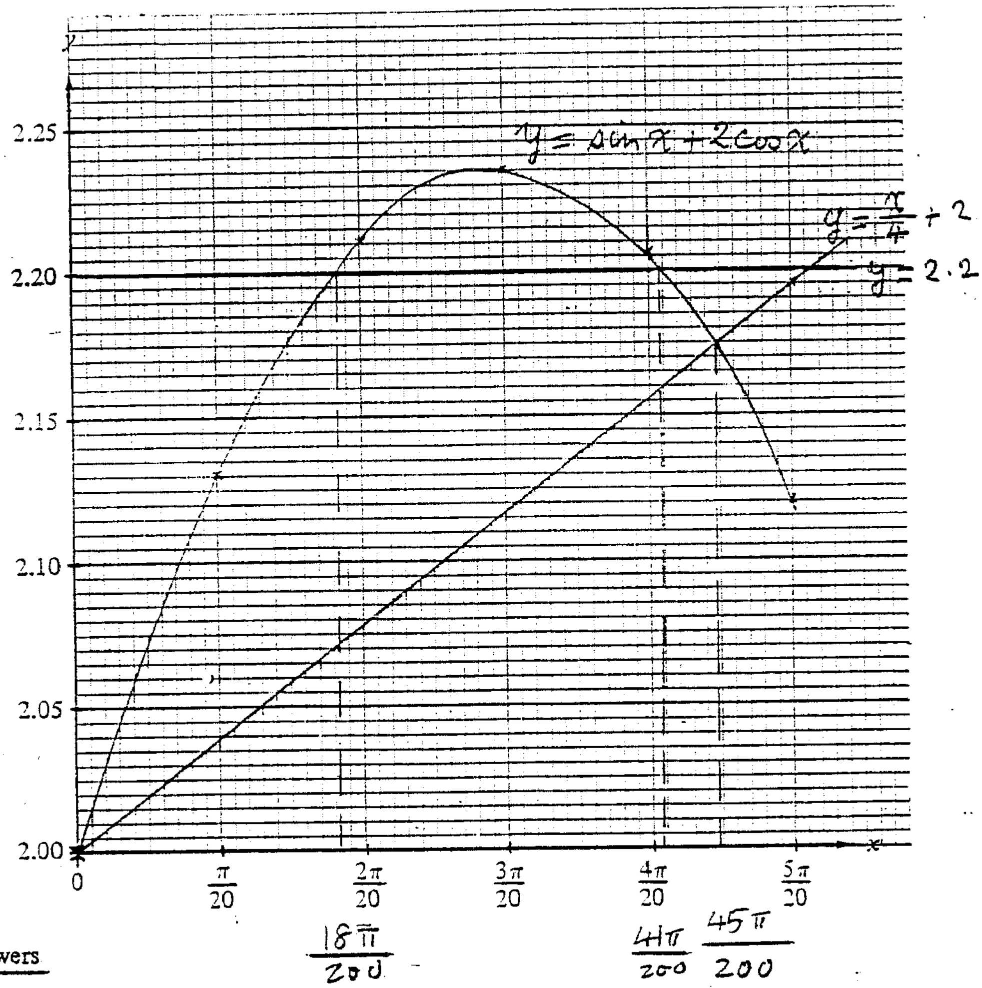
SOLUTION	MARKS	REMARKS
3. (a) $\sin 2\theta + \sin 8\theta = \sin 5\theta$		
$2\sin 5\theta \cos 3\theta = \sin 5\theta$	1A	
$\underline{\sin 5\theta\ (2\cos 3\theta - 1) = 0}$		must be correct
$\sin 5\theta = 0$ or $\cos 3\theta = \frac{1}{2}$	1+1A	
$5\theta = n\pi$ or $3\theta = 2n\pi \pm \frac{\pi}{3}$		
$\therefore\ \theta = \frac{n\pi}{5}$ $(36n°)$	1A	awarded if either one answer
or $\frac{(6n\pm1)\pi}{9}$ $(120n° \pm 20°)$, $\underline{n = 0, \pm1, \pm2, \ldots}$	1A+$\underline{1A}$	for θ is correct.
	6	
(b) $x = \frac{4\pi}{20}$, $y = 2.206$	1A	
$x = \frac{5\pi}{20}$, $y = 2.121$	1A	
Curve of $y = \sin x + 2\cos x$	3	Shape 2 curved line 1
(i) $5\sin x + 10\cos x = 11$		
$\Leftrightarrow \sin x + 2\cos x = 2.2$		
Consider the line $y = 2.2$	1A+1A	1A for equation 1A for line
The solutions are:		
$x = \frac{18\pi}{200}$ (or $\frac{19\pi}{200}$), $\frac{41\pi}{200}$ 0.267-0.299 0.628-0.660	1+1A	[illegible] $\frac{\text{decimal}}{200}\pi$ etc.
(ii) Consider the line $y = \frac{x}{4} + 2$	1A+1A	1A for equation 1A for line
$x = 0$, $y = 2.000$		
$x = \frac{5\pi}{20}$, $y = 2.196$		
The solutions are $x = 0$,	1A	
$\frac{44\pi}{200}$ $(\frac{45\pi}{200})$ 0.675 - 0.707	2A	
	14	

Candidate Number	Centre Number	Seat Number	Total Marks on this page	

8.(b) If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your answer book.

Table 1

x	0	$\frac{\pi}{20}$	$\frac{2\pi}{20}$	$\frac{3\pi}{20}$	$\frac{4\pi}{20}$	$\frac{5\pi}{20}$
$y = \sin x + 2\cos x$	2.000	2.132	2.211	2.236	2.206	2.121



Answers

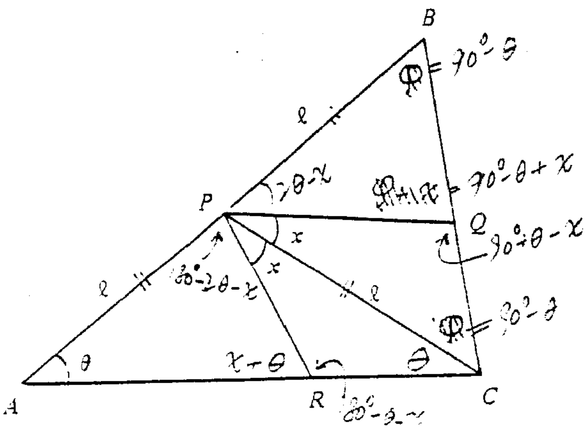
(i) ____________________

(ii) ____________________

SOLUTION	MARKS	REMARKS
9. (a) Equation of L is		
$y - 3 = m(x - 0)$ or $y = mx + 3$	1A	
Substituting in $x^2 + 4y^2 = 4$	1M	
$x^2 + 4(mx + 3)^2 = 4$		
$(4m^2+1)x^2 + 24mx + 32 = 0$	1A	
Discriminant $= (24m)^2 - 4(4m^2+1)32$	1M	
L cuts C at two real points iff		
$(24m)^2 - 4(4m^2+1)32 > 0$	1M	
$64m^2 - 128 > 0$		
$m^2 > 2$	1A	
$\therefore\ m > \sqrt{2}$ or $m < -\sqrt{2}$ 2A	~~1+1A~~	no mark for "and"; comma — C.
If L touches C, $m = \pm\sqrt{2}$	1A	
Equations of tangents from P are		
$y = \sqrt{2}x + 3$ and $y = -\sqrt{2}x + 3$	1A	
	10	
(b) $2x + 8y\dfrac{dy}{dx} = 0$	1A	
$\dfrac{dy}{dx} = -\dfrac{x}{4y}$	1A	
At $(2\cos\theta, \sin\theta)$,		
gradient $= -\dfrac{\cos\theta}{2\sin\theta}$	1M+1A	
$= -\dfrac{1}{2}\cot\theta$		
$\therefore$ the equation of the tangent T is		
$y - \sin\theta = -\dfrac{1}{2}\cot\theta\,(x - 2\cos\theta)$	1A	
or $x\cos\theta + 2y\sin\theta = 2$		
Distance from P(0, 3) to the tangent		for any equivalent form
is $d = \left\|\dfrac{6\sin\theta - 2}{\sqrt{\cos^2\theta + 4\sin^2\theta}}\right\| = \left\|\dfrac{6\sin\theta - 2}{\sqrt{3\sin^2\theta + 1}}\right\|$	2A	Abs. value optional
(i) when $\theta = \dfrac{3\pi}{2}$, $d = \left\|\dfrac{6(-1) - 2}{\sqrt{3(-1)^2 + 1}}\right\| = 4$	1A	Accept −4
(ii) when $\sin\theta = \dfrac{1}{3}$, $d = 0$	1A	
i.e. P lies on the tangent	1A	
	10	

SOLUTION	MARKS	REMARKS
10. (a) Put $x = a\sin\phi$, $dx = a\cos\phi\, d\phi$	1A	
when $x = a$, $\phi = \frac{\pi}{2}$; when $x = -a$, $\phi = -\frac{\pi}{2}$	1A	must be in radians
$\int_{-a}^{a}\sqrt{a^2 - x^2}\,dx$ $= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sqrt{a^2-a^2\sin^2\phi}\,.\,a\cos\phi\,d\phi$		
$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} a^2\cos^2\phi\,d\phi = \frac{a^2}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(1 + \cos 2\phi)\,d\phi$	1A+1A	For integrands only
$= \frac{a^2}{2}\left[\phi + \frac{\sin 2\phi}{2}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ $= \frac{\pi a^2}{2}$	1A	
	5	
(b) (i) Equation of the circle is $x^2 + (y-b)^2 = a^2$ or $(y-b)^2 = a^2-x^2$	1A	
∴ equation of APB is $y - b = \sqrt{a^2 - x^2}$ or $y = b + \sqrt{a^2 - x^2}$	1A	
Equation of AQB is $y - b = -\sqrt{a^2 - x^2}$ or $y = b - \sqrt{a^2 - x^2}$	1A	
	3	
(ii) Volume = $\int_{-a}^{a}\pi(b + \sqrt{a^2-x^2})^2dx - \int_{-a}^{a}\pi(b - \sqrt{a^2-x^2})^2dx$	1+1M +1A	1M for $V = \int_{\alpha}^{\beta}\pi y^2dx$ 1M for " – " 1A for limits
$= \pi\int_{-a}^{a}[b^2 + 2b\sqrt{a^2-x^2} + (a^2-x^2)]dx - \pi\int_{-a}^{a}[b^2 - 2b\sqrt{a^2-x^2} + (a^2-x^2)]dx$		
$= \pi\int_{-a}^{a}4b\sqrt{a^2-x^2}\,dx$	1A	
$= 4\pi b \times \frac{\pi a^2}{2} = 2\pi^2a^2b$	1A	
	5	
(c) Volume $= 2\pi^2(2)^2(8) = 64\pi^2$ (mm^3)	1A	Vol $= 64\pi^2(cm^3)$ 1
$V = \int -32\pi^2(2-t)\,dt$	1M	$\int_{64\pi^2}^{V} dV = \int_0^t -32\pi^2(2-t)\,dt$ ......1M- ~~1A~~ 1M (limits)
$= 16\pi^2t^2 - 64\pi^2t + c$	1A	$V - 64\pi^2 = [-64\pi^2 t + 16\pi^2t^2]_0^t$ 1
When $t = 0$, $V = 64\pi^2$	1M	∴ $V = 16\pi^2t^2 - 64\pi^2t + 64\pi^2$ 1
∴ $c = 64\pi^2$	1A	etc.
∴ $V = 16\pi^2t^2 - 64\pi^2t + 64\pi^2$		
Putting $V = 0$ $16\pi^2(t^2 - 4t + 4) = 0$	1M	
$(t - 2)^2 = 0$ $t = 2$	1A	
∴ the piece of sweet dissolves completely in 2 hours		
	7	

SOLUTION	MARKS	REMARKS
11. (a) $PA = PC \Rightarrow \angle PCA = \theta$	1A	Alternatively: $PA = PC \Rightarrow \angle PCA = \theta$ ....
$\therefore \angle PRA = x + \theta$	1A	$\therefore \angle PRC = \pi - (x + \theta)$ ....
In $\triangle PRA$, $\dfrac{PR}{\sin\theta} = \dfrac{l}{\sin(x+\theta)}$	1M	In $\triangle PRC$, $\dfrac{PR}{\sin\theta} = \dfrac{l}{\sin(\pi-(x+\theta))}$ 1
$\therefore PR = \dfrac{l\sin\theta}{\sin(x+\theta)}$	1A	$\therefore PR = \dfrac{l\sin\theta}{\sin(x+\theta)}$ 1
	4	
can be shown on diagram (b) $PC = PB \Rightarrow \angle PCQ = \angle PBQ \quad (= \phi)$	1A	Alternatively: $\angle PCQ = \angle PBQ$ 1
$\therefore \angle PQB = x + \phi$ $\quad \phi = \frac{\pi}{2} - \theta$	1A	$2(\theta+\phi) = \pi \Rightarrow \phi = \frac{\pi}{2} - \theta$ 1
In $\triangle PQB$, $\dfrac{PQ}{\sin\phi} = \dfrac{l}{\sin(x+\phi)}$	1M	(or $\triangle$ in semicircle) In $\triangle PCQ$, $\dfrac{PQ}{\sin(\frac{\pi}{2}-\theta)} = \dfrac{l}{\sin(\pi - x - (\frac{\pi}{2}-\theta))}$ 1
$\therefore PQ = \dfrac{l\sin\phi}{\sin(x+\phi)} = \dfrac{l\cos\theta}{\cos(x-\theta)}$	1A	$\therefore PQ = \dfrac{l\cos\theta}{\cos(x-\theta)}$ ....
	4	
(c) Area of $\triangle PQR = \frac{1}{2} PQ \cdot PR \sin 2x$	1M	
$= \dfrac{l^2 \sin\theta \cos\theta \sin 2x}{2\sin(x+\theta)\cos(x-\theta)}$	1A	
$= \dfrac{l^2}{2} \cdot \dfrac{\sin 2\theta \sin 2x}{\sin 2x + \sin 2\theta}$	1A	
$= \dfrac{l^2 \sin 2\theta}{2}\left(\dfrac{\sin 2x + \sin 2\theta - \sin 2\theta}{\sin 2x + \sin 2\theta}\right)$ $= \dfrac{l^2 \sin 2\theta}{2}\left(1 - \dfrac{\sin 2\theta}{\sin 2x + \sin 2\theta}\right) \ldots (*)$	1A	working necessary
	4	



No penality if 180° is written as

SOLUTION	MARKS	REMARKS
11. (d) (i) Let $\theta = \frac{\pi}{8}$		
$\phi = \frac{\pi}{2} - \theta = \frac{3\pi}{8}$	1A	
$0 < x \leqslant \pi - 2\theta$ and $0 < x \leqslant \pi - 2\phi$	1A	
$0 < x \leqslant \frac{\pi}{4}$	1A	Accept $0 \leqslant x \leqslant \frac{\pi}{4}$, $x \leqslant \frac{\pi}{4}$
$0 < \sin 2x \leqslant 1$		
The maximum area of $\triangle PQR$ is		
$= \frac{l^2 \sin 2\theta}{2}\left(1 - \frac{\sin 2\theta}{1 + \sin 2\theta}\right)$	1M	Check candidate's range of x
$= \frac{l^2 \sin\frac{\pi}{4}}{2}\left(1 - \frac{\sin\frac{\pi}{4}}{1 + \sin\frac{\pi}{4}}\right)$		
$= \frac{l^2}{2(1+\sqrt{2})}$ $\left(\frac{l^2(\sqrt{2}-1)}{2}\right.$ or $\left.0.207 l^2\right)$	1A	Any figure roundable to $0.207 l^2$
(ii) If $\theta = \frac{\pi}{12}$, then		
$\phi = \frac{5\pi}{12}$ and $0 < x \leqslant \frac{\pi}{6}$	1A	
$\therefore$ the maximum area of $\triangle PQR$		
$= \frac{l^2 \sin\frac{\pi}{6}}{2}\left(1 - \frac{\sin\frac{\pi}{6}}{\sin\frac{\pi}{3} + \sin\frac{\pi}{6}}\right)$	1M	
$= \frac{l^2}{4}\left(1 - \frac{1}{\sqrt{3}+1}\right)$		5 for either (i) or (ii)
$= \frac{l^2\sqrt{3}}{4(\sqrt{3}+1)}$ $\left(\frac{l^2\sqrt{3}(\sqrt{3}-1)}{8}\right.$ or $\left.0.158 l^2\right)$	1A	3 for the other
	8	