香 港 考 試 局

HONG KONG EXAMINATIONS AUTHORITY

一 九 八 二 年 香 港 中 學 會 考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1982

附加數學
試卷一

二小時完卷

上午八時三十分至十時三十分

本試卷必須用英文作答

ADDITIONAL MATHEMATICS
PAPER I

Two hours

8.30 a.m.—10.30 a.m.

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

SECTION A (40 marks)
Answer ALL questions in this section.

1. If $2^x = 10^{x+1}$, find x, giving your answer correct to 3 significant figures. (5 marks)

2. Without using tables or calculators, simplify

$$\frac{\log\sqrt[3]{4} + \log\sqrt[3]{25} - \log\sqrt[3]{9}}{\log 8 + \log 5 - \log 12} .$$

(5 marks)

3. Given that $\theta = \dfrac{\sin kt}{3 + 2\cos kt}$, where k is a non-zero constant, find the value of $\dfrac{d\theta}{dt}$ when $t = \dfrac{3\pi}{2k}$.

(6 marks)

4. Express the two complex numbers $-1 - i$ and $1 - i$ in polar form. Hence simplify $\dfrac{-1 - i}{(1 - i)^5}$.

(6 marks)

5. Show that the tangents to the curve $y = x^3 - 9x^2 + 30x + 4$ cannot be parallel to the x-axis.

(6 marks)

6. Given: $x + x^2 + x^3 + \ldots + x^{n-1} + x^n = \dfrac{x(x^n - 1)}{x - 1}$,

where $x \neq 1$ and n is a positive integer. By differentiating both sides of the above identity with respect to x, find the sum

$$1 + 2x + 3x^2 + \ldots + (n - 1)x^{n-2} + nx^{n-1}.$$

Hence find the value of $1 + 2(2) + 3(2)^2 + \ldots + 9(2)^8 + 10(2)^9$. (6 marks)

7. Let x be a real number satisfying $|x - 2| \leq 1$. Solve the inequality and hence find the greatest value of $|x^2 - 6|$. (6 marks)

SECTION B (60 marks)
Answer any THREE questions from this section.
Each question carries 20 marks.

8. A boy sits by the side of a shallow pool and plays with a radio-controlled boat. The boat starts from A in a direction which makes an angle of 45° with the side SS' of the pool, as shown in Figure 1. However, the control is not working properly. As a result, the boat moves in a straight line but with reducing speed given by

$$\frac{ds}{dt} = \sqrt{2}\left(2 - \frac{t}{30}\right),$$

where s metres is the distance covered by the boat t seconds after starting. The boat finally stops at B.

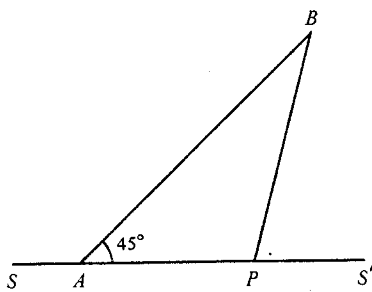

Figure 1

(a) Find the time taken by the boat to reach B.
Using integration, show that the distance between A and B is $60\sqrt{2}$ m. (8 marks)

(b) To get the boat back, the boy runs from A along the side of the pool to a point P and then across the pool along PB (see Figure 1). If he can run at 5 m/s on shore and 3 m/s in water, find the least time he needs to reach the point B. (12 marks)

9. Let $f(x) \equiv x^3 - (p + 1)x^2 + (p - q)x + q$, where p and q are constants. ABC is a triangle such that $\sin A$, $\sin B$ and $\sin C$ are the three roots of the equation $f(x) = 0$.

(a) By factorising $f(x)$, deduce that ΔABC has a right angle, and show that $q \neq 0$. (7 marks)

(b) Let R_1 and R_2 be the remainders when $f(x)$ is divided by x and $(x - p)$, respectively. If $R_1 - R_2 = \dfrac{2q}{p}$, find the possible values of p.
Hence show that ABC is an isosceles triangle and find the value of q. (13 marks)

10. (a) Sketch the graph of $f(x) = 2x^3 - 9x^2 + 12x - 5$. (10 marks)

(b) The graph of a function Y defined in the interval $1 < X < 6$ passes through the points (2, 10), (3, 15) and (5, 0). The graphs of $\dfrac{dY}{dX}$ and of $\dfrac{d^2Y}{dX^2}$ are shown in Figure 2 and Figure 3, respectively. Without finding the equation of the graph of Y:

(i) determine the maximum and minimum points of the graph of Y,

(ii) sketch the graph of Y.

(10 marks)

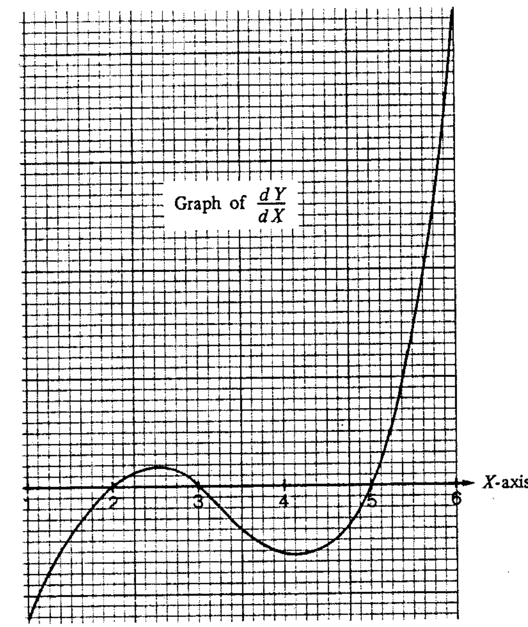

Figure 2

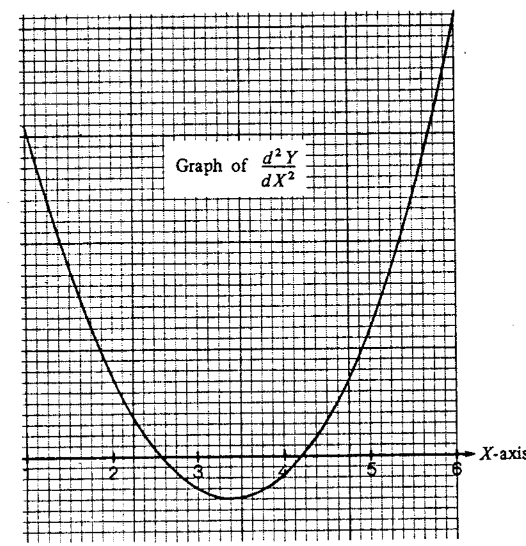

Figure 3

Go on to the next page

11. (a) Let $|z|$ and $\bar{z}$ denote, respectively, the modulus and conjugate of the complex number z. Show that

(i) $|z|^2 = z\bar{z}$,

(ii) the imaginary part of $z = -\frac{1}{2}i(z - \bar{z})$.

(5 marks)

(b) Let p and q be non-zero, distinct complex numbers such that $|p - q| = |p + q|$.

(i) Using the results in (a), or otherwise, show that

$$p\bar{q} + \bar{p}q = 0$$

and the imaginary part of $\frac{ip}{q} = 0$.

(ii) Let O, P and Q be three points on the Argand plane representing the complex numbers 0, p and q, respectively. By considering the argument of $\frac{p}{q}$, or otherwise, show that $OP \perp OQ$.

(15 marks)

12. Let λ_1 and λ_2 be the roots of the quadratic equation

$$t^2 - (b + 1)t + (b - 1) = 0 \quad \text{..........................} \quad (*),$$

where b is a real number.

(a) (i) Show that λ_1 and λ_2 are real and distinct.

(ii) By proving $(1 - \lambda_1)(1 - \lambda_2) < 0$, deduce that either $\lambda_1 < 1 < \lambda_2$ or $\lambda_2 < 1 < \lambda_1$.

(7 marks)

(b) Let λ be one of the roots of $(*)$. Find b in terms of λ and hence express $(1 - \lambda)[(x^2 + 2x + b) - \lambda(x^2 + 1)]$ as a perfect square.

$b = \frac{\lambda^2 - \lambda - 1}{\lambda - 1}$

$= \{x(1-\lambda)+1\}^2$

(5 marks)

(c) Using the results of (a) and (b), show that if $\lambda_1 < \lambda_2$, then

$$\lambda_1 < \frac{x^2 + 2x + b}{x^2 + 1} < \lambda_2 \quad \text{for all real values of } x.$$

(8 marks)

END OF PAPER

香 港 考 試 局

HONG KONG EXAMINATIONS AUTHORITY

一 九 八 二 年 香 港 中 學 會 考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1982

附加數學 試卷二

二小時完卷

上午十一時十五分至下午一時十五分

本試卷必須用英文作答

ADDITIONAL MATHEMATICS PAPER II

Two hours

11.15 a.m.—1.15 p.m.

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.