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ADD. MATHEMATICS PAPER 2
MARKING SCHEME

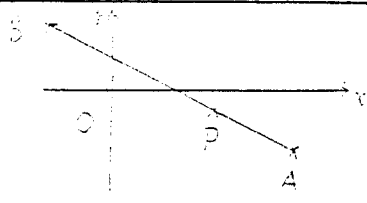
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SOLUTIONS	MARKS	REMARKS
1. (a) $\vec{AB} = \vec{OB} - \vec{OA}$ $= (-\vec{i} - \vec{j}) - (3\vec{i} - 2\vec{j})$ $= -4\vec{i} + 3\vec{j}$ The unit vector $= \frac{-4\vec{i} + 3\vec{j}}{\sqrt{(-4)^2 + 3^2}}$ $= -\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}$	1 1A 1M 1A	 <p>Method to find unit vector</p>
(b) $\vec{OP} = \vec{OA} + m\vec{AB}$ $= \vec{OA} + m\vec{AB}$ $= (3\vec{i} - 2\vec{j}) + (-4m\vec{i} + 3m\vec{j})$ $= (3 - 4m)\vec{i} + (3m - 2)\vec{j}$	1 1M 1A	<p>Alternatively: $\vec{OP} = m\vec{b} + (1-m)\vec{a}$.....2A $= m(-\vec{i} + \vec{j}) + (1-m)(3\vec{i} - 2\vec{j})$ $= (3-4m)\vec{i} + (3m-2)\vec{j}$.....1A</p>
7		
2. $S = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$ $\frac{dS}{dt} = \frac{dS}{dr} \cdot \frac{dr}{dt} = 8\pi r \cdot \frac{dr}{dt}$ $3 = 8\pi r \cdot \frac{dr}{dt}$ At $S = 36\pi$, $r = 3$ $\frac{dr}{dt} = \frac{3}{8\pi r} = \frac{1}{3\pi}$ $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$ $= 4\pi r^2 \cdot \frac{dr}{dt}$ At $S = 36\pi$, $\frac{dV}{dt} = 36\pi \cdot \frac{1}{3\pi}$ $= 12$	1A+1A 1M 1A 1A 1A 1M	<p>Equate $\frac{dS}{dt} = 3$</p> <p>Sub for r</p>
The volume is increasing at a rate of 12 cm ³ /s	1A	
8		
<p><u>Alternatively</u></p> $S = 4\pi r^2$ $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \cdot \left(\frac{S}{4\pi}\right)^{\frac{3}{2}}$ $\frac{dV}{dt} = \frac{dV}{dS} \cdot \frac{dS}{dt}$ $= \frac{4}{3}\pi \cdot \left(\frac{1}{4\pi}\right)^{\frac{3}{2}} \cdot \frac{3}{2} S^{\frac{1}{2}} \cdot \frac{dS}{dt}$ $= \frac{4}{3}\pi \left(\frac{1}{4\pi}\right)^{\frac{3}{2}} \cdot \frac{3}{2} (36\pi)^{\frac{1}{2}} \cdot 3$ $= 12$	1M+2A 1 1A 1M+1A 1A	<p>1M for attempt to eliminate r</p> <p>1M for sub $S = 36\pi$, $\frac{dS}{dt} = 3$</p>
The volume is increasing at a rate of 12 cm ³ /s	1A	

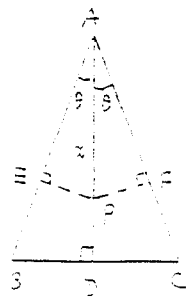
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SOLUTIONS	MARKS	REMARKS
<p>4. $2 x - 1 < 2$</p> $-2 \leq 2 x - 1 \leq 2$ $-1 \leq 2 x \leq 3$ $-\frac{1}{2} \leq x \leq \frac{3}{2} \quad (\text{or } x \leq \frac{3}{2})$ $\therefore -\frac{3}{2} \leq x \leq \frac{3}{2}$	<p>1A-1A</p> <p>1A</p> <p>1A+1A</p>	<p>-1 if strict inequality '$<$' given in any line</p>
3		
<u>Alternatively</u>		
<p>(i) Let $2 x - 1 \geq 0$, then $x \geq \frac{1}{2}$</p> <p>i.e. $x \geq \frac{1}{2}$ or $x \leq -\frac{1}{2}$</p> $ 2 x - 1 \leq 2 \implies 2 x - 1 \leq 2$ $\implies x \leq \frac{3}{2}$ $\implies -\frac{3}{2} \leq x \leq \frac{3}{2}$	<p>1A</p>	
Combining with the assumption,		
$\frac{1}{2} \leq x \leq \frac{3}{2} \text{ or } -\frac{3}{2} \leq x \leq -\frac{1}{2}.$	<p>1A</p>	
<p>(ii) Let $2 x - 1 < 0$, then $x < \frac{1}{2}$</p> <p>i.e. $-\frac{1}{2} < x < \frac{1}{2}$.</p> $ 2 x - 1 \leq 2 \implies 1 - 2 x \leq 2$ $\implies -\frac{1}{2} \leq x $	<p>1A</p>	
This is true for all x .		
$\therefore -\frac{1}{2} < x < \frac{1}{2}.$	<p>1A</p>	
Combining (i) and (ii),		
$-\frac{3}{2} \leq x \leq \frac{3}{2}$	<p>1A</p>	

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SOLUTIONS	MARKS	REMARKS
<p>5. (a) $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$</p> <p>$= 2^2 - 4(m^2 - m + 1)$ (sub $\begin{matrix} \alpha + \beta \\ \alpha\beta \end{matrix}$) \rightarrow</p> <p>$= 4(m^2 - m + 2)$</p> <p>$= 4[(m - \frac{1}{2})^2 + \frac{7}{4}]$ completing square \rightarrow</p> <p>> 0 ($\forall m \in \mathbb{R}$)</p>	<p>1A</p> <p>1M+1M</p> <p>1M+1A</p>	<p><u>Alternatively:</u></p> <p>$D = 4 + 4(m^2 - m + 1)$</p> <p>$= 4(m^2 - m + 2) \dots \dots \dots 1A$</p> <p>$> 0$ because the discriminant of $4(m^2 - m + 2) = 0$ is negative and $\dots \dots \dots 1M$ coeff. of m^2 is positive. $\dots \dots \dots 1M$</p> <p>$\therefore \alpha, \beta$ are real & distinct. $\dots \dots \dots 1M$</p> <p>Hence $(\alpha - \beta)^2 > 0 \dots \dots \dots 1A$</p>
<p>(i) Since $(\alpha - \beta)^2$ is real, $\alpha - \beta = \sqrt{(\alpha - \beta)^2}$</p> <p>Minimum value of $\alpha - \beta$ is $\sqrt{7}$.</p>	<p>1M+1A</p>	<p>1M for $(m - \frac{1}{2})^2 = 0$</p>

<p>Let $AP = x$. Since $AB = AC$, $AD \perp BC$ and $\angle BAD = \angle CAD = \theta$</p> <p>$PE = PF = x \sin \theta$.</p> <p>Product of distances $p = x^2 \sin^2 \theta (h - x)$</p> <p>$\frac{dp}{dx} = \sin^2 \theta (2xh - 3x^2)$</p> <p>$= x \sin^2 \theta (2h - 3x)$</p> <p>$\frac{dp}{dx} = 0 \iff x = 0$ or $\frac{2}{3}h$</p> <p>At $x = \frac{2}{3}h$, $\frac{dp}{dx}$ changes sign from +ve to -ve.</p> <p>$\therefore p$ is a maximum at $x = \frac{2}{3}h$.</p>	<p>1</p> <p>1A</p> <p>1M</p> <p>1M+1A</p> <p>1A</p>	<p><u>Alternatively:</u></p> <p>Let $PD = x$.</p> <p>$AD \perp BC$, $\angle BAD = \angle CAD = \theta$</p> <p>$PE = PF = (h - x) \sin \theta$</p> <p>$p = x(h - x)^2 \sin^2 \theta$</p> <p>$\frac{dp}{dx} = (h^2 - 4hx + 3x^2) \sin^2 \theta$</p> <p>$= (h - 3x)(h - x) \sin^2 \theta = 0$</p> <p>$\frac{dp}{dx} = 0 \iff x = h$ or $\frac{1}{3}h$</p> <p>Max. at $x = \frac{1}{3}h$</p> <p>(working necessary)</p>
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SOLUTIONS	MARKS	REMARKS
3. (a) $f\left(\frac{1}{2}\right) = 5\left(\frac{1}{2}\right)^2 - b\left(\frac{1}{2}\right) - c$	1	
$\therefore \frac{5}{4} - \frac{b}{2} - c < 0$	1A	
Consider $5x^2 + bx + c = 0$		
Discriminant = $b^2 - 20c$	1A	
$> b^2 + 20\left(\frac{5}{4} + \frac{b}{2}\right)$	1M	(sub. c)
$= b^2 + 10b - 25$		
$= (b - 5)^2$	1M	
Thus the discriminant is always positive		
$\therefore f(x) = 0$ has two distinct real roots.	1A	
	6	
(b) (i) $f(x) = 5x^2 + bx + c$		
$= 5(x - \alpha)(x - \beta)$	2A	The omission of the factor 5 will not be penalised again.
Since $f\left(\frac{1}{2}\right) < 0$		
$\left(\frac{1}{2} - \alpha\right)\left(\frac{1}{2} - \beta\right) < 0$	1M	
\therefore either $\beta < \frac{1}{2} < \alpha$ or $\alpha < \frac{1}{2} < \beta$	1A	Accept " , "
Since $\alpha < \beta$,		
$\alpha < \frac{1}{2} < \beta$	1A	Explanation necessary
Further $\alpha\beta = \frac{c}{5}$	1	
$\therefore \alpha\beta > 0$ ($\alpha > 0$)	1A	
and $\alpha > 0$ as $\beta > 0$	1A	
$\therefore 0 < \alpha < \frac{1}{2} < \beta$	8	
(b) (ii) $ \alpha - \frac{1}{2} = \beta - \frac{1}{2} $		
$\Rightarrow \frac{1}{2} - \alpha = \beta - \frac{1}{2}$	1A	
$\Rightarrow \alpha + \beta = 1$	1A	
$\therefore b = -5(\alpha + \beta)$	1M	
$= -5$	1A	
$\frac{5}{4} + \frac{b}{2} - c < 0 \Rightarrow c < \frac{5}{4}$	1M	
$\therefore 0 < c < \frac{5}{4}$	1A	
	6	

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SOLUTION	MARKS	REMARKS
9. (a) (i) $w^3 = 1$ $\Rightarrow w^3 - 1 = 0$ $\Rightarrow (w - 1)(w^2 + w + 1) = 0$ $\Rightarrow w^2 + w + 1 = 0$ since $w \neq 1$	1 1A 1A	must mention $w \neq 1$
(ii) $(w^2)^{3k+1} + w^{3k+1} + 1$ $= (w^3)^{2k} \cdot w^2 + (w^3)^k w + 1$ $= w^2 + w + 1 = 0$	1M 1A	Factorise powers of w^3
$(w^2)^{3k+2} + w^{3k+2} + 1$ $= (w^3)^{2k} \cdot w^4 + w^{3k} w^2 + 1$ $= w^4 + w^2 + 1$ $= w^2 + w + 1 = 0$	1A	
	6	
(b) $ 1 - w\bar{z} ^2 = (1 - w\bar{z})(1 - w\bar{z})$ $= (1 - w\bar{z})(1 - \bar{w}z)$ $= 1 - w\bar{z} - \bar{w}z + w\bar{w}z\bar{z}$ $= 1 - w\bar{z} - \bar{w}z + z\bar{z}$	1 1A	
$ z - w ^2 = (z - w)(\bar{z} - \bar{w})$ $= z\bar{z} - z\bar{w} - \bar{z}w + w\bar{w}$ $= z\bar{z} - z\bar{w} - \bar{z}w + 1$	1 1A	
$\therefore 1 - w\bar{z} = z - w $	1A	
	5	
(c) $ 1 - w\bar{z} = c$ $\Leftrightarrow z - w = c$ which is a circle with centre w and radius c	2 1A 1A	
Let $w_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ ($= \text{cis } \frac{2}{3}\pi$) $w_2 = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$ ($= \text{cis } -\frac{3}{2}\pi$)	1A 1A	
	3	Deduct 1 mark for omission of each of the following features: (i) 2 circles in appropriate quadrants (ii) radius = $\frac{1}{2}$ (iii) touching y -axis (iv) not cutting x -axis note (iii) \Rightarrow (ii)
	9	

(85)

Alternative 1:

Let $z = x + iy$, $w = \frac{-1}{2} + \frac{\sqrt{3}}{2}i$

$w\bar{z} = (\frac{-1}{2} + \frac{\sqrt{3}}{2}i)(x - iy)$
 $= (\frac{-x}{2} + \frac{\sqrt{3}y}{2}) + (\frac{y}{2} + \frac{\sqrt{3}x}{2})i$ 1A

$|1 - w\bar{z}|^2 = (1 + \frac{x}{2} - \frac{\sqrt{3}y}{2})^2 + (\frac{y}{2} + \frac{\sqrt{3}x}{2})^2$
 $= 1 + \frac{x^2}{4} + \frac{3y^2}{4} + x - \sqrt{3}y - \frac{\sqrt{3}xy}{2} + \frac{y^2}{4} + \frac{3x^2}{4} + \frac{\sqrt{3}xy}{2}$
 $= x^2 + y^2 + x - \sqrt{3}y + 1$ 1A

$|z - w|^2 = (x^2 - x + \frac{1}{4}) + (y^2 - \sqrt{3}y - \frac{3}{4})$
 $= x^2 + y^2 + x - \sqrt{3}y - 1$ 1A

$\therefore |1 - w\bar{z}| = |z - w|$ 1A

Let $w = \frac{-1}{2} - \frac{\sqrt{3}}{2}i$,

$|1 - w\bar{z}|^2 = \text{etc.}$

Answer1A

Alternative 2:

Let $z = r\text{cis } \theta$

$w = \text{cis } \phi$

$|1 - w\bar{z}|^2 = 1 - 2|w\bar{z}| \cos(\phi - \theta)$ 1A
 $= 1 - 2|z| \cos(\phi - \theta)$ 1A

$|w - z|^2 = |w|^2 + |z|^2 - 2|w||z| \cos(\phi - \theta)$ 1A
 $= 1 + |z|^2 - 2|z| \cos(\phi - \theta)$ 1A

$\therefore |1 - w\bar{z}| = |w - z|$ 1A

Alternative 3:

$|1 - w\bar{z}| = |w(\frac{1}{w} - \bar{z})|$ 1A

$= |\frac{1}{w} - \bar{z}|$ ($|w| = 1$)1A

$= |\bar{w} - \bar{z}|$ 1A

$= |w - z|$ 1A

$= |z - w|$ 1A

Alternative 4:

$|1 - w\bar{z}| = |w\bar{w} - w\bar{z}|$ 1A

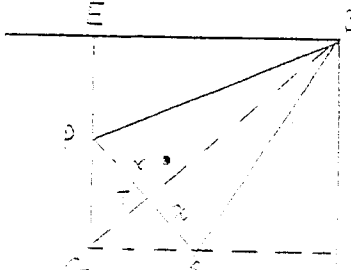
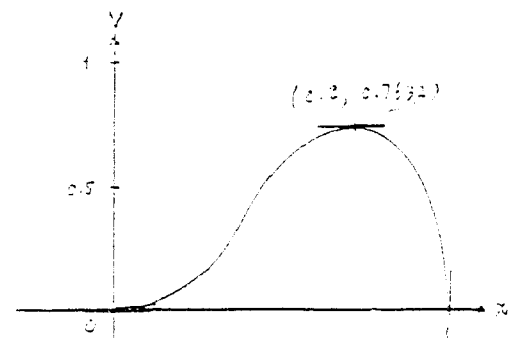
$= |w||\bar{w} - \bar{z}|$ 1A

$= |\bar{w} - \bar{z}|$ 1A

$= |w - z|$ 1A

$= |z - w|$ 1A

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SOLUTIONS	MARKS	REMARKS
<p>10. (a) Let G be the centre of the two squares and BG bisects PQ at T</p> $BG = \frac{1}{2} \sqrt{AB^2 - BC^2}$ $= \frac{1}{2} \sqrt{8 + 8}$ $= 2$ <p>$\therefore BT = 2 - x$</p> <p>The height of the pyramid</p> $= \sqrt{BT^2 - TG^2}$ $= \sqrt{(2 - x)^2 - x^2}$ $= 2\sqrt{1 - x} \text{ metres}$ <p>Volume = $\frac{1}{3}$ base area \times height</p> $V = \frac{1}{3} \times (2x)^2 \times 2\sqrt{1 - x}$ $= \frac{8}{3} x^2 \sqrt{1 - x}$	<p>1</p> <p>1A</p> <p>1A</p> <p>2</p> <p>1A</p> <p>1M</p> <p>1A</p> <p>3</p>	<p>Alternatively:</p> $PG = \frac{1}{2} \sqrt{(4x^2 + 4x^2)^2}$ $= x\sqrt{2} \dots\dots\dots 1A$ $PE = \sqrt{2}(1 - x) \dots\dots\dots 1A$ $BP = \sqrt{BE^2 + PE^2}$ $= \sqrt{4 - 4x + 2x^2} \dots\dots 1A$ $\text{Height} = \sqrt{BP^2 - PG^2} \dots\dots 2$ $= \sqrt{4 - 4x + 2x^2 - 2x^2} \dots\dots 1A$ $= 2\sqrt{1 - x}$ 
<p>(b) $\frac{dV}{dx} = \frac{8}{3} \left(2x\sqrt{1-x} - \frac{x^2}{\sqrt{1-x}} \right)$</p> $= \frac{4x(4 - 5x)}{3\sqrt{1-x}}$ <p>$\frac{dV}{dx} = 0$ iff $x = 0$ or $\frac{4}{5}$</p> <p>\therefore the stationary points are</p> $\left(0, 0 \right), \left(\frac{4}{5}, \frac{128}{75\sqrt{5}} \right)$ <p>At $x = 1$, $V = 0$,</p> <p>and the slope is infinite</p> <p>\therefore Equation of tangent at $x = 1$ is $x - 1 = 0$</p> <p>Equations of tangent at stationary points are $V = 0$, $V = 0.763$</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A</p> <p>1A+1A</p>	<p>Follow if constant factor of V incorrect.</p> <p>Accept $(0.3, 0.763)$</p>
<p>Graph of $V(x)$</p> 	<p>4</p> <p>12</p>	<p>1 mark for slope at $(0, 0)$, 1 mark for slope at $(1, 0)$, 1 mark for range, 1 mark for maximum with coordinates labelled.</p>

SOLUTIONS	MARKS	REMARKS	
11. (a) $CP = h \sec \theta$ ($\frac{h}{\cos \theta}$) $PB = h \tan \theta$ $AP = 50 - h \tan \theta$ $N = 2h \sec \theta + (50 - h \tan \theta)$	1A 1A 1M+1A 4		
(b) If $h = 50$, $N = 100 \sec \theta + 50 - 50 \tan \theta$ $\frac{dN}{d\theta} = 2h \sec \theta \tan \theta - h \sec^2 \theta$ $= 100 \sec \theta \tan \theta - 50 \sec^2 \theta$ $\frac{dN}{d\theta} = 0 \Rightarrow 50 \sec \theta (2 \tan \theta - \sec \theta) = 0$ $\Rightarrow 2 \tan \theta - \sec \theta = 0$ $\Rightarrow \sin \theta = \frac{1}{2}$ $\Rightarrow \theta = \frac{\pi}{6} \quad (30^\circ)$	1A 1M 1A 1A		
$\frac{d^2N}{d\theta^2} = 100(\sec^3 \theta + \sec \theta \tan^2 \theta - \sec^2 \theta \tan \theta)$ $= \frac{200}{\sqrt{3}} > 0$ at $\theta = \frac{\pi}{6}$	1M 1A		
$\therefore N$ is least at $\theta = \frac{\pi}{6}$. The least transportation cost from C to A is $\$ (100 \times \frac{2}{\sqrt{3}} - 50 - 50 \times \frac{1}{\sqrt{3}})$ $= \$ 50 (\frac{3 + \sqrt{3}}{\sqrt{3}}) = \$ 50 (\sqrt{3} + 1)$	1A 7		
(c) (i) As $(0 \leq) \theta \leq \angle ACB$ $\tan \theta \leq \tan \angle ACB$ $= \frac{50}{h}$	1 1A		$AP = 50 - h \tan \theta \geq 0 \dots\dots\dots 1M$ $\tan \theta \leq \frac{50}{h} \dots\dots\dots 1A$
For $h > 50\sqrt{3}$, $\tan \theta < \frac{1}{\sqrt{3}}$ $\therefore \theta < \frac{\pi}{6}$	1A 1A		
Hence $\frac{dN}{d\theta} = h \sec \theta (2 \tan \theta - \sec \theta)$ $= \frac{h}{\cos^2 \theta} (2 \sin \theta - 1)$ < 0 as $\sin \theta < \frac{1}{2}$	1A 1A		
(ii) If $h = 200$ then $h > 50\sqrt{3}$ $\therefore \frac{dN}{d\theta} < 0$	1A 1		
$\therefore N$ decreases as θ increases Goods should be transported directly from C to A by track for minimum cost.	1A 9		