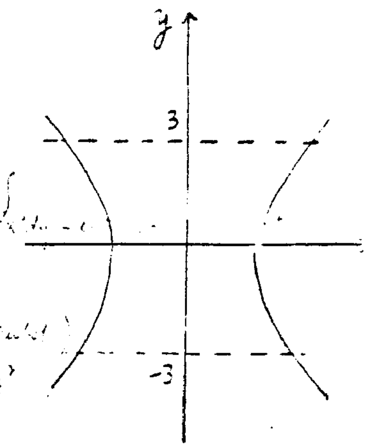
| Solution | Marks | Notes |
|---|---|---|
| 1. $(1+2x)^3(1+3x)^4 = (1+6x+12x^2+\dots)(1+12x+54x^2+\dots)$ | 2A+1A | 2A for 1st correct expansion (3 terms only) −1 if omit "+…" |
| $= (1+18x+138x^2+\dots)$ | 2A | −1 if include $x^3$, etc. −1 for each wrong term |
| | 5 | |
| 2. Let $u = x-1$ | | |
| $\int (x+2)\sqrt{x-1}\,dx = \int (u+3)\sqrt{u}\,du$ | 1A | Full marks given if no intermediate steps … |
| $= \int (u^{\frac{3}{2}} + 3u^{\frac{1}{2}})\,du$ | 1M | |
| $= \frac{2}{5}u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + C$ | 1+1A | −1 if omit C. |
| $= \frac{2}{5}(x-1)^{\frac{5}{2}} + 2(x-1)^{\frac{3}{2}} + C$ | 1A | |
| | 5 | |
| 3. Diff. w.r.t. $x$, | | |
| $4xy + 2x^2y' + 2x + 2yy' = 0$ | 3A | −1 for each wrong term |
| $y' = -\frac{2xy+x}{x^2+y}$ | 1A | |
| Putting $x=2$, $y=0$, | 1M | |
| slope of tangent at $(2,0) = -\frac{1}{2}$ | 1A | |
| | 6 | |
| 4. $\frac{dy}{dx} = -\sin(\sin x)\frac{d}{dx}\sin x$ | 1A/1M | optional |
| $= -[\sin(\sin x)]\cos x$ | 1A | |
| $\frac{d^2y}{dx^2} = -\cos x\frac{d}{dx}[\sin(\sin x)] - [\sin(\sin x)]\frac{d}{dx}\cos x$ | 1A+1M | optional. |
| $= -[\cos(\sin x)]\cos^2 x + [\sin(\sin x)]\sin x$ | 1+1A | |
| | 6 | |

| Solution | Marks | Notes |
|---|---|---|
| 5. (I) $\log_4 x - \log_x 16 = 1$ | | |
| $\log_4 x - \dfrac{\log_4 16}{\log_4 x} = 1$ $\longrightarrow \dfrac{\log x}{\log 4} - \dfrac{\log 16}{\log x} = 1$ (2A) | 2A | Cand. not expected to write $\log_4 x \neq 0$ |
| $(\log_4 x)^2 - \log_4 x - 2 = 0$ | 1A | $\log x = \dfrac{0.602 \pm 1.806}{2}$ (1A) |
| $(\log_4 x + 1)(\log_4 x - 2) = 0$ | | $\to x = 16.0$ or $0.250$ (1A+1A) |
| $\log_4 x = -1$ or $2$ | 1A | For any one correct |
| $\therefore\ x = \frac{1}{4}$ or $16$ | 1+1A | |
| | 6 | |
| 6. Eqn. of AB is $y - 2 = \dfrac{2 - \frac{1}{2}}{-4 - 2}(x + 4)$ | 1A | |
| or $y = -\frac{1}{4}(x - \ldots)$ | | |
| Area required $= \int_{-4}^{2} \left[-\frac{1}{4}(x-4) - \frac{1}{8}x^2\right] dx$ | 1M+1M | |
| $= \int_{-4}^{2} \left(-\frac{1}{8}x^2 - \frac{x}{4} + 1\right) dx$ | | |
| $= \left[-\frac{x^3}{24} - \frac{x^2}{8} + x\right]$ | | |
| $= \left(-\frac{8}{24} - \frac{4}{8} + 2\right) - \ldots$ | | |
| $= \frac{9}{2}$ | | |
| Alternatively | | |
| Area under AB | 1A | |
| $= \left(\frac{1}{2} + 2\right)(2 - -4) \times \frac{1}{2}$ | | |
| $= \frac{15}{2}$ | | |
| Area under curve | | |
| $= \int_{-4}^{2} y\,dx = \int_{-4}^{2} \frac{x^2}{8}\,dx = \ldots$ | 1M+1A | |
| $\therefore$ area required $= \frac{15}{2} - \ldots$ | 1M | |
| $= \frac{9}{2}$ | 1A | |
| | 6 | |

| Solution | Marks | Notes |
|---|---|---|
| 7. Let $P = (a, b)$, $M = (x, y)$. | | |
| $x = \dfrac{a-1}{2}$ | 1A | |
| $y = \dfrac{b+2}{2}$ | 1A | |
| $\therefore\ a = 2x+1$, $b = 2y-2$ | 1M | |
| P lies on the circle | | |
| $\Rightarrow (2x+1)^2 + (2y-2)^2 - 2(2x+1) - 4(2y-2) - 5 = 0$ | 1M+1A | for wrong a, b |
| $2x^2 + 2y^2 - 8y + 3 = 0$ | 1A | for $4x^2+4y^2-16y+6=0$ acc |
| | 6 | |
| 8. (a) $\dfrac{dy}{dx} = \dfrac{2}{y}$ | | |
| Putting $x=1$, $y=2$, $\dfrac{dy}{dx} = 1$ | 1M+1A | |
| Eqn. of tangent is $y-2 = x-1$ | 1A | optional |
| $x - y + 1 = 0$ <br> $y = x+1$ | 1A | −1 if not simplified |
| | 4 | |
| Alternatively | | |
| $y_1 y = 2(x + x_1)$ | 1A | |
| $2y = 2(x+1)$ | 1M + | |
| $x - y + 1 = 0$ | 1A | |

| Solution | Marks | Notes |
|---|---|---|
| 8. (b) (i) Expanding C | | |
| (I) $x^2+y^2-(2+k)x-(4-k)y+(5-k)=0$ | 2A | |
| $\left[x-(1+\frac{k}{2})\right]^2+\left[y-(2-\frac{k}{2})\right]^2=\frac{k^2}{2}$ | 1M+1A (R.S) <br> 1M+1A (L.S.) | If $r^2$ given $\neq \frac{k^2}{2}$, ... awarded for checking $r$ |
| (1, 2) satisfies C | 1M | |
| $\therefore$ C represents a circle passing thro' (1, 2). | | Alt. soln. for (b) (i) <br> Expanding (optional) <br> coeff $x^2$ = coeff $y^2$ <br> coeff. $xy=0$ <br> $g^2+f^2-c=\frac{k^2}{2}$ 1M 1A <br> (1, 2) satisfies C |
| (ii) Centre of C $=\left(1+\frac{k}{2},\ 2-\frac{k}{2}\right)$ | 1+1A | |
| (iii) (II) $2(x-1)+2(y-2)\frac{dy}{dx}+k\left(\frac{dy}{dx}-1\right)=0$ (1M) | 1M | |
| (1A) $\frac{dy}{dx}=\frac{k+2-2x}{k-4+2y}$ | 1A | |
| At (1, 2), $\frac{dy}{dx}=1$ | 1A | |
| Eqn. required is $y-x-1=0$ (1A) | 1A <br> 11 | |
| (c) (I) By (a) and (b), C represents a variable circle which shares a common tangent with the parabola $y^2=4x$ at (1, 2) | 1M | |
| If its centre lies on $x-y=3$, from (b)(ii) | | |
| $\left(1+\frac{k}{2}\right)-\left(2-\frac{k}{2}\right)=3$ | 1M | (II) for sub. into $x-y=$ |
| $k=4$ | 1A | |
| $\therefore$ the required eqn. is $(x-1)^2+(y-2)^2+4(y-x-1)=0$ | 1M | |
| $x^2+y^2-6x+1=0$ | 1A <br> 5 | |

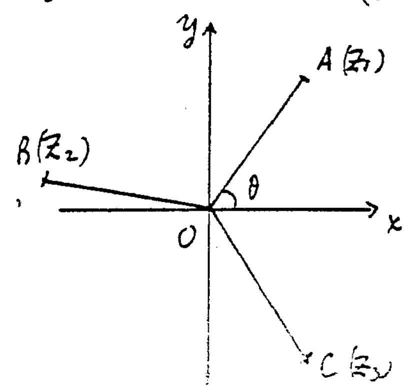
| Solution | Marks | Notes |
|---|---|---|
| 9. (a) Let $P=(x,y)$. | | |
| Slope of $PA = \dfrac{y-3}{x-5}$ | 1A | |
| Slope of $PB = \dfrac{y+3}{x+5}$ | 1A | |
| $\dfrac{y-3}{x-5} \times \dfrac{y+3}{x+5} = k$ | 2M | |
| Locus of $P$ is $kx^2-y^2=25k-9$ | 1A | |
| | 5 | |
| (b) (i) Circle, $k=-1$. | 1A | |
| (ii) Ellipse but not circle, $\underline{k<0}$, $\underline{k\neq -1}$. | 1+1A | |
| (iii) Hyperbola, $\underline{k>0}$, $\underline{k\neq \frac{9}{25}}$. | 1+1A | |
| If $k=0$, two parallel lines | 1A | |
| If $k=\frac{9}{25}$, two intersecting lines | 2A | |
| | 8 | |
| (c) When $k=1$, the locus is the hyperbola $x^2-y^2=16$ | 1A |  |
| Vol. req. $= \displaystyle\int_{-3}^{3} \pi x^2\,dy$ | 1M+1M +1A | |
| $= \displaystyle\int_{-3}^{3} \pi(16+y^2)\,dy$ | 1M | |
| $= \left[\pi\left(16y+\frac{y^3}{3}\right)\right]_{-3}^{3}$ | 1A | |
| $= 114\pi$ | 1A | For volume, without "π" — give 5 |
| | 7 | |

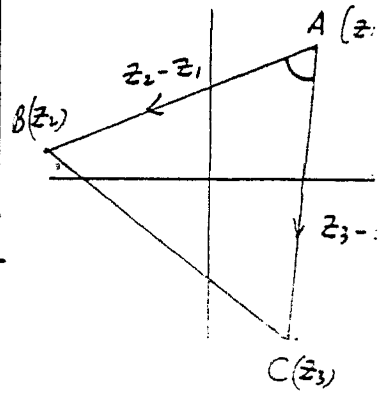
| Solution | Marks | Notes |
|---|---|---|
| 10. $x^2-2x-1=0$ | | |
| $x=\dfrac{2\pm\sqrt{8}}{2}$ | 1A | |
| $\therefore\ \alpha=\underset{2.414}{1+\sqrt{2}},\quad \beta=\underset{-0.414}{1-\sqrt{2}}$ $\quad \alpha+\beta=2,\ \alpha\beta=-1$ | | |
| (a) $U_{n+2}=\dfrac{1}{2\sqrt{2}}(\alpha^{n+2}-\beta^{n+2})$ | | 6 marks for 1st given part, 3 m. for 2nd part. |
| $=\dfrac{1}{2\sqrt{2}}\left[\alpha^n(3+2\sqrt{2})-\beta^n(3-2\sqrt{2})\right]$ | 1M+1A | $\alpha^2, \beta^2$ correct |
| $=\dfrac{1}{2\sqrt{2}}\left[3(\alpha^n-\beta^n)+2\sqrt{2}(\alpha^n+\beta^n)\right]$ | 1A | |
| $\therefore\ 2U_{n+1}+U_n=\dfrac{2}{2\sqrt{2}}(\alpha^{n+1}-\beta^{n+1})+\dfrac{1}{2\sqrt{2}}(\alpha^n-\beta^n)$ | 1A | |
| $=\dfrac{1}{2\sqrt{2}}\left[2\alpha^n(1+\sqrt{2})-2\beta^n(1-\sqrt{2})+\alpha^n-\beta^n\right]$ | 1A | |
| $=\dfrac{1}{2\sqrt{2}}\left[3(\alpha^n-\beta^n)+2\sqrt{2}(\alpha^n+\beta^n)\right]$ | 1A | |
| $=U_{n+2}$ | | |
| $V_{n+2}=\dfrac{1}{2\sqrt{2}}(\alpha^{n+2}+\beta^{n+2})$ | | |
| $=\dfrac{1}{2\sqrt{2}}\left[\alpha^n(3+2\sqrt{2})+\beta^n(3-2\sqrt{2})\right]$ | | |
| $=\dfrac{1}{2\sqrt{2}}\left[3(\alpha^n+\beta^n)+2\sqrt{2}(\alpha^n-\beta^n)\right]$ | 2A | |
| $\therefore\ 2V_{n+1}+V_n=\dfrac{2}{2\sqrt{2}}\left[\alpha^n(1+\sqrt{2})+\beta^n(1-\sqrt{2})\right]+\dfrac{1}{2\sqrt{2}}(\alpha^n+\beta^n)$ | | |
| $=\dfrac{1}{2\sqrt{2}}\left[3(\alpha^n+\beta^n)+2\sqrt{2}(\alpha^n-\beta^n)\right]$ | | |
| $=V_{n+2}$ | 1A | |
| | 10 | |

| Solution | Marks | Notes |
|---|---|---|
| 10. (b) (i) $U_1 = \frac{1}{2\sqrt{2}}(\alpha - \beta) = 1$ | 2A | |
| $U_2 = \frac{1}{2\sqrt{2}}(\alpha^2 - \beta^2)$ | | |
| $= \frac{1}{2\sqrt{2}}\left[(3+2\sqrt{2}) - (3-2\sqrt{2})\right] = 2$ | 2A | |
| (ii) If $U_n$, $U_{n+1}$ are integers, | | |
| $U_{n+2} = 2U_{n+1} + U_n$ must also be an integer. | 1+1M | → Quote result in (a); → for integers. |
| (iii) Since $U_1, U_2$ are integers and from (b)(ii), if $U_k$, $U_{k+1}$ are integers, $U_{k+2}$ is also an integer, by induction, $U_n$ is an integer for all $n$. | Yes<br>1+1 | 1 for answer<br>1 for reasons. |
| | 8 | |
| (c) $V_n$ is not an integer for some $n$ | | 1 for answer |
| e.g. $V_1 = \frac{1}{\sqrt{2}}$ | 1+1 | 1 for reasons. |
| | 2 | |

$V_1 = \frac{1}{2\sqrt{2}}(1+\sqrt{2}+1-\sqrt{2})$

$= \frac{1}{\sqrt{2}}$

| Solution | Marks | Notes |
| --- | --- | --- |
| 11. $\omega^3-1=(\omega-1)(\omega^2+\omega+1)=0$ | 1A | |
| $\omega \neq 1 \Rightarrow \omega = \frac{-1\pm\sqrt{3}i}{2}$ | 1A | Accept $\frac{-1+\sqrt{3}i}{2}$ |
| $= \operatorname{cis}\frac{2\pi}{3}$ ($\because 0<\arg(\omega)<\pi$) | 1A | |
| Let $z_1 = 3\operatorname{cis}\theta$, | | |
| then $z_2 = \omega z_1 = 3\operatorname{cis}(\theta+\frac{2}{3}\pi)$ 120° acceptable. | 1A | |
| $z_3 = \omega z_2 = 3\operatorname{cis}(\theta+\frac{4}{3}\pi)$ | 1A | |
| (a)   Show ∠AOB = ∠BOC = ∠COA app. equal | 2M | 2M |
| (b) $\lvert z_2\rvert = \lvert z_3\rvert = 3$ | 1A | |
| | 8 | |
| (c) Since $\lvert z_1\rvert = \lvert z_2\rvert = \lvert z_3\rvert$, | | |
| A, B, C lie on a circle centred O | 2M | 4 marks for any correct method. |
| $\angle AOB = \angle AOC = \angle BOC = \frac{2}{3}\pi$ | | |
| $\therefore$ $\Delta$s AOB, AOC, BOC are congruent | 2M | |
| $\therefore$ $\Delta ABC$ is equilateral. | | |
| | 4 | |

| Solution | Marks | Notes |
|---|---|---|
| 11. (d) $z_1^2+z_2^2+z_3^2+z_1z_2+z_2z_3+z_3z_1$ | | |
| $=z_1^2+(\omega z_1)^2+(\omega^2 z_1)^2+z_1(\omega z_1)+(\omega z_1)(\omega^2 z_1)+(\omega^2 z_1)z_1$ | 2M | write in polar form – |
| $=z_1^2(1+\omega^2+\omega^4+\omega+\omega^3+\omega^2)$ | 1A | |
| $=2z_1^2(1+\omega+\omega^2)$ | 1A | |
| $=0$, (since $1+\omega+\omega^2=0$) | | $-1$ if omit "$=0$" |
| | 4 | |
| (e) $\text{Amp}\,\dfrac{z_3-z_1}{z_2-z_1}=\text{amp}(z_3-z_1)-\text{amp}(z_2-z_1)$ | 1A |  |
| $=\angle BAC$ | 2A | |
| $=60^\circ$ | 1A | |
| | 4 | |
| Alternatively | | |
| $\text{Amp}\,\dfrac{z_3-z_1}{z_2-z_1}=\text{amp}\,\dfrac{(\omega^2-1)z_1}{(\omega-1)z_1}$ | 1A | |
| $=\text{amp}(\omega+1)$ $(\because \omega\neq 1)$ | 1A | |
| $=\text{amp}\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$ | 1A | |
| $=60^\circ$ | 1A | |

| Solution | Marks | Notes |
|---|---|---|
| 12. (a) Let $u = a - x$, $x = a - u$, $f(x) = f(a-u)$, $dx = -du$ | | |
| $\int_0^a x f(x)\,dx = -\int_a^0 (a-u) f(a-u)\,du$ → must be written out | 1A+1A | for limits |
| $= \int_0^a (a-u) f(a-u)\,du$ give (2M) | 1A | for $\int_0^a$ |
| $= \int_0^a (a-u) f(u)\,du$ | 1M | Using $f(x) = f(a-x)$ |
| $= a\int_0^a f(u)\,du - \int_0^a u f(u)\,du$ | 1A 1M. | |
| $f(x) = f(a-x) = f(u)$ $\quad = a\int_0^a f(x)\,dx - \int_0^a x f(x)\,dx$ | 1A | |
| $\therefore \int_0^a x f(x)\,dx = \frac{a}{2}\int_0^a f(x)\,dx$ | 1A | |
| | 6 | |
| (b) Putting $u = x - \frac{\pi}{2}$, by (a), | | |
| $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin^4(u+\frac{\pi}{2})}{\sin^4(u+\frac{\pi}{2}) + \cos^4(u+\frac{\pi}{2})}\,du$ | 1M+1A | limits |
| $= \int_0^{\frac{\pi}{2}} \frac{\cos^4 u}{\cos^4 u + \sin^4 u}\,du$ | 1A | |
| $\therefore \int_0^{\pi} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx = \int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx + \int_{\frac{\pi}{2}}^{\pi} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx$ | 2A | |
| $= \int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\,dx + \int_0^{\frac{\pi}{2}} \frac{\cos^4 x}{\sin^4 x + \cos^4 x}\,dx$ | 1A | |
| $= \int_0^{\frac{\pi}{2}} \frac{\sin^4 x + \cos^4 x}{\sin^4 x + \cos^4 x}\,dx$ | 2A | |
| $= \int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2}$ | 1A | |
| | 9 | |

| Solution | Marks | Notes |
|---|---|---|
| 12. (c) Let $f(x) = \frac{\sin^4 x}{\sin^4 x + \cos^4 x}$, then $f(x) = f(\pi - x)\ \forall x$. | 2M | |
| $\int_0^\pi \frac{x \sin^4 x}{\sin^4 x + \cos^4 x} dx = \frac{\pi}{2} \int_0^\pi \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$, by (a) | 2M | |
| $= \frac{\pi}{2} \cdot \frac{\pi}{2}$, by (b) | 1A | |
| $= \frac{\pi^2}{4}$ $(= 2.467)$ | | |
| | 5 | |