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香港考試局

HONG KONG EXAMINATIONS AUTHORITY

一九八〇年香港中學會考

HONG KONG CERTIFICATE OF EDUCATION EXAMINATION, 1980

SUBJECT

Additional Mathematics I

MARKING SCHEME

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Solution	Marks	Notes

1. $2x^2 + x + 5 = k(x+1)^2$

$(2-k)x^2 + (1-2k)x + (5-k) = 0$ — 1A → for RHS = 0

It has no real roots if $(1-2k)^2 - 4(2-k)(5-k) < 0$ — 2M+1A → for setting $\Delta < 0$

i.e. $k < \frac{39}{24}$ (or $\frac{13}{8}$) accept — 1A → deduct if "$\le$" (1A)

5

2. $\lim\limits_{\Delta x \to 0} \frac{1}{\Delta x}\left[\frac{2x+3+2\Delta x}{x+4+\Delta x} - \frac{2x+3}{x+4}\right]$ — −1 if omit "$\lim\limits_{\Delta x\to 0}$" at any line

$= \lim\limits_{\Delta x \to 0} \frac{1}{\Delta x}\left[\frac{(2x+3+2\Delta x)(x+4) - (2x+3)(x+4+\Delta x)}{(x+4+\Delta x)(x+4)}\right]$ — 1M ~~2M~~ → common deno.

$= \lim\limits_{\Delta x \to 0} \frac{1}{\Delta x}\left[\frac{2\Delta x(x+4) - \Delta x(2x+3)}{(x+4+\Delta x)(x+4)}\right]$ (1A) $5\Delta x$ — 1A

$= \lim\limits_{\Delta x \to 0} \frac{5\cancel{\Delta x}}{\cancel{\Delta x}(x+4+\Delta x)(x+4)}$ 1A for cancelling Δx — 1A

$= \frac{5}{(x+4)^2}$ — 2A

5

Alternatively,

$\lim\limits_{\Delta x \to 0}[\quad] = \frac{d}{dx}\left[\frac{2x+3}{x+4}\right]$ 2A

$= \frac{2(x+4) - (2x+3)}{(x+4)^2}$

$= \frac{5}{(x+4)^2}$ 3A

3. Slope of $L_3 = \frac{-5}{3}$

$\therefore$ slope of altitude $= \frac{3}{5}$ (1A) — 1A

System of lines thro' intersection of L_1 and L_2 is $6x + y + 3 + k(x+2y+1) = 0$ combining (1A) — 1M

or $(6+k)x + (1+2k)y + (3+k) = 0$ — equal slopes

It is the altitude iff $-\frac{6+k}{1+2k} = \frac{3}{5}$ — 1M+1A

$\therefore k = -3$ (1A) — 1A

Eqn. of altitude is $(6-3)x + (1-6)y + (3-3) = 0$

$3x - 5y = 0$ (1A) — 1A

accept if no simplification on line

6

Alternatively.

Solving L_1, L_2 — 1M

$x+2y+1-(12x+2y+6)=0$

$x = -\frac{5}{11}$ — 1A

$y = -\frac{3}{11}$ — 1A

Slope of altitude $= \frac{3}{5}$ — 1A

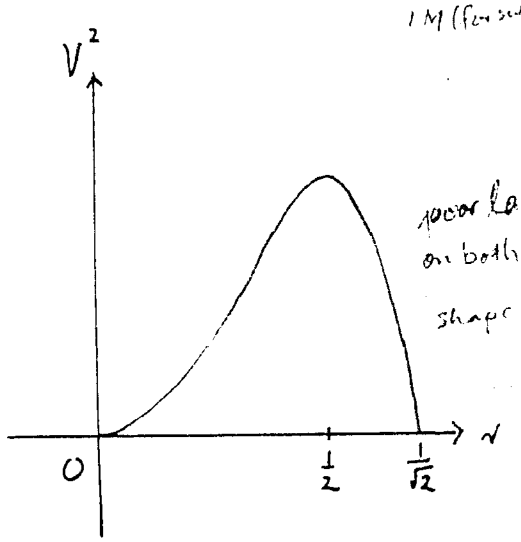
Altitude is ... for pt-slope form

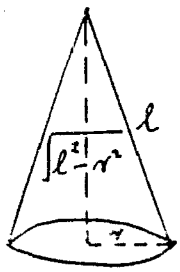
$y + \frac{3}{11} = \frac{3}{5}\left(x + \frac{5}{11}\right)$ — 1M

$3x - 5y = 0$ — 1A

Solution	Marks	Notes
$\int \frac{2}{\cot\frac{x}{2}+\tan\frac{x}{2}}dx = \int \frac{2}{\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}+\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}dx$	~~1A~~	
$= \int 2\sin\frac{x}{2}\cos\frac{x}{2}\,dx$	2A	
$= \int \sin x\,dx$ or $\int 4\sin\frac{x}{2}\,d(\sin\frac{x}{2})$ 2A	2A	for C
$= -\cos x + C$	1A+1M	1M for constant
or $\boxed{2\sin^2\frac{x}{2}}+C$ (1A+1M)	6	
5. $\sin^2 A = 3 + 6\cos^2 A$		
$\sin^2 A = \frac{9}{25}$ (or $\cos^2 A = \frac{16}{25}$, etc)	2A	
$\therefore \sin A = \pm\frac{3}{5}$		
$\cos A = \pm\sqrt{1-\sin^2 A}$		
$= \pm\frac{4}{5}$		
If $90° < A < 180°$, $\sin A = \frac{3}{5}$ (1A)	1A	
$\cos A = -\frac{4}{5}$ (1A)	1A	
$\therefore \frac{\sin A}{1+2\cos A} = \frac{\frac{3}{5}}{1-\frac{8}{5}}$	1M	for sub.
$= -1$	1A	
	6	
6. (i) For $x \le 0$, accept if write $x<0$	1M	$x^2 - x < \lvert x\rvert$
$x^2 - \lvert x\rvert - x < 0$		$x^4 - 2x^3 + x^2 < x^2$
$\Rightarrow x^2 + x - x < 0$		
$\Rightarrow x^2 < 0$, which is impossible	1A	$x^2(x-2)x<0$
(ii) For $x > 0$,	1M	$(x-2)x<0$
$x^2 - \lvert x\rvert - x < 0$		$0<x<2$
$\Rightarrow x^2 - 2x < 0$	1A	Incomplete
$\Rightarrow x(x-2) < 0$		award 3 marks
$\Rightarrow 0 < x < 2$	1+1A	or 0 mark if
(i) and (ii) $\Rightarrow 0 < x < 2$ (-1) if emphasis on set is ϕ	6	any mistake was made in between

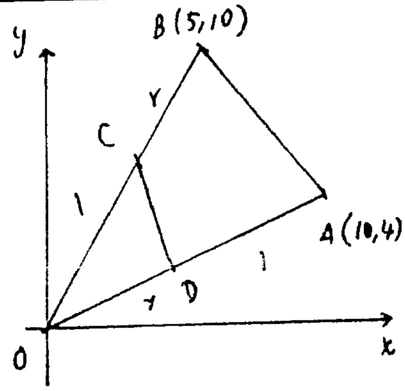
Solution	Marks	Notes
7. $\tan 7\theta + \cot 2\theta = 0$		
$\frac{\sin 7\theta}{\cos 7\theta} + \frac{\cos 2\theta}{\sin 2\theta} = 0$		
$\frac{\sin 2\theta \sin 7\theta + \cos 2\theta \cos 7\theta}{\cos 7\theta \sin 2\theta} = 0$	1A	
$\frac{\cos 5\theta}{\cos 7\theta \sin 2\theta} = 0$	1A	for $\cos 5\theta$
If $\sin 2\theta, \cos 7\theta \neq 0$, i.e. $\theta \neq n\pi, \frac{\pi}{2}+n\pi, \frac{4n+1}{14}\pi, \frac{4n+3}{14}\pi$, $n = 0, \pm 1, \pm 2, \ldots$		
then $\cos 5\theta = 0$ (1A)	1A	general solution for cosine.
$5\theta = 2n\pi \pm \frac{\pi}{2}$ (or $\frac{\pi}{2}+n\pi$)	1M+1A	
$\therefore \theta = \frac{4n\pm 1}{10}\pi$ (1A), $n = 0, \pm 1, \pm 2, \ldots$ or $\frac{2n+1}{10}\pi$ accept if n is not defined.	1A	
	6	
<u>Alternatively</u>,		
$\tan 7\theta + \cot 2\theta = 0$		
$\tan 7\theta + \tan(\frac{\pi}{2} - 2\theta) = 0$	1A	to $\tan(\frac{\pi}{2}-2\theta)$
$\tan 7\theta = \tan(2\theta - \frac{\pi}{2})$ [or $\tan(2\theta+\frac{\pi}{2})$]	2A	"transfer" & sign.
$7\theta = 2\theta - \frac{\pi}{2} + n\pi$, $n = 0, \pm 1, \pm 2, \ldots$ (or $2\theta+\frac{\pi}{2}+n\pi$)	1M+1A	general tan formula.
$5\theta = n\pi - \frac{\pi}{2}$ [or $\frac{\pi}{2}+n\pi$]		accept if in terms of "degree" or mixture
$\therefore \theta = \frac{1}{10}(2n-1)\pi$, $n = 0, \pm 1, \pm 2, \ldots$ [or $\frac{1}{10}(2n+1)\pi$]	1A	(-1) if no degree "o" written.

Solution	Marks	Notes
8. (a) Area of curved surface		
$= \pi r \ell$ (1A)	1A	→ can skip
$\therefore \pi \overset{1M}{=} \pi r^2 + \pi r \ell$ (1A)	1M+1A	
$\ell = \dfrac{1-r^2}{r}$ (1A)	1A	
Height of cone $= \sqrt{\ell^2 - r^2}$ (1A)	1A	
$\therefore$ volume $= \left(\frac{1}{3}\right)(\pi r^2)\sqrt{\ell^2 - r^2}$	1M	For formula of vol. $\frac{1}{3}$(base)(ht.)
$V^2 = \frac{1}{9}\pi^2 r^4(\ell^2 - r^2)$		
$= \frac{1}{9}\pi^2 r^2(1-2r^2)$ (2A)	2A	
	8	
(b) $\dfrac{d(V^2)}{dr} = \dfrac{\pi^2}{9}(2r - 8r^3)$ (1M) (1A)	1M+1A	
$\dfrac{d(V^2)}{dr} = 0$ (1M)	1M	
$\therefore r = 0, \pm\frac{1}{2}$	1A ~~2A~~ must have $r=\frac{1}{2}$ (or more)	Accept $r = 0, \frac{1}{2}$; or $r = \frac{1}{2}$.
Test for max. (1M) → either mention or do it.	1M	
$\therefore V^2$ is max when $r = \frac{1}{2}$	1A	
Max. value of $V = \frac{1}{3}\pi r\sqrt{1-2r^2} = \dfrac{\sqrt{2}}{12}\pi$ or its equivalent; 1M (for subst.) 1A	1M+1A	can leave units.
	~~9~~ 8	
(c) 	1	for label on x, y axes
poor labelling on both x & y axes (−1)	1	for $\frac{1}{2}$ max
shape unreasonable not shown (−1)	1	for 0 origin
3 marks not award if wrong V² …	1	for $\frac{1}{\sqrt{2}}$ on x-axis
		−1 if outside range $(0 \le r \le \frac{1}{\sqrt{2}})$
	~~3~~ 4	



Solution	Marks	Notes
9.(a) $\tan 3\theta = \tan(2\theta+\theta)$		
$= \dfrac{\tan 2\theta + \tan\theta}{1-\tan 2\theta \tan\theta}$ must have this line	2A	
$= \dfrac{\frac{2\tan\theta}{1-\tan^2\theta}+\tan\theta}{1-\frac{2\tan\theta}{1-\tan^2\theta}\cdot\tan\theta}$ " " "	1A	
$= \dfrac{\tan^3\theta - 3\tan\theta}{3\tan^2\theta-1}$ " " "	1A	
	4	
(b) (i) Putting $x=1$ (1M), $m+n+2=0$ (1A) ....(1)	1M+1A	
Putting $x=2$ (1M), $4m+n+11=0$ (1A) ....(2)	1M+1A	
(1) and (2) $\Rightarrow$ $m=-3$, $n=1$	1A	Awarded only if (1), (2) correct.
	5	
(ii) $f(x)=0$ iff $3x^3-3x^2-9x+1=0$ for m, n substituted	1A	
Putting $x=\tan\theta$, subst. (1M)		
$3\tan^3\theta-3\tan^2\theta-9\tan\theta+1=0$	1M	Alternatively,
$3(\tan^3\theta-3\tan\theta)-(3\tan^2\theta-1)=0$		
$3\tan 3\theta(3\tan^2\theta-1)-(3\tan^2\theta-1)=0$ by (a) subst (1A) on tan 3A	1A	$\dfrac{\tan^3\theta-3\tan\theta}{3\tan^2\theta-1}=\dfrac{1}{3}$, 1M+1A
$(3\tan 3\theta-1)(3\tan^2\theta-1)=0$ factorization	1A	
$3\tan 3\theta-1=0$ (1M)	1M	$\therefore \tan 3\theta = \dfrac{1}{3}$ 1M
[$3\tan^2\theta-1\neq 0$ for the validity of identity] accept if not mentioning this line		
$\tan 3\theta = \frac{1}{3}$		
$\Rightarrow$ $3\theta = 18.43°, 198.43°, 378.43°$ accept for gen. solution	(2A)	1 for 1, 1 for 2, 2 for 3; ~~-1 for one wrong~~
allow $\pm 0.01°$ (or $18°26'$, $198°26'$, $378°26'$)		
or allow ±1 min $\theta = 6.14°, 66.14°, 126.14°$	~~3A~~	~~for 2 or more correct~~
(or $6°9'$, $66°9'$, $126°9'$)		
Solutions are $x=\tan\theta$ (1M)	1M	1 mark each (to 2 d.p.)
only here lost 3 marks → $=$ ~~0.108, 2.262, -1.369~~	3A	~~for 2 or more correct~~
$= 0.11, 2.26, -1.37$ (corr. to 2 d.p.)	~~1A~~	
	11	

Solution	Marks	Notes
(a) $\lvert z-(3+4i)\rvert = 4$		
$\lvert (x-3)+(y-4)i \rvert = 4$	1A	mod.
$(x-3)^2+(y-4)^2=16$	2A	either 3rd or 4th line.
$x^2+y^2-6x-8y+9=0 \quad \ldots\ldots (i)$		
	3	
(b) $\dfrac{z-1}{z+1} = \dfrac{(x-1)+yi}{(x+1)+yi}$ (1A)	1A	
$= \dfrac{[(x-1)+yi][(x+1)-yi]}{(x+1)^2+y^2}$ (1M)	1M	kn. conjugate
$= \dfrac{1}{(x+1)^2+y^2}\left[(x^2+y^2-1)+2yi\right]$ (1A)	1A	separate Real & Imag.
If its amp. $= \frac{\pi}{2}$, $\quad x^2+y^2-1=0 \ldots\ldots (ii)$	1M+1A	equate real = 0. Real part = 0
	5	
(c) If $z_1 = x+yi$ satisfies (a) and (b),		
(ii) − (i) $\Rightarrow$ $\quad x = \dfrac{5-4y}{3}$ $\quad \left(\text{or } y = \dfrac{5-3x}{4}\right)$	2M+1A	for combining (i)+(ii) together (solving) for right x or right y.
Sub. in (ii), $\left(\dfrac{5-4y}{3}\right)^2+y^2-1=0$ $\quad \left(x^2+\left(\dfrac{5-3x}{4}\right)^2-1=0\right)$	1M	subst. x or y
$25y^2-40y+16=0$ $\quad (25x^2-30x+9=0)$	1A	find Quad. in x (or y).
$y = \frac{4}{5}$ 1A	1A	
$x = \frac{3}{5}$ 1A	1A	
$\therefore z_1 = \frac{3}{5}+\frac{4}{5}i$ 1A	1A	
If $z_1 = (p+qi)^2$,		
$\frac{3}{5}+\frac{4}{5}i = (p+qi)^2$ 1M	1M	
$= (p^2-q^2)+2pqi$	1A	kn. of ?
$\therefore pq = \frac{2}{5}$	1M+1A	
	12	

Solution	Marks	Notes
(a) Let $C = (x_1, y_1)$, $D = (x_2, y_2)$.		
Then $x_1 = \frac{5}{1+r}$	1A	
$y_1 = \frac{10}{1+r}$	1A	
$x_2 = \frac{10r}{1+r}$	1A	
$y_2 = \frac{4r}{1+r}$	1A	
	4	
(b) Area of $\Delta ODC = \frac{1}{2}(x_2 y_1 - x_1 y_2)$	1M	→ for area formula.
$= \frac{1}{2}\left(\frac{10r}{1+r}\cdot\frac{10}{1+r} - \frac{5}{1+r}\cdot\frac{4r}{1+r}\right)$		
$= \frac{40r}{(1+r)^2}$ 2A	2A	
	3	
(c) Area of $\Delta OAB = \frac{1}{2}(10\times 10 - 4\times 5)$		for area formula.
$= 40$ (1A)	1M+1A	equate 2 areas
Since $\Delta ODC = k \times \Delta OAB$, $\frac{40r}{(1+r)^2} = 40k$	1M+1A	→ correct equation
$kr^2 + (2k-1)r + k = 0$	1A	→ correct quadrate equ.
$r = \frac{(1-2k) \pm \sqrt{(2k-1)^2 - 4k^2}}{2k}$	1A	→ correct r.
$= \frac{(1-2k) \pm \sqrt{1-4k}}{2k}$ (Ans.)		
r is real $\Rightarrow$ $1-4k \geq 0$	2M	For $D \geq 0$
$\therefore k \leq \frac{1}{4}$ 1A	1A	
	9	
(d) Area of ΔODC is max if k is max.	2M	→ may use differentiation
$\therefore$ max. area of $\Delta ODC = \frac{1}{4}\times 40$		
$= 10$ sq. units (accept ~~sq~~ units)	2A	
	4	

Solution	Marks	Notes
12. (a) $AX = \sqrt{k^2s^2+s^2}$	1M+1A	Pythagoras Thm. / For correct AX
$= s\sqrt{1+k^2}$ (can omit)		
	2	
(b) $XY = ks \sin 45°$	1A	> By diag. for XY.
$= \frac{ks}{\sqrt{2}}$	1A	
$\therefore \sin\alpha = \frac{XY}{AX}$		
$= \frac{\frac{ks}{\sqrt{2}}}{s\sqrt{1+k^2}}$	1M	> Subst. XY, AX.
$= \frac{k}{\sqrt{2(1+k^2)}}$ 1A	1A	
	4	
(c) If $\alpha \le 30°$, then $\frac{k}{\sqrt{2(1+k^2)}} \le \sin 30°$	3M	−1 for giving "<" any time
$= \frac{1}{2}$ 1A	1A	If "=" given, award only 3 marks unless followed by explanation.
$\therefore \frac{k^2}{2(1+k^2)} \le \frac{1}{4}$		
(can be omitted) $k^2 \le 1$. 1A	1A	
$(0 \le)\ k \le 1$ 1A	1A	
i.e. $XB \le s$.		
Similarly, if the inclination of XD to the horizontal is not to exceed $30°$, $\boxed{XC \le s}$ 2A	2A	
Under this condition, $XB = BC - XC$		
$= \sqrt{2}s - XC$ (1A)	1A	
$\ge \sqrt{2}s - s$ (2A)	2A	for $\ge$ and subst. of s.
Combining $XB \le s$, $XB \ge \sqrt{2}s - s$, we have	1M	> Rm. …
$\sqrt{2}s - s \le ks \le s$		
$\sqrt{2} - 1 \le k \le 1$	2A	> for whole statements correct not on 1 side only
	14