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Additional Mathematics I

MARKING SCHEME

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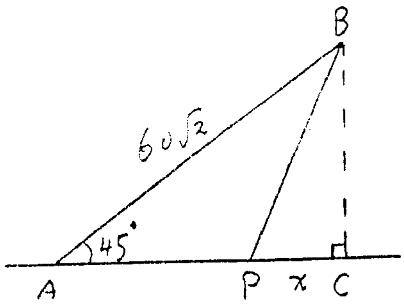
Solutions	Marks	Remarks
$2^x = 10^{x+1}$		
$x \log 2 = (x+1)\log 10$	1M+1A	1M for taking log.
$x(\log 2 - 1) = 1$ or $(0.3010-1)x = 1$	1A	
$x = \frac{1}{\log 2 - 1}$		
$= -1.431$	1A	or any figure roundable to -1.431
$= -1.43$ (correct to 3 sig fig.)	1A	
	5	
$\frac{\log\sqrt[3]{4} + \log\sqrt[3]{25} - \log\sqrt[3]{9}}{\log 8 + \log 5 - \log 12}$		
$= \frac{\frac{1}{3}(\log 4 + \log 25 - \log 9)}{\log 8 + \log 5 - \log 12}$	1M	$\log a^p = p\log a$
$= \frac{1}{3}\frac{\log\frac{4\times 25}{9}}{\log\frac{8\times 5}{12}}$	1M + 1M	$\log a + \log b = \log ab$; $\log a - \log b = \log\frac{a}{b}$
$= \frac{1}{3}\frac{\log\frac{2^2\times 5^2}{3^2}}{\log\frac{2\times 5}{3}}$		
$= \frac{2}{3}$	2A	
	5	
Alternatively,		
$\frac{\log\sqrt[3]{4} + \log\sqrt[3]{25} - \log\sqrt[3]{9}}{\log 8 + \log 5 - \log 12} = \frac{\log 2^{\frac{2}{3}} + \log 5^{\frac{2}{3}} - \log 3^{\frac{2}{3}}}{\log 2^3 + \log 5 - \log 2^2\times 3}$	1M	Expressing in powers of 2, 3, 5
$= \frac{\frac{2}{3}[\log 2 + \log 5 - \log 3]}{3\log 2 + \log 5 - 2\log 2 - \log 3}$	1M + 1M	$\log a^p = p\log a$; $\log ab = \log a + \log b$
$= \frac{2}{3}$	2A	
	5	

Solution	Marks	Remarks
(3) $\dfrac{d}{dt}\left(\dfrac{\sin kt}{3+2\cos kt}\right)=\dfrac{(3+2\cos kt)k\cos kt-\sin kt(-2k\sin kt)}{(3+2\cos kt)^2}$	1M + 1M + 1A	Chain Rule Quotient Rule
$=\dfrac{3k\cos kt+2k(\cos^2 kt+\sin^2 kt)}{(3+2\cos kt)^2}$		—
$=\dfrac{3k\cos kt+2k}{(3+2\cos kt)^2}$		
When $t=\frac{3\pi}{2k}$, $\dfrac{d\theta}{dt}=\dfrac{3k\cos\frac{3\pi}{2}+2k}{(3+2\cos\frac{3\pi}{2})^2}$	1M	Substitution
$=\dfrac{2k}{9}$	2A	
	6	
(4) $-1-i=\sqrt{2}\left[\cos(-\frac{3}{4}\pi)+i\sin(-\frac{3}{4}\pi)\right]$ (or $\sqrt{2}\operatorname{cis}(-\frac{3\pi}{4})$)	1A	or $\frac{5}{4}\pi$, $225°$, $-135°$
$1-i=\sqrt{2}\left[\cos(-\frac{1}{4}\pi)+i\sin(-\frac{1}{4}\pi)\right]$	1A	or $\frac{7}{4}\pi$, $315°$, $-45°$
$\dfrac{-1-i}{(1-i)^5}=\dfrac{\sqrt{2}\left[\cos(-\frac{3}{4}\pi)+i\sin(-\frac{3}{4}\pi)\right]}{\left[\sqrt{2}\,(\cos(-\frac{1}{4}\pi)+i\sin(-\frac{1}{4}\pi))\right]^5}$		
$=\dfrac{\sqrt{2}\left[\cos(-\frac{3}{4}\pi)+i\sin(-\frac{3}{4}\pi)\right]}{(\sqrt{2})^5\left[\cos(-\frac{5}{4}\pi)+i\sin(-\frac{5}{4}\pi)\right]}$	1M	De Moivre's Thm
$=\frac{1}{4}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$	1M +1A	$\dfrac{\operatorname{cis}\theta}{\operatorname{cis}\phi}=\operatorname{cis}(\theta-\phi)$
(or $=\dfrac{-1-i}{4(-1+i)}=\dfrac{(-1-i)^2}{4(-1+i)(-1-i)}$)	(1M)	For multiplying $-1-i$ in denominator
$=\frac{1}{4}i$	1 ~~2~~ A	
	6	
(5) $y=x^3-9x^2+30x+4$		
Slope of tangent $=\dfrac{dy}{dx}=3x^2-18x+30$	1M+1A	1M for attempt to diff.
A tangent // x-axis iff $3x^2-18x+30=0$ for some x.		
Since discriminant $=18^2-4\times3\times30$ $=-36<0$	2M+1A	or $3x^2-18x+30$ $=3\{(x-3)^2+1\}$ 2M+1A $\neq 0$
$3x^2-18x+30=0$ has no real roots. Hence tangent are never // x-axis.	1M	Association of "slope = 0" with "tangent // x-axis".
	6	

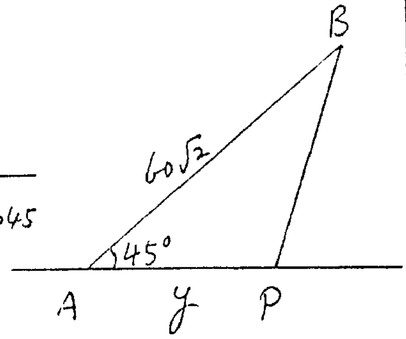
G.F. 405 (2/77)

Solutions	Marks	Remarks
(6) $\frac{d}{dx}[x+x^2+\dots+x^{n-1}+x^n]$		
$= 1+2x+\dots+(n-1)x^{n-2}+nx^{n-1}$	1A	
$\frac{d}{dx}\left[\frac{x(x^n-1)}{x-1}\right] = \frac{(x-1)[(n+1)x^n-1]-(x^{n+1}-x)}{(x-1)^2}$	1M +1A	Quotient Rule
$= \frac{nx^{n+1}-(n+1)x^n+1}{(x-1)^2}$		
$\therefore 1+2x+\dots+(n-1)x^{n-2}+nx^{n-1} = \frac{nx^{n+1}-(n+1)x^n+1}{(x-2)^2}$	1A	
Putting $n=10, x=2$, $1+2(2)+3(2)^2+\dots+9(2)^8+10(2)^9$		
$= \frac{10\times 2^{11}-11\times 2^{10}+1}{(2-1)^2}$	1M	
$= 9\times 2^{10}+1$	1A	no mark for direct calculation
$= 9217$ (9208 from tables)		
	6	
(7) $\lvert x-2\rvert \le 1 \iff -1 \le x-2 \le 1$	1M	e.g. 1A for ① $1<x<3$ ② $1\le x\le a$ ③ $b\le x\le 3$
$\iff 1 \le x \le 3$	2A	
$1\le x\le 3 \Rightarrow 1 \le x^2 \le 9$	1M	Graphical method or Checking end-points + sketching graph to support
$-5 \le x^2-6 \le 3$	1M	
$\therefore \lvert x^2-6\rvert \le 5$	1A	
the max. value of x^2-6 is 5	6	

Solutions	Marks	Remarks
8 (a) At B, $\frac{ds}{dt} = 0$	2M	
$t = 60$ (sec)	1A	
The boat stops after 60 sec		
$S = \int_0^{60} \frac{ds}{dt}\,dt$	1A+1M +1M	1A 0 1M 60 1M $\int \frac{ds}{dt}\,dt$
$= \int_0^{60} \sqrt{2}\left(2 - \frac{t}{30}\right)dt$		Alt. method $s = \int \frac{ds}{dt}\,dt$ 1M $= 2\sqrt{2}t - \frac{\sqrt{2}t^2}{60} + C$ 1A
$= \left[2\sqrt{2}t - \frac{\sqrt{2}t^2}{60}\right]_0^{60}$	1A	$t = 0$ 1M $c = 0$ $t = 60$ 1M
$= 60\sqrt{2}$ m	1A	$S = 60\sqrt{2}$, 1A
$\therefore AB = 60\sqrt{2}$ m	8	
(b) $BC = AB\cos 45°$		
$= 60$	1A	
$AC = BC = 60$	1A	
Let $PC = x$, $AP = 60 - x$		
$PB = \sqrt{PC^2 + BC^2}$		
$= \sqrt{x^2 + 3600}$	1A ~~2A~~	
Time required $t = \frac{60 - x}{5} + \frac{\sqrt{x^2+3600}}{3}$	1M+1M	
$\frac{dt}{dx} = -\frac{1}{5} + \frac{2x}{6\sqrt{x^2+3600}}$	2A	
$\frac{dt}{dx} = 0 \Rightarrow \frac{1}{5} = \frac{x}{3\sqrt{x^2+3600}}$	1M	
$\Rightarrow 25x^2 = 9x^2 + 9 \times 3600$	1A	
$x = 45$ ($x = -45$ rejected)	1A	Accept giving 45 also or $x = \pm 45$



Solutions	Marks	Remarks
8 (cont'd) On checking, it is found that t is a min. at $x=45$		Optional
$t = \dfrac{60-45}{5} + \dfrac{\sqrt{45^2+3600}}{3}$	1M	
$= 28$ sec.	1A	
	12	
Alt		
(b) Let $AP = y$		
$PB = \sqrt{(60\sqrt{2})^2 + y^2 - 120\sqrt{2}\, y\cos 45}$	1A	
$= \sqrt{y^2 - 120y + 7200}$	2A ~~1A~~	
$t = \dfrac{y}{5} + \dfrac{\sqrt{y^2-120y+7200}}{3}$	1M+1M	
$\dfrac{dt}{dy} = \dfrac{1}{5} + \dfrac{1}{6}\dfrac{2y-120}{\sqrt{y^2-120y+7200}}$	2A	
$\dfrac{dt}{dy} = 0 \Rightarrow \dfrac{3\sqrt{y^2-120y+7200} + 5y - 300}{15} = 0$	1M	
$\Rightarrow 9(y^2-120y+7200) = (300-5y)^2$		
$= 90000 - 3000y + 25y^2$		
$y^2 - 120y + 1575 = 0$	1A ~~1A~~	
$(y-15)(y-105) = 0$		
$y = 15$ ($y = 105$ rejected)	1A	Accept "$y=15$" or "$y=15$ or 105"
On checking, t is min. at $y=15$		
$t = \dfrac{15}{5} + \dfrac{\sqrt{15^2-120\times15+7200}}{3}$	1M	
$= 28$ (sec)	1A	If a cand. write: $t = 28$ or 46, no mark.
	12	



Solutions	Marks	Remarks
9 (a) $f(x) \equiv x^3 - (p+1)x^2 + (p-q)x + q$		
$f(1) = 1 - (p+1) + (p-q) + q$ $= 0$	1M	
$\therefore (x-1)$ is a factor of $f(x)$	1A	
$f(x) \equiv (x-1)(x^2 - px - q)$	1A	
$x = 1$ is a solution of $f(x) = 0$		
Let $\sin A = 1$. $A = 90°$	1+1A	
$\therefore \triangle ABC$ is right-angled		
$\sin B$ and $\sin C$ are the roots of $x^2 - px - q = 0$	1M	
Since ABC is a $\triangle$, $\sin B, \sin C \neq 0$, $\therefore q \neq 0$	1A	
	7	
(b) $R_1 = f(0)$ $= q$	1A	
$R_2 = f(p)$ $= p^3 - (p+1)p^2 + (p-q)p + q$ $= q - pq$	1A	
$\frac{2q}{p} = R_1 - R_2$ $= q - (q - pq)$ $= pq$		
$q(p^2 - 2) = 0$	1A	
Since $q \neq 0$, $p = \pm\sqrt{2}$	1+1A	
$\sin B + \sin C = p$	1M	
$= \sqrt{2}$ (-ve root rejected)	1M	rejecting -ve root and subst.

Solutions	Marks	Remarks
9 (Cont'd) As $B+C=90^\circ$		
$C=90-B$		
$\sin B+\cos B=\sqrt{2}$	1M+1A	
$\frac{1}{\sqrt{2}}\sin B+\frac{1}{\sqrt{2}}\cos B=1$		$\sin^2 B+\cos^2 B+2\sin B\cos B=2$
$\sin(45^\circ+B)=1$	1M	$\sin 2B=1$
$45^\circ+B=90^\circ$		$2B=90^\circ$
$B=45^\circ$, $C=45^\circ$	1A	$B=45^\circ$, $C=45^\circ$
$\therefore$ $\triangle ABC$ is isosceles		
$q=-\sin B\sin C$	1A	May be awarded if given at end of (a)
$=-\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}=-\frac{1}{2}$	1A	
	13	
10 (a) $f(x)=2x^3-9x^2+12x-5$		
$f'(x)=6x^2-18x+12$	1A	
$=6(x^2-3x+2)$		
$=6(x-1)(x-2)$		
$f'(x)=0 \Leftrightarrow x=1$ or 2	1M+1A	
$f''(x)=12x-18$	1A	
At $x=1$, $f''(x)=-6<0$		
$\therefore$ $(1,0)$ is a max pt	1A	
At $x=2$, $f''(x)=6>0$		
$\therefore$ $(2,-1)$ is a min. pt.	1A	

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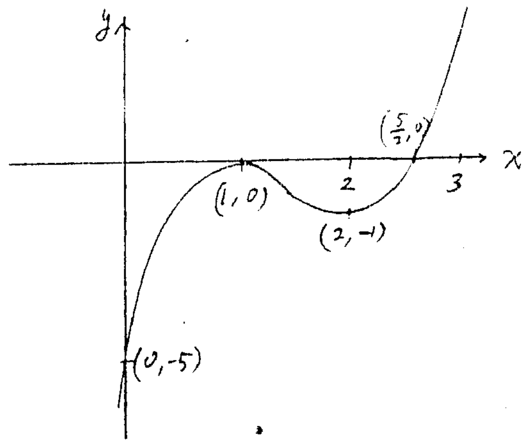
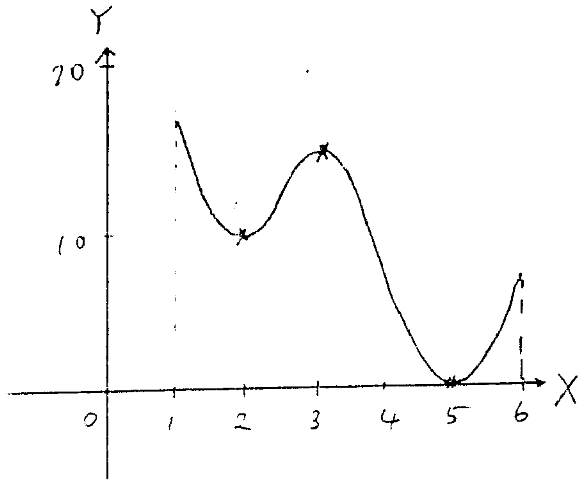
Solutions	Marks	Remarks
10 (Cont'd) $f(x) = 2x^3 - 9x^2 + 12x - 5$		
$= (x-1)(x-1)(2x-5)$		
$(1, 0)$, $(\frac{5}{2}, 0)$ and $(0, -5)$ are the points where the curve & the axes meet	0+1+1A	
	1+1	1 mark for shape 1 mark for marking the four points
	10	
(b) (i) Since $\frac{dY}{dX} = 0$ at $X = 2, 3, 5$		
Y has stationary values there	1 1M	Accept max/min
At $X = 2$, $\frac{d^2Y}{dX^2} > 0$, $(2, 10)$ is a min-pt.	1+1A	1A for sign of $\frac{d^2Y}{dX^2}$ 1A for correct conclusi
At $X = 3$, $\frac{d^2Y}{dX^2} < 0$, $(3, \cancel{14}\ 15)$ is a max-pt.	1+1A	
At $X = 5$, $\frac{d^2Y}{dX^2} > 0$, $(5, 0)$ is a min. pt.	1+1A	
	2+1	2 for shape 1 for $1 < X < 6$
	10	

G.F. 405 (2/77)

Solutions	Marks	Remarks
(11) (a) Let $z = a+bi$		
$\bar{z} = a-bi$	1A	
(i) $\lvert z\rvert^2 = a^2+b^2$	1A	
$z\bar{z} = (a+bi)(a-bi)$		
$= a^2+b^2$	1A	
$= \lvert z\rvert^2$		
(ii) $-\frac{i}{2}(z-\bar{z}) = -\frac{i}{2}\left[(a+bi)-(a-bi)\right]$		
$= -\frac{i}{2}(2bi)$		
$= b$	1A	
$= \operatorname{Im}(z)$	1A	
	5	
(b)(i) $\lvert p-q\rvert = \lvert p+q\rvert \Rightarrow \lvert p-q\rvert^2 = \lvert p+q\rvert^2$	1M	
$\Rightarrow (p-q)(\overline{p-q}) = (p+q)(\overline{p+q})$	1M	
$\Rightarrow (p-q)(\bar{p}-\bar{q}) = (p+q)(\bar{p}+\bar{q})$	1M	
$\Rightarrow p\bar{p}+q\bar{q}-p\bar{q}-\bar{p}q = p\bar{p}+q\bar{q}+p\bar{q}+\bar{p}q$	1A	
$\therefore\ 2(p\bar{q}+\bar{p}q)=0$		
$p\bar{q}+\bar{p}q=0$	1A	
$\operatorname{Im}\left(\frac{ip}{q}\right) = -\frac{i}{2}\left[\frac{ip}{q} - \left(\overline{\frac{ip}{q}}\right)\right]$	1M	*
$= -\frac{i}{2}\left[\frac{ip}{q} - \frac{\bar{i}\,\bar{p}}{\bar{q}}\right]$	1M	
$= -\frac{i}{2}\left[\frac{ip\bar{q}-\bar{i}\bar{p}q}{q\bar{q}}\right]$		
$= -\frac{i^2}{2}\left[\frac{p\bar{q}+\bar{p}q}{q\bar{q}}\right]$	1A	
$= 0$	1M	

Solutions	Marks
11 (Cont'd) (b)(ii) $\operatorname{Im}\left(\frac{ip}{q}\right)=0 \Rightarrow \frac{ip}{q}$ is real	2M
$\therefore \frac{p}{q}$ is purely imaginary	1M
$\arg\left(\frac{p}{q}\right)=\pm\frac{\pi}{2}$ (Accept $\frac{\pi}{2}$)	1A
$\arg(p)-\arg(q)=\pm\frac{\pi}{2}$	1M
$OP \perp OQ$	1A
	15
Alt	
(b)(i) Let $p=a+bi$, $q=c+di$	
$\lvert p-q\rvert=\sqrt{(a-c)^2+(b-d)^2}$	1A
$=\sqrt{a^2+b^2+c^2+d^2-2(ac+bd)}$	~~1A~~
$\lvert p+q\rvert=\sqrt{(a+c)^2+(b+d)^2}$	1A
$=\sqrt{a^2+b^2+c^2+d^2+2(ac+bd)}$	~~1A~~
$\lvert p-q\rvert=\lvert p+q\rvert$	
$\Rightarrow \sqrt{a^2+b^2+c^2+d^2-2(ac+bd)}=\sqrt{a^2+b^2+c^2+d^2+2(ac+bd)}$	1M
$\therefore ac+bd=0$	1A
$p\bar{q}+\bar{p}q=(a+bi)(c-di)+(a-bi)(c+di)$	1A
$=ac+bd+(bc-ad)i+ac+bd+(ad-bc)i$	
$=2(ac+bd)=0$	1A
$\operatorname{Im}\left(\frac{ip}{q}\right)=\operatorname{Im}\left[\frac{i(a+bi)}{c+di}\right]=\operatorname{Im}\left[\frac{(-b+ai)(c-di)}{c^2+d^2}\right]$	1A
$=\operatorname{Im}\left[\frac{\mp(ad\mp bc)+(ac+bd)i}{c^2+d^2}\right]$	1A
$=\frac{ac+bd}{c^2+d^2}=0$	1A

Remarks

Alt (i) $\lvert\vec{p}\rvert^2=a^2+b^2$ (1A)

$\lvert\vec{q}\rvert^2=c^2+d^2$ (1A)

$\lvert\vec{p}-\vec{q}\rvert^2=(a-c)^2+(b-d)^2$ (1A)

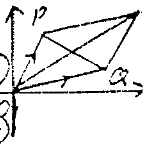
$=a^2+b^2+c^2+d^2-2(ac+bd)$

$=a^2+b^2+c^2+d^2$ (1A)

$\therefore \lvert\vec{p}\rvert^2+\lvert\vec{q}\rvert^2=\lvert\vec{p}-\vec{q}\rvert^2$

$OP\perp OQ$ (2M)

Alt (ii)



$OR=\lvert\vec{p}+\vec{q}\rvert$ (1A)

$PQ=\lvert\vec{p}-\vec{q}\rvert$ (1A)

$\lvert\vec{p}+\vec{q}\rvert=\lvert\vec{p}-\vec{q}\rvert$ (1M)

$\therefore$ OPRQ is a rectangle (2M)

$OP\perp OQ$ (1A)

$\tan(\arg p)=\frac{b}{a}$ (1)A

$\tan(\arg q)=\frac{d}{c}$ (1)A

$\tan(\arg p-\arg q)=\frac{\frac{b}{a}-\frac{d}{c}}{1+\frac{b}{a}\cdot\frac{d}{c}}=\infty$ (1)A

$\arg p-\arg q=\frac{\pi}{2}$ (1)M

$\therefore OP\perp OQ$ (1)A

$\frac{p}{q}=\frac{(ac+bd)+(bc-ad)i}{c^2+d^2}$

$\tan\left(\arg\frac{p}{q}\right)=\frac{bc-ad}{ac+bd}$ (1)

$=\infty$ (1)

$\therefore \arg\frac{p}{q}=\frac{\pi}{2}$ (1)A

$\arg p-\arg q=\frac{\pi}{2}$ (1)M

$\therefore OP\perp OQ$ (1)A

$m_1=\frac{b}{a}$ (1A)

$m_2=\frac{d}{c}$ (1A)

$m_1m_2=\frac{b}{a}\cdot\frac{d}{c}$ (1M)

$=-\frac{ac}{ac}$ (1M)

$=-1$ (1A)

$\therefore OP\perp OQ$ (1A)

G.F. 405 (2/77)

Solutions	Marks	Remarks
12 (a)(i) $t^2-(b+1)t+(b-1)=0$ ......(*)		
Discriminant $=(b+1)^2-4(b-1)$	1A	
$=b^2-2b+5$		
$=(b-1)^2+4$	1A	
Since discriminant >0, λ_1, λ_2 are real and distinct.	1	
(ii) $(1-\lambda_1)(1-\lambda_2)=1-(\lambda_1+\lambda_2)+\lambda_1\lambda_2$		
$=1-(b+1)+(b-1)$	1M	
$=-1$	1A	
<0		
$\therefore$ either $1-\lambda_1>0$ and $1-\lambda_2<0$	1A	
or $1-\lambda_1<0$ and $1-\lambda_2>0$	1A	
i.e. either $\lambda_1<1<\lambda_2$		
or $\lambda_2<1<\lambda_1$		
	7	
(b) If λ is a root of (*),		
$\lambda^2-(b+1)\lambda+(b-1)=0$	1M	
$b(1-\lambda)=1+\lambda-\lambda^2$		
$b=\dfrac{1+\lambda-\lambda^2}{1-\lambda}$	1A	
$(1-\lambda)[(x^2+2x+b)-\lambda(x^2+1)]$		
$=(1-\lambda)\left[\left(x^2+2x+\dfrac{1+\lambda-\lambda^2}{1-\lambda}\right)-\lambda(x^2+1)\right]$	1M	
$=(1-\lambda)\left[\dfrac{(1-\lambda)(x^2+2x)+1+\lambda-\lambda^2-\lambda(1-\lambda)(x^2+1)}{1-\lambda}\right]$		
$=(1-2\lambda+\lambda^2)x^2+2(1-\lambda)x+1$	1A	
$=[(1-\lambda)x+1]^2$	1A	
	5	

Solutions	Marks	Remarks
(12) (c) $(1-\lambda)[(x^2+2x+b)-\lambda(x^2+1)] = [(1-\lambda)x+1]^2$		
≥ 0 (for all real x)	1M	
$\therefore (1-\lambda)\left[\frac{x^2+2x+b}{x^2+1}-\lambda\right] \geq 0 \quad (x^2+1>0)$	1M	
Since λ_1, λ_2 are roots of (*)		
$(1-\lambda_1)\left[\frac{x^2+2x+b}{x^2+1}-\lambda_1\right] \geq 0$	1M	
and $(1-\lambda_2)\left[\frac{x^2+2x+b}{x^2+1}-\lambda_2\right] \geq 0$ for all real x	1M	
If $\lambda_1 < \lambda_2$, by (a) $\lambda_1 < 1 < \lambda_2$	1M	
$1-\lambda_1 > 0 \Rightarrow \frac{x^2+2x+b}{x^2+1} - \lambda_1 \geq 0$	1M	
$\frac{x^2+2x+b}{x^2+1} \geq \lambda_1$		
and $1-\lambda_2 < 0 \Rightarrow \frac{x^2+2x+b}{x^2+1} - \lambda_2 \leq 0$	1M	
$\frac{x^2+2x+b}{x^2+1} \leq \lambda_2 \quad \forall x \in R$		
$\therefore \lambda_1 \leq \frac{x^2+2x+b}{x^2+1} \leq \lambda_2$	1	
	8	