香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八一年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION 1981

附加數學 試卷一

ADDITIONAL MATHEMATICS PAPER I

二小時完卷

Two hours

上午八時三十分至十時三十分

8.30 a.m.—10.30 a.m.

本試卷必須用英文作答

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.

SECTION A (40 marks)

Answer ALL questions in this section.

1. Find the coefficient of x^2 in the expansion of $(1 + 2x)^4 (1 - x)^7$. (5 marks)

2. If $\log_3 2 = a$ and $\log_3 13 = b$, express $\log_{78} 52$ in terms of a and b. (5 marks)

3. If $y = \tan \frac{x-1}{x+1}$, find $\frac{dy}{dx}$. (5 marks)

4. Solve the quadratic equation $E : x^2 - 2x\cos\theta + 1 = 0$.
Hence form a quadratic equation whose roots are the nth powers of the roots of E.
Express the equation in its simplest form. (6 marks)

5. Let $f(x) = x^2 + ax + b$, where a and b are real.
Show that $f(x) \geq f(-\frac{a}{2})$ for all real values of x.
Hence, or otherwise, find the minimum value of $x^2 - \sqrt{13}x + 5$. (6 marks)

6. If a, b, x, y and z are numbers greater than 1 and $a^x = b^y = (ab)^z$,
show that $z = \frac{xy}{x+y}$. (6 marks)

7. Draw the graphs of $y = x^2$ and $y = |x - 2|$ for $-3 \leq x \leq 3$.
Hence solve the inequality $|x - 2| < x^2$. (7 marks)

SECTION B (60 marks)

Answer any THREE questions from this section.
Each question carries 20 marks.

8. Let $y = f(x) = \frac{2x}{x^2 + 1}$.

(a) Show that $f(-x) = -f(x)$. (2 marks)

(b) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. (5 marks)

(c) Find the turning points of $y = f(x)$ and determine whether they are maximum or minimum points. (7 marks)

(d) Sketch the curve $y = f(x)$ for $-\infty < x < \infty$.
Hence sketch (in the same coordinate system) the curve $y = f(x-1) = \frac{2(x-1)}{(x-1)^2 + 1}$. (6 marks)

9. A man is to make a tank of capacity V cubic metres from thin metal sheets. The tank is to consist of a right circular cylinder and two hemispheres, as shown in Figure 1. The cylinder is of length h metres and radius r metres.

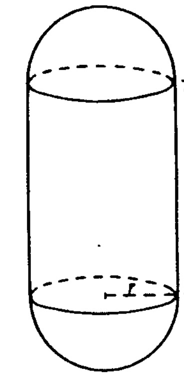

Figure 1

(a) Express h in terms of r and V. (3 marks)

(b) The cost per square metre of the cylindrical surface is k while that of the hemispherical surfaces is $2k$. Let the cost for making the tank be C.

(i) Show that $C = \frac{16}{3}\pi r^2 k + \frac{2kV}{r}$.

(ii) If $\frac{dC}{dr} = 0$, find r in terms of V.
Show that this value of r gives a minimum value of C.

(iii) If C is to be a minimum, find the ratio $r : h$. (17 marks)

10. The function $f(x) = x^3 + ax^2 + bx + c$ has stationary values at $x = \alpha$ and $x = \beta$, where $\alpha \neq \beta$.

(a) Find $\alpha + \beta$ and $\alpha\beta$ in terms of a and b. (6 marks)

(b) Show that $a^2 > 3b$. (3 marks)

(c) Show that $\frac{f(\alpha) - f(\beta)}{\alpha - \beta} = \frac{2}{9}(3b - a^2)$. (7 marks)

(d) Using the results of (b) and (c), find the relation between α and β so that $f(\alpha) < f(\beta)$. (4 marks)

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11. (a) Prove, by mathematical induction, that

$$1^2 + 2^2 + \dots + n^2 = \frac{1}{6}n(n+1)(2n+1)$$

for all positive integers n .

(6 marks)

(b) Identical cubical bricks are piled up in layers to form a pyramid-like solid with a square base of side x metres as shown in Figure 2. The side of the bottom layer consists of n bricks whereas each side of the square layer immediately above has $n - 1$ bricks, and so on. There is only one brick in the top layer.

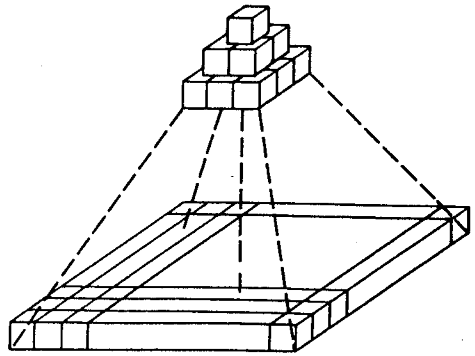

Figure 2

(i) Find the volume of the rth layer counting from the top. Hence find the volume of the solid.

(ii) Using the results of (a) and (b)(i), show that the volume of the solid is always greater than that of a pyramid of the same height, standing on the same base.

When n is very large, what value will the difference in volumes be close to?

(14 marks)

12. Let $\omega = \cos\frac{2k\pi}{5} + i\sin\frac{2k\pi}{5}$, where $i^2 = -1$ and k is a given integer such that $\omega \neq 1$.

(a) Show that $\omega^n + \omega^{-n} = 2\cos\frac{2nk\pi}{5}$ for any integer n . (3 marks)

(b) Prove that $\omega^5 = 1$.
Hence, or otherwise, show that $1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$. (6 marks)

(c) Making use of the results in (b), show that

$$(\omega + \omega^{-1})^2 + (\omega^2 + \omega^{-2})^2 = 3 .$$ (6 marks)

(d) Deduce from (a) and (c) that $\left(\cos\frac{2k\pi}{5}\right)^2 + \left(\cos\frac{4k\pi}{5}\right)^2 = \frac{3}{4}$. (5 marks)

END OF PAPER

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附加數學 試卷二

ADDITIONAL MATHEMATICS PAPER II

二小時完卷

上午十一時十五分至下午一時十五分

本試卷必須用英文作答

Two hours

11.15 a.m.—1.15 p.m.

This paper must be answered in English

Answer ALL questions in Section A and any THREE questions from Section B.

All working must be clearly shown.