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Additional Mathematics II

MARKING SCHEME

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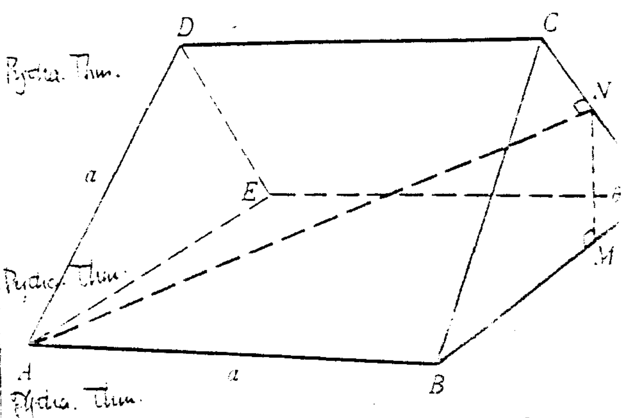
Solution	Marks	Remarks
1. Area of Δ PQR $= \pm\frac{1}{2}[11k + 7 + 21 - 11 - 1 - 3k]$ $= \pm\frac{1}{2}(8k + 16)$	1+1A	or $\left\|\frac{1}{2}(8k + 16)\right\|$
If this is 20 units $\pm 4k + 8) = 20$	1M	
$k = 3$ or -7	1+1A	
	5	
2. Let $u = x^2$ $du = 2xdx$	1A	
$\int x\sin^2(x^2)dx = \frac{1}{2}\int \sin^2 u du$	1A	
$= \frac{1}{2}\int \frac{1 - \cos 2u}{2} du$	1M	for $\sin^2 u = \frac{1-\cos 2u}{2}$
$= \frac{1}{4}u - \frac{1}{8}\sin 2u + c$	1A	-1 if omit either "c"
$= \frac{x^2}{4} - \frac{1}{8}\sin 2x^2 + c$	1A	
	5	
3. Put $u = 1 + 3x^2$, $du = 6xdx$	1A	
$x = 0 \Rightarrow u = 1$ $x = 1 \Rightarrow u = 4$	1A	
$\therefore \int_0^1 x^3\sqrt{1 + 3x^2}dx = \int_1^4 \frac{u-1}{3}\frac{\sqrt{u}}{6}du$	1A	for integrand
$= \frac{1}{18}\int_1^4 (u^{\frac{3}{2}} - u^{\frac{1}{2}})du$		
$= \frac{1}{18}\left[\frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}}\right]_1^4$	1A	
$= \frac{58}{135}$ (0.4296)	1A	Any figure roundable to 0.43.
	5	
4. (a) Equation of L is $y - (-3) = \frac{5 - -3}{5 - 1}(x - 1)$	1M	
or $y = 2x - 5$	1A	
(b) Area between curves $= \int_a^b (y_1 - y_2)dx$	1M	For other methods Area between curves $= \int_a^b (y_1 - y_2)dx$
Area req'd $= \int_0^1 [(x^2-4x) - (2x-5)]dx + \int_1^5 [(2x-5) - (x^2-4x)]dx$	1M	Required area $= A_1 + A_2 + \dots + A_n$
$= \int_0^1 [x^2 - 6x + 5]dx + \int_1^5 [-x^2 + 6x - 5]dx$		Answer.
$= \left[\frac{x^3}{3} - 3x^2 + 5x\right]_0^1 + \left[-\frac{x^3}{3} + 3x^2 - 5x\right]_1^5$		
$= \left[\frac{1}{3} - 3 + 5\right] - \frac{125}{3} + 75 - 25 + \frac{1}{3} - 3 + 5 = 13$	2A	
	6	

Solution	Marks	Remarks
5. Let $y = \frac{3}{2}x + c$ be a tangent. (or $3x - 2y + c = 0$)	1A	
Substituting in equation of ellipse	1M	
$4x^2 + (\frac{3}{2}x + c)^2 = 16$		
$(4 + \frac{9}{4})x^2 + 3cx + (c^2 - 16) = 0$	1A	
For tangency, $9c^2 - 4(4 + \frac{9}{4})(c^2 - 16) = 0$	1M	
$16c^2 = 25 \times 16$		
$c = \pm 5$	1+1A	
equations of tangents are $y = \frac{3}{2}x \pm 5$		
	6	
5. <u>Alternatively</u>		
$8x + 2yy' = 0$		
Slope of tangent is $y' = -\frac{4x}{y}$	1A	
But slope of line $= \frac{3}{2}$		
$\therefore -\frac{4x}{y} = \frac{3}{2}$	1M	$y = \sqrt{16 - 4x^2}$
or $y = -\frac{8}{3}x$		
Substituting in equation of ellipse	1M	
$4x^2 + (-\frac{8}{3}x)^2 = 16$		
$100x^2 = 144$		
$x = \pm\frac{6}{5}$		
$y = \mp\frac{16}{5}$	1A	For either x or y.
$\therefore$ equation of tangents required are		
$y \pm \frac{16}{5} = \frac{3}{2}(x \mp \frac{6}{5})$		
i.e. $3x - 2y - 10 = 0$	1A	
and $3x - 2y - 10 = 0$	1A	
	6	

Solution	Marks	Remarks
6. (a) The family of circles passing through the points of intersection of C_1 and C_2 is		
$x^2 + y^2 - 3x + 2y - 2 + k(x^2 + y^2 + x + 3y - 10) = 0$	1M	<u>Note</u> $kC_1 + C_2 = 0$ $\Rightarrow k = -\frac{1}{2}$
or $(1+k)x^2 + (1+k)y^2 + (k-3)x + (3k+2)y - (10k+2) = 0$		
Substituting $P(1, 2)$ in the equation	1M	
$(1+k) + (1+k)4 + (k-3) + (3k+2)2 - (10k+2) = 0$		
$2k + 4 = 0$		
$k = -2$	1A	
equation of C is $x^2 + y^2 + 5x + 4y - 18 = 0$	1A	
	4	
<u>Alternatively</u>		
$C_2 - C_1$: $4x + y - 8 = 0$.		
Substituting $y = 8 - 4x$ in C_1,	1M	
$x^2 + (8 - 4x)^2 - 3x + 2(8 - 4x) - 2 = 0$		
$17x^2 - 75x + 78 = 0$		
$x = \frac{75 \pm \sqrt{321}}{34}$ $\begin{pmatrix} 2.7328, \\ 1.6789 \end{pmatrix}$		
$y = \frac{-14 \mp 2\sqrt{321}}{17}$ $\begin{pmatrix} -2.9313, \\ 1.2843 \end{pmatrix}$		
Let C : $x^2 + y^2 + ax + by + c = 0$		
Substituting the three points in C and solving,	1M	
$a = 5,\ b = 4,\ c = -18.$	2A	
	4	
(b) Equation of tangent at P is		
$1x + 2y + \frac{5}{2}(x + 1) + 2(y + 2) - 18 = 0$	1A	
or $7x + 8y - 23 = 0$	1A	
	2	
<u>Alternatively</u>		
$2x + 2yy' + 5 + 4y' = 0$		
$y' = -\frac{2x + 5}{2y + 4}$		
At $P(1, 2)$, slope $= -\frac{7}{8}$	1A	
$y - 2 = -\frac{7}{8}(x - 1)$		
$7x + 8y - 23 = 0$	1A	
	2	

Solution	Marks	Remarks
7. $\sin(n+m)\theta \sin(n-m)\theta$		
$= [\sin n\theta \cos m\theta + \cos n\theta \sin m\theta] \times [\sin n\theta \cos m\theta - \cos n\theta \sin m\theta]$	1A	
$= \sin^2 n\theta \cos^2 m\theta - \cos^2 n\theta \sin^2 m\theta$		
$= \sin^2 n\theta(1-\sin^2 m\theta) - (1-\sin^2 n\theta)\sin^2 m\theta$		
$= \sin^2 n\theta - \sin^2 n\theta \sin^2 m\theta - \sin^2 m\theta + \sin^2 n\theta \sin^2 m\theta$		
$= \sin^2 n\theta - \sin^2 m\theta$	1A	
$\sin^2 3\theta - \sin^2 2\theta - \sin\theta = 0$		
$\Rightarrow \sin(3+2)\theta \sin(3-2)\theta - \sin\theta = 0$	1M	
$\Rightarrow \sin 5\theta \sin\theta - \sin\theta = 0$	1A	
$\Rightarrow \sin\theta(\sin 5\theta - 1) = 0$		
$\Rightarrow \sin\theta = 0$ or $\sin 5\theta = 1$	1A	
$\Rightarrow \theta = 0$ or π or $5\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$		
$\because 0 \leqslant \theta \leqslant \pi$		
$\theta = 0, \frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}$ or π.	3A	−1 for each missing or wrong answer
$(0°, 18°, 90°, 162°, 180°)$		
	7	
<u>Alternatively</u>		
(i) $\sin^2 n\theta - \sin^2 m\theta = \frac{1}{2}(1-\cos 2n\theta) - \frac{1}{2}(1-\cos 2m\theta)$	1A	
$= \frac{1}{2}(\cos 2m\theta - \cos 2n\theta)$		
$= \sin(n+m)\theta \sin(n-m)\theta$	1A	
(ii) $\sin^2 n\theta - \sin^2 m\theta$		
$= (\sin n\theta - \sin m\theta)(\sin n\theta + \sin m\theta)$		
$= \left[2\cos\frac{n+m}{2}\theta \sin\frac{n-m}{2}\theta\right]\left[2\sin\frac{n+m}{2}\theta \cos\frac{n-m}{2}\theta\right]$	1A	
$= 4\sin\frac{n-m}{2}\theta \sin\frac{n+m}{2}\theta \cos\frac{n-m}{2}\theta \cos\frac{n+m}{2}\theta$		
$= \sin(n-m)\theta \sin(n+m)\theta$	1A	

Solution	Marks	Remarks
3. (a) $BM = 2a\cos\theta - x\cos\theta$	2A+1A	
$AM = \sqrt{AB^2 + BM^2}$	1M	Pytha. Thm.
$= \sqrt{a^2 + (2a - x)^2 \cos^2\theta}$	1A	
	4	
(b) $AF^2 = AB^2 + BF^2$	2M	Pytha. Thm.
$= a^2 + 4a^2\cos^2\theta$	1A	
$AN = \sqrt{AF^2 - NF^2}$	1M	Pytha. Thm.
$= \sqrt{a^2 + 4a^2\cos^2\theta - x^2}$	1A	
	5	



Solution	Marks	Remarks
(c) $NM = x\sin\theta$	1A	
Consider △ AMN,		
$AN^2 = NM^2 + AM^2$	1M	Pytha. Thm.
$a^2 + 4a^2\cos^2\theta - x^2 = x^2\sin^2\theta + a^2 + (2a - x)^2\cos^2\theta$		
$2x^2 - 4ax\cos^2\theta = 0$	2A ~~1A~~	any equivalent step
$x = 2a\cos^2\theta$ (∵ $x \neq 0$)	1A	(∵ $x \neq 0$) optional
	5	
(d) If $x = \frac{a}{2}$, by (c) $\cos^2\theta = \frac{1}{4}$	1A	
$\theta = \frac{\pi}{3}$ $60°$	1A	
Let β be the inclination.		Or
$\tan\beta = \frac{NM}{AM}$ 可以錯,但必須已交待	1A	$\sin\beta = \frac{NM}{AN}$
$= \frac{x\sin\theta}{\sqrt{a^2 + (2a - x)^2\cos^2\theta}}$	1M	$= \frac{x\sin\theta}{\sqrt{a^2+4a^2\cos^2\theta - x^2}}$
$= \frac{\frac{a}{2} \times \frac{\sqrt{3}}{2}}{\sqrt{a^2 + \frac{9}{4}a^2 \cdot \frac{1}{4}}}$		$= \frac{\sqrt{3}}{2\sqrt{7}}$ $(= 0.327)$
$= \frac{\sqrt{3}}{5}$ $(= 0.346)$	1A	Any fig. roundable to 0.346
$\beta = 19.1°$		
$\beta \doteqdot 19°$ (correct to the nearest degree)	1A	
只有可,但不可只寫		
	6	

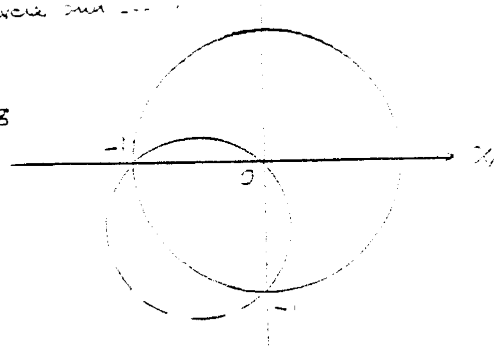
Solution	Marks	Remarks
9. (a) $y^2 = 4x$		
$2yy' = 4$		
Slope of tangent $= \frac{2}{y}$	1A	
Slope of $L_1 = 1$,	1A	
of $L_2 = -2$	1A	
Equation of L_1 is $x - y - 3 = 0$	1A	
of L_2 is $2x + y - 12 = 0$	1A	
Solving the above, the coordinates of N are $x = 5$, $y = 2$.	1+1A	
Slope of ON $= \frac{2}{5}$	1A	
	8	
(b) Coordinates of P are $x = \frac{4+k}{1+k}$, $y = \frac{4-2k}{1+k}$	1+1A	
Slope of OP $= \frac{4-2k}{4+k}$	1A	
$\tan \angle PON = \pm \left\lvert \dfrac{\frac{4-2k}{4+k} - \frac{2}{5}}{1 + \frac{4-2k}{4+k} \times \frac{2}{5}} \right\rvert$	2M	for $\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$
$= \pm \left\lvert \frac{12-12k}{28+k} \right\rvert$ according as $\angle$PON is acute or obtuse	1+1A	
	7	
(c) (i) If $\left\lvert \frac{12-12k}{28+k} \right\rvert = 1$ $\underbrace{\pm\frac{12-12k}{28+k}}_{\text{都自的}} = 1$ 亦可	1M	
$k = -\frac{16}{13}$ or $\frac{40}{11}$	1A	
By inspection, $k = -\frac{16}{13}$ corresponds to the case $\angle$ PON $= 135°$.		
$\therefore$ if $\angle$ PON $= 45°$, $k = \frac{40}{11}$	1A	
(ii) When PON is a straight line		
$\frac{12-12k}{28+k} = 0$	1M	
$k = 1$	1A	need not be $\pm\left\lvert\frac{12-12k}{28+k}\right\rvert$ ...
	5	

Solution	Marks	Remarks
10. (a) Let the line be		計錯
$y + 1 = m(x + 1)$	1M	例如: $y = mx + c$
$y = mx + (m - 1)$	1A	x or y as subject / permit unsimplified form of st. line eqn.
Substituting in the circle,		
$x^2 + [mx + (m-1)]^2 = 1$	1M	
$(1 + m^2)x^2 + 2m(m-1)x + (m-1)^2 - 1 = 0$ $(1+m^2)y^2 + (2-2m)y + 1-2m = 0$	1A	
If $A = (x_1, y_1)$, $B = (x_2, y_2)$		
$x_1 + x_2 = -\dfrac{2m(m-1)}{1+m^2}$	1M	also for $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$
$\therefore$ the coordinates of P are		
$x = \dfrac{x_1 + x_2}{2}$	1M	
$= -\dfrac{m(m-1)}{1+m^2}$ .... (i)	1A	
$y = mx + (m-1)$		
$= -\dfrac{m^2(m-1)}{1+m^2} + (m-1)$	1M	
$= \dfrac{m-1}{1+m^2}$ .... (ii)	1A	
	9	
(b) (i) $\div$ (ii) $\dfrac{x}{y} = -m$		
Substituting in (ii) $y = \dfrac{-\frac{x}{y} - 1}{1 + \frac{x^2}{y^2}}$	2M (attempt 1M + m eliminated 1M)	Attempt to eliminate m between x, y.
$x^2 + y^2 + x + y = 0$ ← may be multiplied by y or y^2	3A	
which is a circle. (or "circle and ...")	1A	
	6	
(c)	3	award 3 or 0 for circle passing through (0,0), (−1,0)(0,−1)
	+ 1	for labelling. This also pts
	+ 1	for indicating correc part of circle as loc
	5	

(c) Or

Sketch by joining mid-points. 2A

Proof for circle. 3A



Solution	Marks	Remarks

11. (a) $\dfrac{\sin 3\theta}{\sin\theta} = \dfrac{\sin 2\theta\cos\theta + \cos 2\theta\sin\theta}{\sin\theta}$ 1A

$= \dfrac{2\sin\theta\cos^2\theta + \cos 2\theta\sin\theta}{\sin\theta}$

$= 2\cos 2\theta + 1$ 1A

Or

$\dfrac{\sin 3\theta}{\sin\theta} = \dfrac{3\sin\theta - 4\sin^3\theta}{\sin\theta}$ 1A

$= 3 - 4\sin^2\theta$

$= 3 - 4\left(\dfrac{1 - \cos 2\theta}{2}\right)$

$= 2\cos 2\theta + 1$ 1A

Putting $\theta = \dfrac{\pi}{4} + \phi$,

L.S. $= \dfrac{\sin 3\theta}{\sin\theta}$

$= \dfrac{\sin\left(\dfrac{3\pi}{4} + 3\emptyset\right)}{\sin\left(\dfrac{\pi}{4} + \phi\right)}$

$= \dfrac{\sin\dfrac{3\pi}{4}\cos 3\emptyset + \cos\dfrac{3\pi}{4}\sin 3\emptyset}{\sin\dfrac{\pi}{4}\cos\emptyset + \cos\dfrac{\pi}{4}\sin\emptyset}$ 1A, 1A

$= \dfrac{\cos 3\emptyset - \sin 3\emptyset}{\cos\emptyset + \sin\emptyset}$ 1A

R.S. $= 2\cos\left(\dfrac{\pi}{2} + 2\emptyset\right) + 1$

$= 1 - 2\sin 2\emptyset$ 1A

$\therefore \dfrac{\cos 3\emptyset - \sin 3\emptyset}{\cos\emptyset + \sin\emptyset} = 1 - 2\sin 2\emptyset$

7

(b) Putting $\emptyset = \dfrac{\pi}{2} - u$, $d\emptyset = -du$ 1A

when $\emptyset = 0$, $u = \dfrac{\pi}{2}$ 1A

$\emptyset = \dfrac{\pi}{2}$, $u = 0$

$\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos 3\emptyset}{\cos\emptyset + \sin\emptyset}\, d\emptyset = -\int_{\frac{\pi}{2}}^{0} \frac{\cos\left(\frac{3\pi}{2} - 3u\right)}{\cos\left(\frac{\pi}{2} - u\right) + \sin\left(\frac{\pi}{2} - u\right)}\, du$

$\displaystyle= -\int_{\frac{\pi}{2}}^{0} \frac{-\sin 3u}{\sin u + \cos u}\, du$

$\displaystyle= \int_{\frac{\pi}{2}}^{0} \frac{\sin 3u}{\cos u + \sin u}\, du$ 2A

Solution	Marks	Remarks
11. (b) $\therefore \int_0^{\frac{\pi}{2}} \frac{\cos 3\theta - \sin 3\theta}{\cos\theta + \sin\theta} d\theta = \int_0^{\frac{\pi}{2}} \frac{\cos 3\theta}{\cos\theta + \sin\theta} d\theta - \int_0^{\frac{\pi}{2}} \frac{\sin 3\theta}{\cos\theta + \sin\theta} d\theta$	2M	must check steps
$= 2\int_0^{\frac{\pi}{2}} \frac{\cos 3\theta}{\cos\theta + \sin\theta} d\theta$	2M	
$\int_0^{\frac{\pi}{2}} \frac{\cos 3\theta}{\cos\theta + \sin\theta} d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\cos 3\theta - \sin 3\theta}{\cos\theta + \sin\theta} d\theta$		
	8	
(c) $\int_0^{\frac{\pi}{2}} \frac{\cos 3\theta}{\cos\theta + \sin\theta} d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\cos 3\theta - \sin 3\theta}{\cos\theta + \sin\theta} d\theta$	1A	
$= \frac{1}{2}\int_0^{\frac{\pi}{2}} (1 - 2\sin 2\theta) d\theta$	2A	
$= \frac{1}{2}[\theta + \cos 2\theta]_0^{\frac{\pi}{2}}$	1A	
$= \frac{1}{2}[\frac{\pi}{2} - 1 - 1]$		
$= \frac{\pi}{4} - 1 \quad (\doteqdot -0.215)$	1A	Any figure roundable -0.215.
	5	

Solution	Marks	Remarks
12. (a) Putting $y = x + ks$, $dy = dx$	1A	
when $x = 0$, $y = ks$; $x = s$, $y = (k+1)s$	1A	
$\int_0^s f(x+ks)\,dx = \int_{ks}^{(k+1)s} f(y)\,dy$	2A	
$= \int_{ks}^{(k+1)s} f(x)\,dx$	1A	
	5	
$\int_0^s [f(x) + f(x+s) + \dots + f(x+(n-1)s)]\,dx$		
$= \int_0^s f(x)\,dx + \int_0^s f(x+s)\,dx + \dots + \int_0^s f(x+(n-1)s)\,dx$	1A	
$= \int_0^s f(x)\,dx + \int_s^{2s} f(x)\,dx + \dots + \int_{(n-1)s}^{ns} f(x)\,dx$	0+1A+2A	
$= \int_0^{ns} f(x)\,dx$	1A	
	5	
(b) Putting $x = \sin\theta$, $dx = \cos\theta\,d\theta$	1A	
when $x = 0$, $\theta = 0$; $x = \frac{1}{2}$, $\theta = \frac{\pi}{6}$	1A	
$\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}} = \int_0^{\frac{\pi}{6}} \frac{\cos\theta\,d\theta}{\cos\theta}$	1A	any form in θ only.
$= [\theta]_0^{\frac{\pi}{6}}$	1A	
$= \frac{\pi}{6}$ (0.524)	1A	Any figure roundable 0.524.
Putting $f(x) = \frac{1}{\sqrt{1-x^2}}$, $s = \frac{1}{2n}$, by (a)	1+1A	may be omitted.
$\int_0^{\frac{1}{2n}} \left[\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-\left(x+\frac{1}{2n}\right)^2}} + \dots + \frac{1}{\sqrt{1-\left(x+\frac{n-1}{2n}\right)^2}}\right] dx$		
$= \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$	2A	
$= \frac{\pi}{6}$	1A	
	10	