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HONG KONG EXAMINATIONS AUTHORITY

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HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1982

# Additional Mathematics II

## MARKING SCHEME







F. 405 (2/77)

| Solutions | Marks | Remarks |
|---|---|---|
| (1) Let $u=\sqrt{x+9}$ | | |
| $u^2=x+9$ | | |
| $2u\,du=dx$ | 1A | |
| $\int \frac{x}{\sqrt{x+9}}dx = \int \frac{(u^2-9)}{u}2u\,du$ | 1A | no mark for $\int \ldots dx$ but may proceed to mark remaining part |
| $=2\int(u^2-9)du$ | | |
| $=\frac{2}{3}u^3 \overset{1A}{-18u} \overset{1A}{+} C$ | 1+1A | −1 if omit C throughout |
| $=\frac{2}{3}(x+9)^{\frac{3}{2}}-18(x+9)^{\frac{1}{2}}+C$ | 1A | |
| | 5 | |
| (2) The line through A and B is given by | | Alternatively |
| $y+1=\frac{-1-1}{3+1}(x-3)$ | | Let $(a,b)$ divides AB in the ratio 1: |
| i.e. $x+2y-1=0$ ($2x+4y-2=0$ etc.) | 1A | $a=\frac{3r-1}{1+r}$ (1A) |
| Solving this with $x-y-1=0$ | 1M | $b=\frac{-r+1}{1+r}$ (1A) |
| $y=0$ | In case of a graph, this is awarded for drawing 2 lines (may be wrong) | Sub. in given line |
| $x=1$ | | $\frac{3r-1}{1+r}-\frac{-r+1}{1+r}-1=$ (2) |
| ∴ the two lines meet at $C=(1,0)$ ($\begin{cases}x=1\\y=0\end{cases}$) | 1A | $r=1$ (1) |
| If C divides AB in the ratio $1:r$, | | |
| $1=\frac{3r-1}{1+r}$ ( or $0=\frac{1-r}{1+r}$ ) | 1M | Alt $\frac{AC}{CB}=\frac{\sqrt{(3-1)^2+(-1-0)^2}}{\sqrt{(-1-1)^2+(1-0)^2}}$ (1M) |
| $r=1$ | 1A | $=\frac{\sqrt{5}}{\sqrt{5}}$ |
| ∴ C divides AB in the ratio 1:1 | 5 | $=1$ (1A) |
| Alt | | |
| Graphical method acceptable. | | or $AC=\ldots$ $CB=\ldots$ |

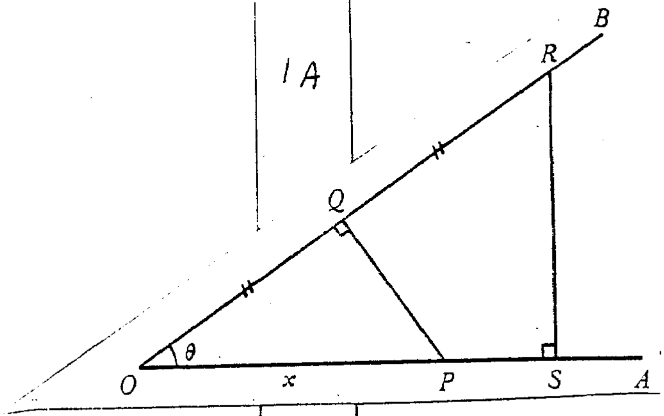
| Solutions | Marks | Remarks |
|---|---|---|
| (3) $\cos 2\theta - \sqrt{3}\cos\theta + 1 = 0$ | | |
| $2\cos^2\theta - \sqrt{3}\cos\theta = 0$ | 1M | Attempt to express $\cos 2\theta$ in terms of $\cos^2\theta$. |
| $\cos\theta(2\cos - \sqrt{3}) = 0$ | | |
| $\cos\theta = 0$ or $\frac{\sqrt{3}}{2}$ | 1+1A | |
| For $0 \le \theta \le \frac{\pi}{2}$, $\theta = \frac{\pi}{2}$ or $\frac{\pi}{6}$ $(90°$ or $30°)$ | | |
| The general soln. is $\theta = 2n\pi \pm \frac{\pi}{2}$ | 1M + 1A | for $2n\pi \pm \alpha$ |
| or $2n\pi \pm \frac{\pi}{6}$, | 1A | |
| where $n = 0, \pm 1, \pm 2, \ldots$ | 6 | -1 if mixing degree with radian. |
| Note other variations of answers. e.g. $(2n+1)\pi \pm \frac{\pi}{2}$, $(2n+1)\frac{\pi}{2}$, etc. | | |
| (4) Volume $= \pi\int_0^{2\pi} x^2\,dy$ | 1A | wrong limits |
| $= \pi\int_0^{2\pi}(4 + 4\sin y + \sin^2 y)\,dy$ | | |
| $= \pi\int_0^{2\pi}\left(4 + 4\sin y + \frac{1-\cos 2y}{2}\right)dy$ | 1M | Attempt to express $\sin^2 y$ in terms of $\cos 2y$. |
| $= \pi\left[\frac{9}{2}y - 4\cos y - \frac{\sin 2y}{4}\right]_0^{2\pi}$ | 1+1+1A | provided limits correct |
| $= 9\pi^2$ | 1A | -2 if omit $\pi$ but otherwise correct |
| must have the above part. | 6 | |

| Solutions | Marks | Remarks |
|---|---|---|
| ⑤ $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\cos x+\sin x}\,dx + \int_0^{\frac{\pi}{2}} \frac{\cos^3 x}{\cos x+\sin x}\,dx$ | | |
| $= \int_0^{\frac{\pi}{2}} \frac{\sin^3 x+\cos^3 x}{\cos x+\sin x}\,dx$ | 1A | |
| $= \int_0^{\frac{\pi}{2}} (\sin^2 x-\sin x\cos x+\cos^2 x)\,dx$ | 1A | |
| $= \int_0^{\frac{\pi}{2}} (1-\frac{\sin 2x}{2})\,dx$ | 1A | |
| $= \left[x+\frac{\cos 2x}{4}\right]_0^{\frac{\pi}{2}}$ | 1A | |
| $= \frac{\pi}{2}-\frac{1}{2}$ | 1A | |
| $\therefore \int_0^{\frac{\pi}{2}} \frac{\sin^3 x}{\cos x+\sin x}\,dx = \frac{1}{2}(\frac{\pi}{2}-\frac{1}{2})$ | 1M | |
| | 6 | |
| ⑥ Let $Q=(x,y)$ | | |
| $x=\frac{2x_1+9}{5}$ | 1A | |
| $y=\frac{2y_1}{5}$ | 1A | |
| $\therefore x_1=\frac{5x-9}{2}$, $y_1=\frac{5y}{2}$ | 1M+1A | 1M for attempt to change subject |
| Since $(x_1,y_1)$ lies on the circle | | |
| $\left(\frac{5x-9}{2}\right)^2+\left(\frac{5y}{2}\right)^2=4$ | 1M | |
| $5x^2+5y^2-18x+13=0$ | 1A | $5a^2+5b^2-18x+13=$ [illegible] acceptable. |
| or $(x-\frac{9}{5})^2+y^2=\frac{16}{25}$ | | |



G.F. 405 (2/77)

| Solutions | Marks | Remarks |
|---|---|---|
| 7 (a) $\Delta_1 = $ area of $\Delta OPQ$ | | |
| $= \frac{1}{2} PQ \times OQ$ | | |
| $= \frac{1}{2} x^2 \sin\theta \cos\theta$ | 1A | |
| $\Delta_2 = $ area of $\Delta ORS$ | | |
| $= \frac{1}{2} RS \times OS$ | | |
| $= \frac{1}{2} (2x\cos\theta\sin\theta)(2x\cos^2\theta)$ | | |
| $= 2x^2 \sin\theta \cos^3\theta$ | 1A | |
| (b) $\frac{d\Delta_1}{dx} = x\cos\theta\sin\theta$ | 1A | $-1$ if write $\frac{d\Delta_1}{dt}$ |
| $\frac{d\Delta_2}{dx} = 4x\sin\theta\cos^3\theta$ | 1A | $-1$ if write $\frac{d\Delta_2}{dt}$ |
| $\frac{d\Delta_1}{dx} = \frac{d\Delta_2}{dx}$ | 1M | $\Rightarrow \frac{d\Delta_1}{dt} = \frac{d\Delta_2}{dt}$ |
| $\Rightarrow x\cos\theta\sin\theta = 4x\sin\theta\cos^3\theta$ | | |
| $\Rightarrow \cos^2\theta = \frac{1}{4}$ | | |
| $\Rightarrow \cos\theta = \frac{1}{2}$ | | |
| $\theta = 60^\circ \ (\frac{\pi}{3})$ | 1A | |
| | 6 | |



| Solutions | Marks | Remarks |
|---|---|---|
| (8) (a) Distance from M to L $= \left\lvert \dfrac{5x+12y-32}{\sqrt{25+144}} \right\rvert$ | 1M | |
| $= \dfrac{65}{13} = 5 \ (r)$ | 1A | |
| Equation of C is $(x-5)^2+(y-6)^2=25$ | 1M+1A | for $r^2$ |
| or $x^2+y^2-10x-12y+36=0$ ....(*) | 4 | |
| (b) Since distance from M to y-axis $=5$, | 2A | |
| C also touches the y-axis | 2 | |
| (c) Let $y=mx$ be a tangent to C through O. | 1A | Alt<br>$y=mx$ (1A) |
| Solving with (*), $x^2+m^2x^2-10x-12mx+36=0$<br>$(1+m^2)x^2-(10+12m)x+36=0$ | 1M+1A | Dist. from M to $y=mx$<br>$= \left\lvert \dfrac{5m-6}{\sqrt{m^2+1}} \right\rvert$ (1M) |
| For tangency, $(10+12m)^2-4(1+m^2)36=0$ | 1M | $=5$ (1A)<br>$\dfrac{(5m-6)^2}{m^2+1}=25$ (1M)<br>$25m^2-60m+36=25m^2+2$ |
| $240m-44=0$<br>$m=\dfrac{11}{60}$ | 1A | $m=\dfrac{11}{60}$ (1A) |
| ∴ the other tangent is $y=\dfrac{11}{60}x$ | 1A | $y=\dfrac{11}{60}x$ (1A) |
| or $11x-60y=0$ | 6 | |
| (d) Slope of PQ $=\dfrac{6-2}{5-2}=\dfrac{4}{3}$ | 1A | Alt<br>$P=(2,2)$, $M=(5,6)$<br>∴ $Q=(8,10)$ |
| Eqn of PQ is $y-2=\dfrac{4}{3}(x-2)$ | | Let circle be |
| $4x-3y-2=0$ $\quad \begin{cases} x^2+y^2+ax+by+c=0 \\ c=0 \end{cases} \rightarrow x^2+y^2+ax+by=0$ | 1A | $x^2+y^2+ax+by=0$ |
| Eqn of family of circle is | | Sub. P, Q. |
| $x^2+y^2-10x-12y+36+k(4x-3y-2)=0$<br>[or $4x-3y-2+k(x^2+y^2-10x-12y+36)=0$] | 2M+1A | $4+4+2a+2b=0$<br>$64+100+8a+10b=$ |
| Putting $(x,y)=(0,0)$, $36-2k=0$ | 1M | $b=-66$ (1A)<br>$a=62$ (1A) |
| $k=18$ (or $\frac{1}{18}$) | 1A | ∴ $x^2+y^2+62x-66y$ |
| ∴ req'd eqn is $x^2+y^2+62x-66y=0$ | 1A | |
| | 8 | |

 (6)

| Solutions | Marks | Remarks |
|---|---|---|
| 9(a) $y^2=4x$ | | Alt |
| $2yy'=4$ | | Eqn. of tangent |
| $y'=\frac{2}{y}$. | 1A | is $yy_1=2(x+x_1)$ 1M |
| Eqn of PR is $y-2s=\frac{2}{2s}(x-s^2)$ | 1M | At $(s^2,2s)$, |
| i.e. $y=\frac{1}{s}x+s$ (or. $x-sy+s^2=0$) | 1A | $2sy=2(x+s^2)$ or $x-sy+s^2=0$ (1A) |
| Similarly, eqn of QR is | | |
| $y=\frac{1}{t}x+t$ (or $x-ty+t^2=0$) | 1A | |
| Solving these two eqns | | |
| $\frac{1}{s}x+s=\frac{1}{t}x+t$ | | |
| $x=st$ | 1A | |
| $y=s+t$ | 1A | |
| $\therefore R=(st,s+t)$ | 6 | |
| (b) If $\frac{1}{s}+\frac{1}{t}=2$ | 1M+1A | |
| $\frac{s+t}{st}=2$ | | |
| $\frac{y}{x}=2$ | 2A | |
| $\therefore R$ must lie on the line $y=2x$ | 4 | |
| (c) Solving $\begin{cases} y^2=4x \\ y=2x \end{cases}$ | | |
| $y=0$ or $2$ | | |
| ($x=0$ or $1$) | | |
| $(x,y)=(0,0)$ or $(1,2)$ | 1+1A | |



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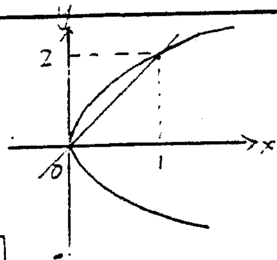
| Solutions | Marks | Remarks |
|---|---|---|
| (9) (Cont'd) Area enclosed $= \int_0^2 \left(\frac{y}{2} - \frac{y^2}{4}\right) dy$ | 1M | for ~ |
| $= \left[\frac{y^2}{4} - \frac{y^3}{12}\right]_0^2$ Insert the correct | 1A+1A | Alt $\int_0^1 (2\sqrt{x} - 2x)\,dx$ |
| $= \frac{1}{3}$ | 1A | $= \left[\frac{4}{3}x^{\frac{3}{2}} - x^2\right]_0^1$ Insert above correct limits |
| Alt $A = \int_0^2 \frac{y}{2}dy - \int_0^2 \frac{y^2}{4}dy$ (1M) $= \left[\frac{y^2}{4}\right]_0^2 - \left[\frac{y^3}{12}\right]_0^2$ (1+1A) $= \frac{1}{3}$ (1A) | | $= \frac{1}{3}$ |
| | 6 | |
| (d) Vol. generated $= \pi \int_0 [4x - (2x)^2]\,dx$ | 1M | for $V_1 \sim V_2$ |
| $= \pi \left[2x^2 - \frac{4}{3}x^3\right]_0^1$ Insert the correct | 1A+1A | $-2$ if omit $\pi$ but otherwise correct |
| $= \frac{2}{3}\pi$ | 1A | |
| Alt (1M) $V = \pi\int_0^1 4x\,dx - \pi\int_0^1 (2x)^2 dx$ $= \pi\left[2x^2\right]_0^1 - \pi\left[\frac{4}{3}x^3\right]_0^1$ (1+1A) $= \frac{2}{3}\pi$ (1A) | | |
| | 4 | |
| (10) (a) Solving $\begin{cases} 3x - 2y - 8 = 0 \\ x - y - 2 = 0 \end{cases}$ | 1M | Alt $3x - 2y - 8 + k(x - y - 2$ |
| $x = 4,\ y = 2$ | 1+1A | Slope $= \frac{3+k}{2+k}$ |
| $\therefore P = (4, 2)$ | | $\frac{3+k}{2+k} = \frac{1}{2} \Rightarrow k = -4$ either one (1M |
| Eqn. of $L_1$ is $y - 2 = \frac{1}{2}(x - 4)$ | 1M | $\frac{3+k}{2+k} = 2 \Rightarrow k = -1$ |
| $x - 2y = 0$ | 1A | $L_1$: $x - 2y = 0$ ( |
| Eqn. of $L_2$ is $y - 2 = 2(x - 4)$ | | $L_2$: $2x - y - 6 = 0$ (1 |
| $2x - y - 6 = 0$ | 1A | |
| | 6 | |
| (b) Eqn. of $L$ is $y = m(x - 2)$ | 1A | |
| Solving with $L_1$, $2m(x-2) = x$ | | |
| $x = \frac{4m}{2m-1}$, $y = \frac{2m}{2m-1}$ | 1+1A | |

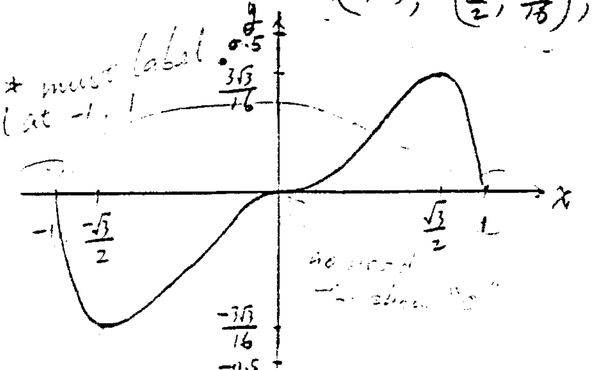


| Solutions | Marks | Remarks |
|---|---|---|
| 13 (Cont'd) $\therefore A = \left(\dfrac{4m}{2m-1}, \dfrac{2m}{2m-1}\right)$ | | |
| Solving $L$ with $L_2$, | | |
| $m(x-2) = 2x-6$ | | |
| $x = \dfrac{2m-6}{m-2}$ | 1A | |
| $y = m\left[\dfrac{2m-6}{m-2} - 2\right]$ | | |
| $= \dfrac{-2m}{m-2}$ | 1A | |
| $\therefore B = \left(\dfrac{2m-6}{m-2}, \dfrac{-2m}{m-2}\right)$ | | |
| Area of $\Delta PAB = \dfrac{1}{2}\begin{vmatrix} 4 & 2 \\ \dfrac{4m}{2m-1} & \dfrac{2m}{2m-1} \\ \dfrac{2m-6}{m-2} & \dfrac{-2m}{m-2} \\ 4 & 2 \end{vmatrix}$ | 1M + 2A | $-1$ if omit $\frac{1}{2}$ |
| $= \dfrac{1}{2}\left[\dfrac{8m}{2m-1} - \dfrac{8m^2}{(2m-1)(m-2)} + \dfrac{2(2m-6)}{m-2} - \dfrac{2m(2m-6)}{(2m-1)(m-2)} - \dfrac{8m}{2m-1} + \dfrac{8m}{m-2}\right]$ | | |
| $= \dfrac{6(m^2-2m+1)}{(m-2)(2m-1)}$ | | |
| $= \dfrac{6(m-1)^2}{(m-2)(2m-1)}$ | | |
| $\dfrac{d\Delta}{dm} = 6\,\dfrac{(m-2)(2m-1)(2m-2) - (m-1)^2(4m-5)}{(m-2)^2(2m-1)^2}$ | 1M | Attempt to diff. |
| $= \dfrac{6(1-m^2)}{(m-2)^2(2m-1)^2}$ | 1A | |
| $\dfrac{d\Delta}{dm} = 0 \Rightarrow m = \pm 1$ | 1A | |
| If $m=1$, $L$ is the line $PQ$, rejected. Take $m=-1$ | 1A | |
| Checking that $m=-1$, $\Delta$ is a min. | 1M. | Attempt to check |
| Eqn. of $L$ is $y = -(x-2)$, $x+y-2=0$ | 1A | |
| | 14 | |



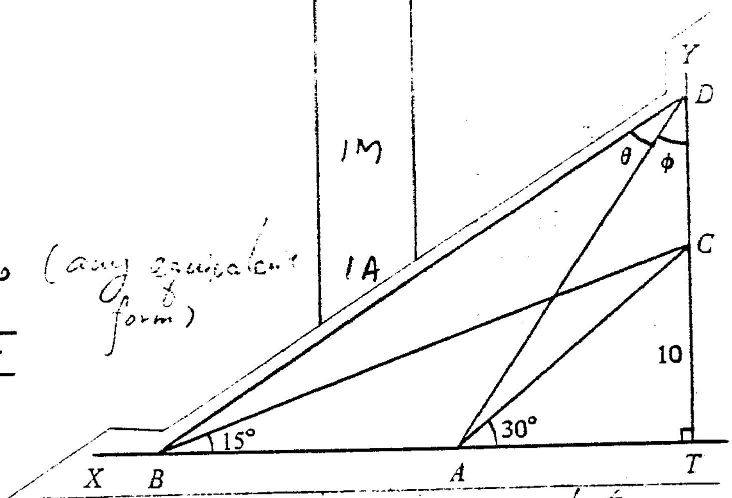
G.F. 405 (2/77)

| Solution | Marks | Remarks |
|---|---|---|
| 11 (a) Putting $u = \cos\theta$, $du = -\sin\theta\, d\theta$ | 1A | |
| When $\theta = 0$, $u = 1$; $\theta = \frac{\pi}{2}$, $u = 0$. | 1A | |
| $\int_0^{\frac{\pi}{2}} \sin^3\theta \cos^2\theta\, d\theta = -\int_1^0 (1-u^2)u^2\, du$ | 1M + 1A | for sub. of limits; for integrand |
| $= \int_0^1 (u^2 - u^4)\, du$ | | Alt $I = -\int_0^{\frac{\pi}{2}} \sin^2\theta \cos^2\theta\, d\cos\theta$ (1A) + (1 |
| $= \left[\frac{u^3}{3} - \frac{u^5}{5}\right]_0^1$ | 1A | $= -\int_0^{\frac{\pi}{2}} (1-\cos^2\theta)\cos^2\theta\, d\cos\theta$ (1 |
| $= \frac{2}{15}$   be correct | 1A | $= -\int_0^{\frac{\pi}{2}} (\cos^2\theta - \cos^4\theta)\, d\cos\theta$ (1 |
| | 6 | $= -\left[\frac{\cos^3\theta}{3} - \frac{\cos^5\theta}{5}\right]_0^{\frac{\pi}{2}}$ |
| (b) (i) The range is $-1 \le x \le 1$ | 1A | $= \frac{2}{15}$ (1A) |
| (ii) Putting $y = 0$, $x = 0$ or $\pm 1$ | | |
| $\therefore$ C meets the x-axis at $(0,0)$, $(1,0)$, $(-1,0)$ | 1A | |
| (iii) $y = x^3\sqrt{1-x^2}$ | | |
| $\frac{dy}{dx} = 3x^2(1-x^2)^{\frac{1}{2}} + \frac{1}{2} \cdot \frac{x^3(-2x)}{(1-x^2)^{\frac{1}{2}}}$ | | |
| $= \frac{x^2(3-4x^2)}{(1-x^2)^{\frac{1}{2}}}$ | | |
| $\boxed{\frac{dy}{dx} = 0} \Rightarrow x = 0$ or $\pm\frac{\sqrt{3}}{2}$ $(\pm 0.8660)$; $y = 0$ or $\pm\frac{3\sqrt{3}}{16}$ $(\pm 0.3248)$ | 1M | |
| $\therefore$ the 3 pts. are $(0,0)$, $\left(\frac{\sqrt{3}}{2}, \frac{3\sqrt{3}}{16}\right)$, $\left(-\frac{\sqrt{3}}{2}, -\frac{3\sqrt{3}}{16}\right)$ | 1+1+1A | $\begin{cases} x = \dots \\ y = \dots \end{cases}$ |
| | 6 | |
| (c) * must label end at $-1$, 1 | 1 | general shape |
| | 1* | range |
| | 1 | tangent at $(0,0)$ |
| | 3 | |




G.F. 405 (2/77)

| Solutions | Marks | Remarks |
|---|---|---|
| 11 (cont'd) | | |
| (d) Putting $x = \sin\theta$, $dx = \cos\theta\, d\theta$ | 1A | |
| when $x = 0$, $\theta = 0$; $x = 1$, $\theta = \frac{\pi}{2}$. | 1A | |
| Area bounded $= 2\int_0^1 x^3\sqrt{1-x^2}\,dx$ | 1M | for $2 \times$ area $\int_0^1 y\,dx$ or sum of areas $\left\lvert\int_{-1}^0 y\,dx\right\rvert + \left\lvert\int_0^1 y\,dx\right\rvert$ |
| $= 2\int_0^{\frac{\pi}{2}} \sin^3\theta\cos^2\theta\, d\theta$ | 1A | |
| $= \frac{4}{15}$ | 1A | |
| | 5 | |
| 12 (a) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ | 1A | |
| $\frac{1}{\sqrt{3}} = \tan 30^\circ = \dfrac{2\tan 15^\circ}{1-\tan^2 15^\circ}$ | 1M | |
| $\therefore \tan^2 15^\circ + 2\sqrt{3}\tan 15^\circ - 1 = 0$ (any equivalent form) | 1A | |
| $\tan 15^\circ = \dfrac{-2\sqrt{3} \pm\sqrt{12+4}}{2}$ | | |
| $= -\sqrt{3} \pm 2$ | | |
| $= 2-\sqrt{3}$ (−ve root rejected) | 1A | no need to simplify |
| | 4 | |
| (b) (i) $BT = \dfrac{10}{\tan 15^\circ} = \boxed{\dfrac{10}{2-\sqrt{3}}}$ $(= \boxed{37.32})$ | 1A | **Alt** $AC = \dfrac{10}{\sin 30^\circ}$ $= 20$ (1A) |
| $AT = \dfrac{10}{\tan 30^\circ} = \boxed{10\sqrt{3}}$ $(= \boxed{17.32})$ | 1A | $\angle ACB = (30^\circ - 15^\circ)$ $= 15^\circ$ (1A) |
| $\therefore AB = \dfrac{10}{2-\sqrt{3}} - 10\sqrt{3}$ | | |
| $= 20$ | 1A | $\therefore AB = AC = 20$ (1A) |



| Solutions | Marks | Remarks |
|---|---|---|

12 (Cont'ed)

(b)(ii) $\tan\phi = \dfrac{10\sqrt{3}}{h}$ $\left(\dfrac{17.3}{h}\right)$ — 1A

$\tan(\theta+\phi) = \dfrac{10}{(2-\sqrt{3})h}$ — 1A

$\tan(\theta+\phi) = \dfrac{\tan\theta + \tan\phi}{1-\tan\theta\tan\phi}$ — 1A

$\dfrac{10}{(2-\sqrt{3})h} = \dfrac{\tan\theta + \frac{10\sqrt{3}}{h}}{1-\tan\theta\times\frac{10\sqrt{3}}{h}}$ — 1M + 1A

$= \dfrac{h\tan\theta + 10\sqrt{3}}{h - 10\sqrt{3}\tan\theta}$

$(2-\sqrt{3})h^2\tan\theta + 10\sqrt{3}(2-\sqrt{3})h = 10h - 100\sqrt{3}\tan\theta$

$[(2-\sqrt{3})h^2 + 100\sqrt{3}]\tan\theta = 10h - (20\sqrt{3}-30)h$

$\tan\theta = \dfrac{(40-20\sqrt{3})h}{(2-\sqrt{3})h^2 + 100\sqrt{3}}$

$= \dfrac{20h}{h^2 + 100(3+2\sqrt{3})}$

(iii) If AB subtends equal angles at D & C,

since $\angle ACB = \angle ABC = 15^\circ$

$\theta = \angle ADB = 15^\circ$ — 1A

$\therefore \tan\theta = \dfrac{20h}{h^2+100(3+2\sqrt{3})}$

$= 2-\sqrt{3}$ — 1M

$(2-\sqrt{3})h^2 - 20h + 100\sqrt{3} = 0$ (any equivalent form) — 1A

$h = \dfrac{20 \pm \sqrt{400 - 400\sqrt{3}(2-\sqrt{3})}}{2(2-\sqrt{3})}$

$= \dfrac{10 \pm 10\sqrt{4-2\sqrt{3}}}{2-\sqrt{3}}$

$= \dfrac{10 \pm 10(\sqrt{3}-1)}{2-\sqrt{3}}$ (64.64 or 10)

$= \dfrac{10\sqrt{3}}{2-\sqrt{3}}$ (64.64, $h=10$ rejected) — 1A

$= 10(3+2\sqrt{3})$

Remarks:

Alt

If $\angle ADB = \angle ACB$, ABDC are concyclic

$\angle\phi = \angle ABC$

$= 15^\circ$ — 1A

$h = \dfrac{AT}{\tan 15^\circ}$ — 1M

$= \dfrac{10\sqrt{3}}{2-\sqrt{3}}$ — 1A

$= 10(3+2\sqrt{3})$



G.F. 405 (2/77)

| Solutions | Marks | Remarks |
|---|---|---|
| (12) (Cont'd) | | |
| $\theta$ is greatest when $\tan\theta$ is greatest. | | |
| $\dfrac{d(\tan\theta)}{dh} = \dfrac{20(h^2+100(3+2\sqrt{3}))-40h^2}{[h^2+100(3+2\sqrt{3})]^2}$ | 1M | Attempt to diff |
| $= \dfrac{2000(3+2\sqrt{3})-20h^2}{[h^2+100(3+2\sqrt{3})]^2}$ | | |
| $\dfrac{d(\tan\theta)}{dh}=0 \Rightarrow h = 10\sqrt{3+2\sqrt{3}}$ $\fallingdotseq \boxed{25.42}$ (-ve root rejected) | 2A | |
| If $h \lessgtr 10\sqrt{3+2\sqrt{3}}$ slightly, $\dfrac{d(\tan\theta)}{dh} \gtrless 0$ | 1M | Attempt to check |
| $\therefore$ when $\theta$ is max., $h = 10\sqrt{3+2\sqrt{3}}$ | | |
| | 16 | |



G.F. 405 (2/77)