香港考試局

HONG KONG EXAMINATIONS AUTHORITY

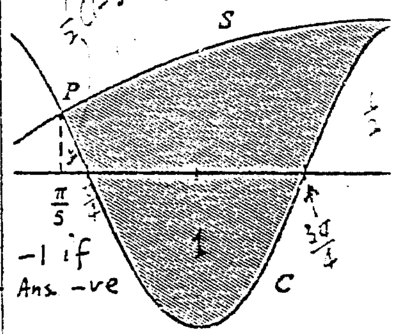
一九八一年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1981

Additional Mathematics II

MARKING SCHEME

評卷參考

Solutions	Marks	Remarks
(1) $\int(1+\cos\theta)^2 d\theta = \int(1+2\cos\theta+\cos^2\theta)\,d\theta$	1A	
$= \int\left(1+2\cos\theta+\frac{\cos 2\theta+1}{2}\right)d\theta$	1A	
$= \frac{3}{2}\theta + 2\sin\theta + \frac{\sin 2\theta}{4} + C$	1+1+1A	-1 if omit C
	5	
(2) Area of region $= \int_{\frac{\pi}{5}}^{\pi}\left(\sin\frac{x}{2}-\cos 2x\right)dx$	1M	
$= \left[-2\cos\frac{x}{2}-\frac{\sin 2x}{2}\right]_{\frac{\pi}{5}}^{\pi}$	1+1A	
$= 2\cos\frac{\pi}{10}+\frac{1}{2}\sin\frac{2\pi}{5}$	1+1A	-1 if Ans $-$ve
$= \frac{5}{2}\cos\frac{\pi}{10}$ $(\doteqdot 2.378)$		
	5	
Alt		
$\int_{\frac{\pi}{5}}^{\pi}\sin\frac{x}{2}\,dx = -2\cos\frac{x}{2}\Big\vert_{\frac{\pi}{5}}^{\pi}$	1A	
$= 2\cos\frac{\pi}{10}$	1A	
$\int_{\frac{\pi}{5}}^{\pi}\cos 2x\,dx = \frac{\sin 2x}{2}\Big\vert_{\frac{\pi}{5}}^{\pi}$	1A	
$= -\frac{1}{2}\sin\frac{2\pi}{5}$ $[-0.476$	1A	
Area $= 2\cos\frac{\pi}{10}+\frac{1}{2}\sin\frac{2\pi}{5}$	1M	-1 for $-$ve answer
(3) Putting $u^2 = 9-x$, $2u\,du = -dx$	1A	
When $x=0$, $u=3$; $x=9$, $u=0$.	1A	
$\int_0^9 \frac{x}{\sqrt{9-x}}dx = \int_3^0 \frac{-2u(9-u^2)}{u}du$	1M+1A	1M for change of limit 1A for integrand
$= \int_0^3 2(9-u^2)\,du$		
$= \left[18u-\frac{2}{3}u^3\right]_0^3$	1A	
$= 36$	1A	
	6	

Solutions	Marks	Remarks
(4) $12\cos 3x - 5\sin 3x = r\cos(3x+\theta)$		
$= r(\cos 3x\cos\theta - \sin 3x\sin\theta)$		
Putting $r\cos\theta = 12$, $r\sin\theta = 5$		
$r = \sqrt{12^2+5^2} = 13$	1A	
$\cos\theta = \frac{12}{13}$ $(=0.9231)$		
$\therefore \theta = \cos^{-1}\frac{12}{13}$		
$= 22.62^\circ$	1A	Accept 22.6°
$= 22^\circ 37'$		Accept $\pm 1'$
$12\cos 3x - 5\sin 3x = 13$		
$13\cos(3x+22.62^\circ) = 13$	1M	for sub.
$\cos(3x+22.62^\circ) = 1$		
$3x+22.62^\circ = 360^\circ \times n$, $n = 0, \pm1, \pm2, \ldots$	1A	$n = 0, \ldots$ optional
$\therefore x = 120^\circ \times n - 7.54^\circ$ $(7^\circ 32')$	1A	
$= 120^\circ \times n - 8^\circ$, $n = 0, \pm1, \pm2, \ldots$	1A	$n = 0, \ldots$ optional
(corr. to the nearest degree)	6	
(5) $\sin\theta + \cos\theta = \frac{h}{2}$	1A	
$\sin\theta\cos\theta = \frac{1}{2}$	1A	Alt
$h^2 = 4(\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta)$ for h^2	1M	$\sin\theta\cos\theta = \frac{1}{2} \Rightarrow \sin 2\theta =$
$= 4(1+1)$	1M+1A for sub.	$\therefore \theta = 45^\circ$, as $0^\circ < \theta < 90^\circ$
$h = 2\sqrt{2}$, since $0^\circ < \theta < 90^\circ$	1A	$\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$
	6	$\therefore h = 2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$
		$= 2\sqrt{2}$

5 Alt

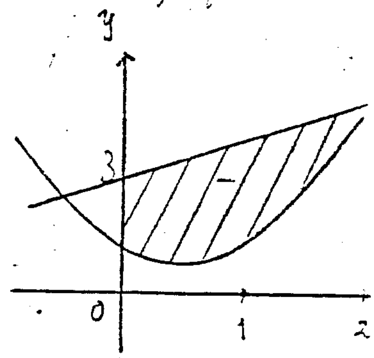
$2\sin^2\theta - h\sin\theta + 1 = 0 \quad \ldots (i)$ $2\cos^2\theta - h\cos\theta + 1 = 0 \quad \ldots (ii)$	1M + 1A	1M for sub.
(i) + (ii) $\quad 2 - h(\sin\theta + \cos\theta) + 2 = 0$ $h = \dfrac{4}{\sin\theta + \cos\theta}$ (i) − (ii) $\quad 2(\sin^2\theta - \cos^2\theta) - h(\sin\theta - \cos\theta) = 0$ $(\sin\theta - \cos\theta)[2(\sin\theta + \cos\theta) - h] = 0$	1M	
$\therefore \quad \sin\theta - \cos\theta = 0 \quad$ or $\quad \sin\theta + \cos\theta = \frac{h}{2}$	1+1A	
In both cases $\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$		
$\therefore h = 2\sqrt{2}$	1A	

Solutions	Marks	Remarks
3) (a) $C_2 - C_1$, $6x-3y-3=0$	1M	
$y=2x-1$		
Sub. in C_1, $x^2+(2x-1)^2+7(2x-1)+11=0$	1M	
$5x^2+10x+5=0$		
$\therefore x=-1$, $y=-3$	1A	
(b) Centre of $C_2=(-3,-2)$ [of $C_1=(0,-\frac{7}{2})$]	1A	(b) Alt. I Tangent to C_1 at P: $-1x+-3y+\frac{7}{2}(y+-3)+11=0$ or $2x-y-1=0$
Slope of line joining P and centre $=-\frac{1}{2}$ Slope of tangent $=2$	2A	
$\therefore$ common tangent at P is $y+3=2(x+1)$		
or $2x-y-1=0$	1A	(b) Alt III $\left.\frac{dy}{dx}\right\|_{x=-1}=2$ (2A)
	6	
(b) Alt II When C_1 and C_2 meet, the common chord is $6x-3y-3=0$ or $2x-y-1=0$	2A	Tangent is $2x-y-1=0$ (1A)
Since they touch each other ext., the above equation is that of the common tangent at P	1A	
4) Let $P=(x,y)$.		
$x=\frac{s+t}{2}$	1A	
$y=\frac{3}{2}(s-t)$	1A	
$s=x+\frac{y}{3}$, $t=x-\frac{y}{3}$ $\left(\text{or } \begin{matrix} s+t=2x \\ s-t=\frac{2}{3}y \end{matrix}\right)$	1A	
$ST=\sqrt{(s-t)^2+9(s+t)^2}=2$	1A	
$\Rightarrow \left(\frac{2y}{3}\right)^2+9(2x)^2=4$	1M	
$\therefore \frac{y^2}{9}+9x^2=1$	1A	
i.e. $81x^2+y^2-9=0$	6	

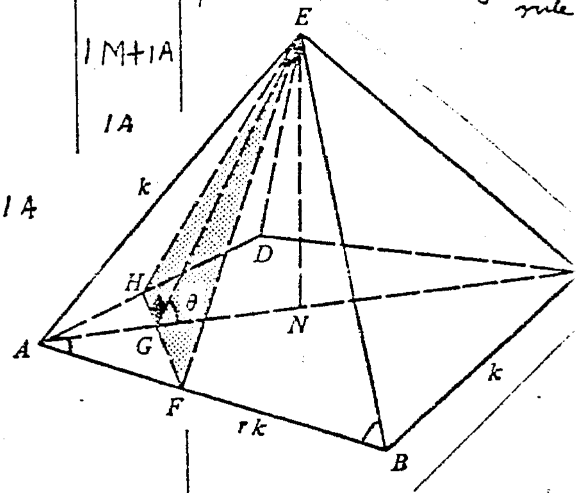
P.

Solutions	Marks	Remarks
8 (a) Putting $y=\sin x$, $dy=\cos x\,dx$	1A	
when $x=0$, $y=0$; $x=\frac{\pi}{2}$, $y=1$. $\}$	1A	
$\int_0^{\frac{\pi}{2}}\cos^3 x\sin^2 x\,dx=\int_0^1(1-y^2)y^2\,dy$	1M+1A	
$=\left[\frac{y^3}{3}-\frac{y^5}{5}\right]_0^1$	1A	
$=\frac{2}{15}\quad(\doteqdot 0.1333)$	1A	
	6	
(b) (i) $\frac{1}{x^2+3}-\frac{1}{(x+1)^2}=\frac{(x+1)^2-(x^2+3)}{(x^2+3)(x+1)^2}$		
$=\frac{2(x-1)}{(x^2+3)(x+1)^2}$, $(x\neq -1.)$	2A	
(ii) Putting $x=\sqrt{3}\tan\theta$, $dx=\sqrt{3}\sec^2\theta\,d\theta$	1A	
when $x=0$, $\theta=0$; $x=3$, $\theta=\frac{\pi}{3}$. $\}$	1A	
$\therefore\int_0^3\frac{dx}{x^2+3}=\int_0^{\frac{\pi}{3}}\frac{\sqrt{3}\sec^2\theta}{3(\tan^2\theta+1)}\,d\theta$	1M+1A	
$=\frac{\sqrt{3}}{3}\int_0^{\frac{\pi}{3}}d\theta$	1A	
$=\frac{\pi\sqrt{3}}{9}$	1A	
(iii) $\int_0^3\frac{2(x-1)}{(x^2+3)(x+1)^2}\,dx=\int_0^3\left[\frac{1}{x^2+3}-\frac{1}{(x+1)^2}\right]dx$	2M	Using result in (b)(i)
$=\int_0^3\frac{dx}{x^2+3}-\int_0^3\frac{dx}{(x+1)^2}$	1A	
$=\frac{\pi\sqrt{3}}{9}+\left[\frac{1}{x+1}\right]_0^3$	0+2A	
$=\frac{\pi\sqrt{3}}{9}-\frac{3}{4}\quad(\doteqdot -0.1454)$	1A	
	14	

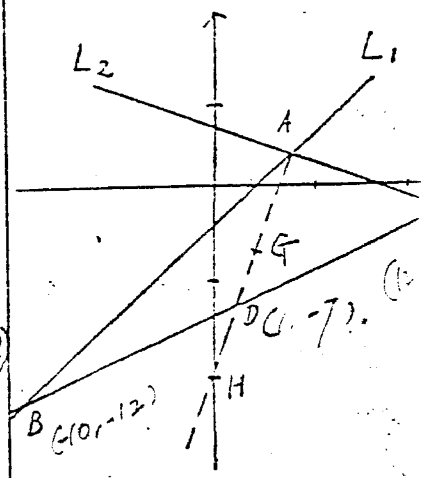
C

Solutions	Marks	Remarks
9 (a) $y = \int k(x-\frac{1}{4})dx$		
$= k(\frac{x^2}{2} - \frac{x}{4}) + C$	1A+1A	
Sub. $(x,y) = (-1,4), (0,1)$,	1M	
$\begin{cases} 4 = k(\frac{1}{2}+\frac{1}{4})+C \\ 1 = C \end{cases}$	1A	
$\therefore k = 4$	1A	
Equation of curve is $y = 2x^2 - x + 1$	1A	
	6	
(b) Solving $y = 2x^2 - x + 1$ with $y = 2x+3$	1M	
$2x^2 - 3x - 2 = 0$		
$\therefore x = 2$ or $-\frac{1}{2}$	1A	
Area of region $= \int_0^2 [(2x+3)-(2x^2-x+1)]dx$	1M+1M	Alt Area $= \int_a^b y\,dx$
$= \int_0^2 [-2x^2+3x+2]dx$	2A	$\int_0^2 (2x+3)dx = x^2+3x\Big\vert_0^2 = 10$
$= \left[-\frac{2}{3}x^3 + \frac{3}{2}x^2 + 2x\right]_0^2$		$\int_0^2 (2x^2-x+1)dx = \frac{2}{3}x^3 - \frac{x^2}{2} + x \ldots = \frac{16}{3}$
$= \frac{14}{3}$ $(\doteqdot 4.667)$	1A	Area req'd $= 10 - \frac{16}{3} = \frac{14}{3}$ (1M
	7	
(c) Volume $= \pi\int_0^2 (2x+3)^2 dx - \pi\int_0^2 (2x^2-x+1)^2 dx$	1M +1M	for $V = \pi\int_a^b y^2 dx$ for $V_1 - V_2$
$= \pi\int_0^2 (4x^2+12x+9)dx - \pi\int_0^2 [4x^4-4x^3+5x^2-2x+1]dx$		Alt $V_1 = \pi\int_0^2 (2x+3)^2 dx$
$= \pi\left[-\frac{4}{5}x^5 + x^4 - \frac{x^3}{3} + 7x^2 + 8x\right]_0^2$ −1 for 1 wrong max. 3	3A	$= \pi\int_0^2 (4x^2+12x+9)dx$
$= \pi\left[-\frac{4}{5}\times 32 + 16 - \frac{8}{3} + 7\times 4 + 8\times 2\right]$		$= \pi\left[\frac{4}{3}x^3 + 6x^2 + 9x\right]_0^2$
$= 31\frac{11}{15}\pi$ $(\doteqdot 31.73\pi \doteqdot 99.69)$	2A	$= 52\frac{2}{3}\pi$
	7	$V_2 = \pi\int_0^2 (2x^2-x+1)^2 dx$
If omit π, award at most 5 marks		$= \pi\left[\frac{4}{5}x^5 - x^4 + \frac{5}{3}x^3 - x^2 + x\right]$
		$= 20\frac{14}{15}\pi$
		$V = V_1 - V_2 = 31\frac{11}{15}\pi$ (1M+1A)

Solutions	Marks	Remarks
10(a) In $\triangle BFE$,		
$FE^2 = BE^2 + BF^2 - 2BE \cdot BF \cos FBE$	1M+1A	1M for idea of using cos rule
$= k^2 + r^2k^2 - 2rk^2 \cos 60°$	1A	
$= k^2(1 - r + r^2)$	1A	
In $\triangle AFG$, $FG \perp AC$	1A	
$FG^2 = (AF \sin FAG)^2$	1A	
$= [(1-r)k \sin 45°]^2$	1A	
$= \frac{k^2}{2}(1 - 2r + r^2)$	1A	
$\frac{k^2}{2}(1-r)^2$		
	8	
(b) In $\triangle EFG$,		
$EG = \sqrt{FE^2 - FG^2}$	1M	
$= \sqrt{k^2(1-r+r^2) - \frac{k^2}{2}(1-2r+r^2)}$	1A	
$= k\left(\frac{1+r^2}{2}\right)^{\frac{1}{2}}$	1A	
$EN = AE \cos 45°$	1M+1A	1A for 45°
$= \frac{k}{\sqrt{2}}$	1A	
$\therefore \sin\theta = \frac{EN}{EG}$	1M	
$= \frac{1}{\sqrt{1+r^2}}$	1A	
	8	
(c) Since $0 \le r \le 1$, $\frac{1}{\sqrt{2}} \le \sin\theta \le 1$	1+1A	Accept strict inequalities
$\therefore 45° \le \theta \le 90°$	1+1A	
	4	



Signs reversed no mark

Solutions	Marks	Remarks
1)(a) Solving L_1 and L_2, $3y-6=0$, $y=2$, $x=4$	1M	
	1A	
$\therefore A=(4,2)$.		
Let $D=(x_1,y_1)$, then $AG:GD=2:1$ (or $AD:DG=-3:1$)	1A (optional)	
$\therefore \begin{cases} t=\frac{2x_1+4}{2+1} \\ t-6=\frac{2y_1+2}{2+1}\end{cases}$		
$x_1=\frac{3t-4}{2}$	1A	
$y_1=\frac{3t-20}{2}$	1A	
	5	
(b) Slope of $AH=\frac{2--10}{4}=3$	1A	Alt: Eqn of AH is
Slope of $AG=\frac{2-(t-6)}{4-t}=3$	1M	$y+10=\frac{2+10}{4}x$ or $3x-y-10=0$ (1A)
$\therefore t=2$	1A	sub. $G(t,t-6)$ (1M) $t=2$ (1A)
$D=(1,-7)$	1A	
$AD=\sqrt{3^2+9^2}$, $\sqrt{90}$		
$=3\sqrt{10}$ ($\doteqdot 9.487$)	1A	
$\tan\angle BAD=\frac{3-1}{1+3\times1}$	1M	
$=\frac{1}{2}$	1A	
	7	
(c) $\sin BAD=\frac{1}{\sqrt{1^2+2^2}}=\frac{1}{\sqrt{5}}$ 0.447	1A	
Area of $\triangle ABD=\frac{1}{2}AB\times AD\sin BAD$	1M	
$=\frac{1}{2}\times14\sqrt{2}\times3\sqrt{10}\times\frac{1}{\sqrt{5}}$		
$=42$	1A	
$\therefore$ area of $\triangle ABC=84$ units	1A	
	4	

Solutions	Marks	Remarks
(11)(d) Let $P = (x, y)$.		
Area of $\triangle APD = \pm\frac{1}{2}[(2x-4y)+(-28-2)+(y+7x)]$	1+1A	for + and − signs optional
$= 42$	~~2A~~	
$\therefore$ locus of P is $9x-3y-30=\pm 84$		
i.e. $3x-y+18=0$	1A	
$3x-y-38=0$	1A	
Alt	4	
(d) Let $P=(x,y)$, $h=$ height of $\triangle ADC$ with AD as base.		
$\frac{1}{2}\times h \times 3\sqrt{10} = 42$		
$h = \frac{28}{\sqrt{10}}$	1A	
$\frac{3x-y-10}{\pm\sqrt{10}} = \frac{28}{\sqrt{10}}$	1A	
Locus of P is $3x-y+18=0$	1A	
$3x-y-38=0$	1A	

Solutions	Marks	Remarks
12) $L: y = mx + 2$		$x_1 + x_2 = -\frac{4m}{m^2+1}$
(a) $C: x^2 + y^2 = 1$		$x_1 x_2 = \frac{3}{m^2+1}$
Sub. L in C,		$y_1 + y_2 = \frac{4}{m^2+1}$
$x^2 + (mx+2)^2 = 1$	1M	$y_1 y_2 = \frac{4-m^2}{m^2+1}$
$(m^2+1)x^2 + 4mx + 3 = 0$	1A	Alt
$x = \frac{-4m \pm \sqrt{16m^2 - 12(m^2+1)}}{2(m^2+1)}$		$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$
$= \frac{-2m \pm \sqrt{m^2-3}}{m^2+1}$	1A	$= \frac{16m^2}{(m^2+1)^2} - \frac{12}{(m^2+1)}$
		$= \frac{4(m^2-3)}{(m^2+1)^2}$ (1A)
$y = mx + 2$		$(y_1 - y_2)^2 = m^2(x_1 - x_2)^2$
$= \frac{2 \pm m\sqrt{m^2-3}}{m^2+1}$	1A	$= \frac{4m^2(m^2-3)}{(m^2+1)^2}$ (1A)
Let $A = \left(\frac{-2m + \sqrt{m^2-3}}{m^2+1}, \frac{2 + m\sqrt{m^2-3}}{m^2+1}\right)$		
$B = \left(\frac{-2m - \sqrt{m^2-3}}{m^2+1}, \frac{2 - m\sqrt{m^2-3}}{m^2+1}\right)$		
$AB = \sqrt{\left(\frac{2\sqrt{m^2-3}}{m^2+1}\right)^2 + \left(\frac{2m\sqrt{m^2-3}}{m^2+1}\right)^2}$	1M	
$= \sqrt{\frac{4(m^4 - 2m^2 - 3)}{m^2+1}}$		
$= 2\sqrt{\frac{m^2-3}{m^2+1}}$	1A	
	6	

Solutions	Marks	Remarks
12 (b)(i) L meets C at two distinct points		In (i), (ii), (iii), 2M for 1st correct idea about length of AB.
iff $2\sqrt{\dfrac{m^2-3}{m^2+1}} > 0$	2M	or $\Delta = m^2-3$
i.e. $m < -\sqrt{3}$ or $m > \sqrt{3}$	1A	
(ii) L is a tangent to C iff $AB = 0$		
i.e. $m = \pm\sqrt{3}$	1A	
(iii) L does not meet C iff $AB \not< 0$ imaginary		
i.e. $-\sqrt{3} < m < \sqrt{3}$	1A	
	5	
(c) Since $m = \pm\sqrt{3}$, sub. in A or B of (a)		In this case, A, B are identical
$P = \left(\dfrac{-2m+\sqrt{m^2-3}}{m^2+1}, \dfrac{2+m\sqrt{m^2-3}}{m^2+1}\right)$	~~1A~~	
$= \left(-\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$	1+1A	
$Q = \left(\dfrac{-2m-\sqrt{m^2-3}}{m^2+1}, \dfrac{2-m\sqrt{m^2-3}}{m^2+1}\right)$ vice versa	~~1A~~	
$= \left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$	1+1A	
$\therefore$ eqn. of PQ is $y = \dfrac{1}{2}$ (or $2y-1=0$)	1A	
	5	
(d) Eqn. req'd is $x^2+y^2-1+k(2y-1)=0$.	2M+2A	
	4	