HONG KONG EXAMINATIONS AUTHORITY

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1984

附加數學　試卷二
ADDITIONAL MATHEMATICS PAPER II

11.15 am–1.15 pm (2 hours)

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

SECTION A (39 marks)

Answer ALL questions in this section.

1. In the expansion of $\left(x^2 + \frac{a}{x}\right)^8$, where $a \neq 0$, the coefficient of x^r is denoted by B_r. Find the value of a if $B_7 = 4B_{10}$.

(5 marks)

2. Prove by mathematical induction that, for all positive integers n, $4n^3 - n$ is divisible by 3.

(6 marks)

3. The slope at any point (x, y) of a curve is given by

$$\frac{dy}{dx} = 4\sin^2 x + 1 .$$

If the curve cuts the x-axis at $x = \frac{\pi}{2}$, find the equation of the curve.

(6 marks)

4. The area of the triangle bounded by the two lines $x + y = 4$ and $x - y = 2p$ and the y-axis is 9. Find the two values of p.

(6 marks)

5. Making use of the derivative of $\tan^3\theta$, find

$$\int \tan^2\theta \sec^2\theta \, d\theta .$$

Hence evaluate $\int_0^{\frac{\pi}{3}} \tan^4\theta \, d\theta$

(8 marks)

6. Given the equation $x^2 + y^2 - 2kx + 4ky + 6k^2 - 2 = 0$.

(a) Find the range of values of k so that the equation represents a circle with radius greater than 1 .

(b) Find the locus of the centre of the circle as k varies within the range in (a) .

(8 marks)

SECTION B (60 marks)

Answer any THREE questions from this section.
Each question carries 20 marks.

7. (a) Prove that $\frac{1}{x^3} + \frac{3}{(2-3x)^2} = \frac{3x^3 + 9x^2 - 12x + 4}{9x^5 - 12x^4 + 4x^3}$.

Hence find the value of $\int_1^2 \frac{3x^3 + 9x^2 - 12x + 4}{9x^5 - 12x^4 + 4x^3} \, dx$.

(7 marks)

(b) (i) Using the substitution $u = \sin\phi$, find $\int \frac{\cos\phi}{\sin^4\phi} \, d\phi$.

(ii) Using the substitution $x = \tan\phi$ and the result of (i) , evaluate

$$\int_{\frac{1}{\sqrt{3}}}^{1} \frac{3\sqrt{1+x^2}}{x^4} \, dx .$$

(13 marks)

8. **If you attempt this question, you should refer to the separate supplementary leaflet provided.**

(a) Find the general solution of the equation

$$\sin 2\theta + \sin 8\theta = \sin 5\theta .$$

(6 marks)

(b) Let $y = \sin x + 2\cos x$. Complete Table 1 on the separate answer sheet provided and use the data to plot the graph of

$$y = \sin x + 2\cos x .$$

By adding two suitable straight lines to the graph, find the solutions of the equations

(i) $5\sin x + 10\cos x = 11$,

(ii) $\sin x + 2\cos x = \frac{x}{4} + 2$.

Give your answers correct to the nearest $\frac{\pi}{200}$.

(14 marks)

9. Given the curve $C : x^2 + 4y^2 = 4$ and the point $P(0, 3)$.

(a) L is a line of variable slope m through P. If L cuts C at two distinct real points, find the possible range of values of m.

If L touches C, what are the possible values of m?

Hence write down the equations of the two tangents from P to C.

(10 marks)

(b) $Q(2\cos\theta, \sin\theta)$ is a point on C. Find by differentiation the gradient of C at Q and hence show that the equation of the tangent T at Q is

$$x\cos\theta + 2y\sin\theta = 2 .$$

Express the distance from P to the tangent T in terms of θ.

Find the distance when

(i) $\theta = \frac{3\pi}{2}$,

(ii) $\sin\theta = \frac{1}{3}$.

Interpret case (ii) geometrically.

(10 marks)

10. (a) Use the substitution $x = a\sin\phi$ to show that

$$\int_{-a}^{a} \sqrt{a^2 - x^2}\, dx = \frac{\pi a^2}{2} .$$

(5 marks)

(b) Figure 1 shows two semicircles APB and AQB with a common centre $C(0, b)$ and equal radii a. AB is parallel to the x-axis.

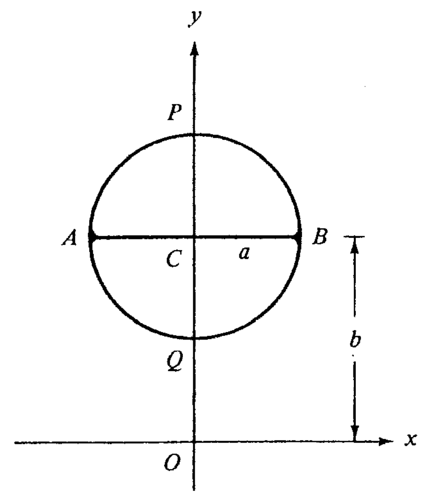

Figure 1

(i) Show that the equation of APB is

$$y = b + \sqrt{a^2 - x^2}$$

and that of AQB is

$$y = b - \sqrt{a^2 - x^2} .$$

(ii) The region bounded by the two semicircles is revolved about the x-axis to generate a solid (called an anchor-ring). Use the result in (a) to prove that the volume of the anchor-ring is $2\pi^2 a^2 b$.

(8 marks)

(c) A sweet has the form of an anchor-ring with $a = 2$ mm and $b = 8$ mm. Write down its volume in terms of π.

The sweet is now dropped into water and it dissolves with a rate of change of volume given by

$$\frac{dV}{dt} = -32\pi^2(2-t) \text{ mm}^3/\text{h},$$

where V is the volume in mm^3, t is the time in hours.

Find V in terms of t and hence find the time required to dissolve the whole sweet completely.

(7 marks)

11. In Figure 2, ABC is a triangle with $\angle A = \theta$. P is a point on AB such that $PA = PB = PC = \ell$. R and Q are points on AC and BC, respectively, such that $\angle QPC = \angle RPC = x$.

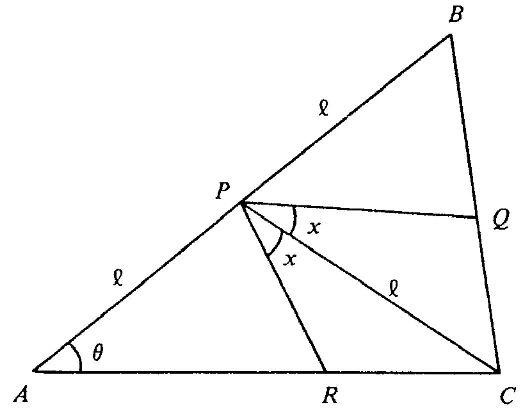

Figure 2

(a) Show that $PR = \dfrac{\ell \sin\theta}{\sin(x+\theta)}$.

(4 marks)

(b) Find $\angle PCQ$ in terms of θ and hence find PQ in terms of ℓ, x and θ.

(4 marks)

(c) Show that the area of $\Delta PQR = \dfrac{\ell^2 \sin\theta \cos\theta \sin 2x}{2\sin(x+\theta)\cos(x-\theta)}$,

and show that it can be expressed as

$$\frac{\ell^2 \sin 2\theta}{2}\left(1 - \frac{\sin 2\theta}{\sin 2x + \sin 2\theta}\right) \quad \text{..........................}(*)$$

(4 marks)

(d) (i) If $\theta = \frac{\pi}{8}$, find the possible range of values of x.

Hence use (*) to deduce the maximum area of ΔPQR and express it in terms of ℓ.

(ii) If $\theta = \frac{\pi}{12}$, what is the possible range of values of x?

Express the maximum area of ΔPQR in terms of ℓ.

(8 marks)

END OF PAPER

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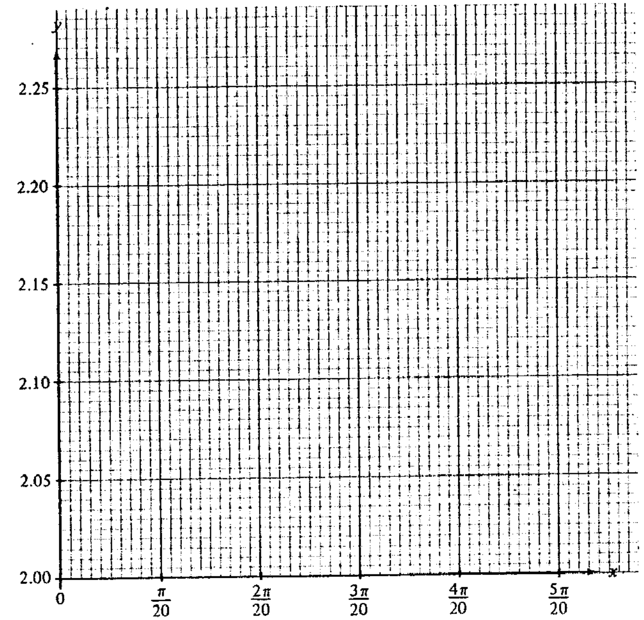
附加數學　試卷二(附頁)
ADDITIONAL MATHEMATICS PAPER II
(SUPPLEMENTARY LEAFLET)

Candidate Number	Centre Number	Seat Number	Total Marks on this page

8.(b) If you attempt this question, fill in the details in the first three boxes above and tie this sheet into your answer book.

Table 1

x	0	$\frac{\pi}{20}$	$\frac{2\pi}{20}$	$\frac{3\pi}{20}$	$\frac{4\pi}{20}$	$\frac{5\pi}{20}$
$y = \sin x + 2\cos x$	2.000	2.132	2.211	2.236		



Answers

(i)

(ii)

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Additional Mathematics I

1. (a) $-\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{j}$
   (b) $(3 - 4m)\mathbf{i} + (3m - 2)\mathbf{j}$

2. $12\ \text{cm}^3/\text{s}$

3. (a) $41 + 38i$
   (b) $\text{Re}(\frac{1}{z}) = \frac{41}{3125}$
   $\text{Re}(z + \frac{1}{z}) = 41$
   (correct to the nearest integer)

4. $-\frac{3}{2} \leq x \leq \frac{3}{2}$

5. (b) $\sqrt{7}$

6. $AP = \frac{2}{3}h$

7. (a) $\frac{k - m}{\sqrt{(m^2 + 2m + 2)(k^2 + 2k + 2)}}$
   (b) (i) $\frac{1+k}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}$
   (ii) $r\mathbf{i} + r(1 + m)\mathbf{j}$
   (iii) $r = \frac{2}{5}$
   $m = k = 1$

8. (b) (ii) $b = -5$
   $0 < c < \frac{5}{4}$

10. (a) $V = \frac{8}{3}x^2\sqrt{1 - x}$
    (b) $(0, 0)$, $(\frac{4}{5}, \frac{128}{75\sqrt{5}})$
    $V = 0$, $V = \frac{128}{75\sqrt{5}}$, $x = 1$

11. (a) $N = 2h\sec\theta + (50 - h\tan\theta)$
    (c) (ii) Goods should be transported directly from C to A by truck.

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Additional Mathematics II

1. 2

3. $y = 3x - \sin 2x - \frac{3\pi}{2}$

4. $p = 1$ or -5

5. $\frac{1}{3}\tan^3\theta + c$
   $\frac{\pi}{3}$

6. (a) $-1 < k < 1$
   (b) The locus is a line segment with end-points $(-1, 2)$ and $(1, -2)$ excluded.

7. (a) $\frac{9}{8}$
   (b) (i) $-\frac{1}{3\sin^3\phi} + c$
   (ii) $8 - 2\sqrt{2}$

8. (a) $\theta = \frac{n\pi}{5}$ or $\frac{(6n \pm 1)\pi}{9}$,
   $n = 0, \pm 1, \pm 2, \ldots$
   (b) $x = \frac{4\pi}{20}$, $y = 2.206$
   $x = \frac{5\pi}{20}$, $y = 2.121$
   (i) $\frac{18\pi}{200}$, $\frac{41\pi}{200}$
   (ii) 0, $\frac{44\pi}{200}$

9. (a) $m > \sqrt{2}$ or $m < -\sqrt{2}$
   $m = \pm\sqrt{2}$
   $y = \sqrt{2}x + 3$, $y = -\sqrt{2}x + 3$
   (b) $d = \left|\frac{6\sin\theta - 2}{\sqrt{3\sin^2\theta + 1}}\right|$
   (i) 4
   (ii) 0, P lies on the tangent

10. (c) $64\pi^2\ \text{mm}^3$
    $V = 16\pi^2 t^2 - 64\pi^2 t + 64\pi^2$
    2 hours

11. (b) $\angle PCQ = \frac{\pi}{2} - \theta$
    $PQ = \frac{\ell\cos\theta}{\cos(x - \theta)}$
    (d) (i) $0 < x \leq \pi - 2\theta$ and
    $0 < x \leq \pi - 2\phi$
    Maximum area $= \frac{\ell^2}{2(1 + \sqrt{2})}$
    (ii) $0 < x \leq \frac{\pi}{6}$
    Maximum area $= \frac{\ell^2\sqrt{3}}{4(\sqrt{3} + 1)}$