香港考試及評核局 HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2 0 1 2 年 香 港 高 級 程 度 會 考 HONG KONG ADVANCED LEVEL EXAMINATION 2012

數學及統計學 高級補充程度 MATHEMATICS AND STATISTICS AS-LEVEL

評 卷 參 考

MARKING SCHEME

本評卷參考乃香港考試及評核局專爲今年本科考試而編寫,供閱卷員參考之用。閱卷員在完成閱卷工作後,若將本評卷參考提供其任教會考班的本科同事參閱,本局不表反對,但須切記,在任何情況下均不得容許本評卷參考落入學生手中。學生若索閱或求取此等文件,閱卷員/教師應嚴詞拒絕,因學生極可能將評卷參考視爲標準答案,以致但知硬背死記,活剝生吞。這種落伍的學習態度,既不符現代教育原則,亦有違考試着重理解能力與運用技巧之旨。因此,本局籲請各閱卷員/教師通力合作,堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for markers' reference. The Authority has no objection to markers sharing it, after the completion of marking, with colleagues who are teaching the subject. However, under no circumstances should it be given to students because they are likely to regard it as a set of model answers. Markers/teachers should therefore firmly resist students' requests for access to this document. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, should be considered outmoded and pedagogically unsound. The Authority is counting on the co-operation of markers/teachers in this regard.

General Instructions To Markers

- 1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits all the marks allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
- 2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.
- 3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.
- 4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.
- 5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.
- 6. In the marking scheme, marks are classified into the following three categories:

'M' marks - awarded for applying correct methods
'A' marks - awarded for the accuracy of the answers

Marks without 'M' or 'A' - awarded for correctly completing a proof or arriving at an answer given in the question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should NOT be awarded, unless otherwise specified.

- 7. In the marking scheme, steps which can be skipped are enclosed by dotted rectangles, whereas alternative answers are enclosed by solid rectangles.
- 8. Marks may be deducted for poor presentation (pp). The symbol (pp-1) should be used to denote 1 mark deducted for pp.

(a) At most deduct 1 mark for pp in each section.

- (b) In any case, do not deduct any marks for pp in those steps where candidates could not score any marks.
- 9. Marks may be deducted for numerical answers with inappropriate degree of accuracy (a). The symbol (a-1) should be used to denote 1 mark deducted for a.

(a) At most deduct 1 mark for a in each section.

(b) In any case, do not deduct any marks for a in those steps where candidates could not score any marks.

	Solution	Marks	Remarks
. (a)	$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$	1A	
	$\frac{-1}{2}$		landi terresigne vites at ti silde erne R. e satablaria
(b)	(i) $(1+4x)^{\frac{-1}{2}}$		iron mitrag sylameta
	$=1+\left(\frac{-1}{2}\right)(4x)+\frac{1}{2!}\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)(4x)^2+\frac{1}{3!}\left(\frac{-1}{2}\right)\left(\frac{-1}{2}-1\right)\left(\frac{-1}{2}-2\right)(4x)^3+\cdots$	1M	
	$=1-2x+6x^2-20x^3+\cdots$	1A	e le mestiones ses sits
	$(1+64x^3)^{\frac{1}{2}}$		ers block broken Josep.
	$=1+\left(\frac{1}{2}\right)(64x^3)+\cdots$		hal requires considerable
	$=1+32x^3-\cdots$	1A	w saidaban garifuncil
	(ii) : $1+64x^3 = (1+4x)(1-4x+16x^2)$		
	$\therefore (1-4x+16x^2)^{\frac{1}{2}} = (1+64x^3)^{\frac{1}{2}}(1+4x)^{\frac{-1}{2}}$	Bestque de la	d bury's series and their
	$\therefore (1-4x+16x^2)^2 = (1+64x^3)^2 (1+4x)^2$ $= (1+32x^3-\cdots)(1-2x+6x^2-20x^3+\cdots)$	1M	
	$= 1 - 2x + 6x^2 + 12x^3 + \cdots$	1A	all age or made a sid i
	consequence of the contract of the contract of the contract of	(6)	n jestierioù gribhaiu adrini
(a)	$y = e^{t^2 + 4t + 4}$ and $x = \ln(2t + 4)$	um III	is analy
(a)	$y = e$ and $x = \ln(2t + 4)$		ediam A. Salao M. Bushas sealal
	$\frac{dy}{dt} = e^{t^2 + 4t + 4}(2t + 4)$ and $\frac{dx}{dt} = \frac{1}{t + 2}$	1A	For both
	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$		del mares yusara
	$dx dt dt = 2e^{t^2 + 4t + 4}(t + 2) \cdot (t + 2)$	1M	Lab age
	Alternative Solution		n manetaz artifizira ett di
	$\ln y = (t+2)^2$ and $x = \ln 2 + \ln(t+2)$		OR and $t+2=\frac{1}{2}e^{t}$
	$\therefore x = \ln 2 + \frac{1}{2} \ln(\ln y)$	1A	$OR \ln y = \frac{1}{4}e^{2x}$
		1M	$OR \frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{4}e^{2x} \cdot 2$
	$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{2} \cdot \frac{1}{\ln y} \cdot \frac{1}{y}$	TIVI	$\int \int \frac{1}{y} \cdot \frac{1}{dx} = \frac{1}{4} e^{-x^2}$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2y \ln y$	1A	iced Lipparinali od vjem odraod
			Carolis A proprieta de Carolis (a)
(b)	$\frac{d^2 y}{dx^2} = \left(2y \cdot \frac{1}{y} + 2\ln y\right) \frac{dy}{dx}$	1M	For chain rule
	$=4y\ln y(1+\ln y)$		
	When $x = 0$, $t = \frac{-3}{2}$ and so $y = e^{\frac{1}{4}}$.	1 A	
	$\therefore \frac{d^2 y}{dx^2} = 4e^{\frac{1}{4}} \left(\frac{1}{4} \right) \left(1 + \frac{1}{4} \right)$	í	
	$\frac{\mathrm{d}x^2}{=\frac{5}{4}e^{\frac{1}{4}}}$		
	$=\frac{1}{4}e^4$	1A	OR 1.6050

	Solution	Marks	Remarks
(a)	Let $u = 1 + e^{-0.2t}$.		e Branda de Carlos
` '	$\mathrm{d}u = -0.2e^{-0.2t}\mathrm{d}t$	1A	
	$0.3e^{-0.2t}$		r ×
	$N = \int \frac{0.3e^{-0.2t}}{(1+e^{-0.2t})^2} \mathrm{d}t$	11 11	i meneral — in the transfer
	$_{\rm N}$ (0.3 du		
	$N = \int \frac{0.3}{u^2} \cdot \frac{\mathrm{d}u}{-0.2}$		
	$=\frac{3}{2u}+C$	1A	
			
	$=\frac{3}{2(1+e^{-0.2t})}+C$		day and the second
	When $t = 0$, $N = 0.5$.		
	$\therefore C = \frac{-1}{4}$		
	i.e. $N = \frac{3}{2(1+e^{-0.2t})} - \frac{1}{4}$	1A	
	$\frac{1.6.17}{2(1+e^{-0.2t})} 4$		
		1 2	
(b)	N(4) - N(0)	A A A A	6.19
	$=\frac{3}{2(1+e^{-0.2\times4})}-\frac{1}{4}-0.5$	1M	
	$2(1+e^{-6.24})$ 4 ≈ 0.284961721		
	Hence the increase in the number of people is 285.	1A	
			4-7
(c)	$\frac{dN}{dt} = \frac{0.3e^{-0.2t}}{(1+e^{-0.2t})^2} > 0 \text{ for all } t \ge 0$		1,
2	$dt = (1 + e^{-0.2t})^2$		Withhold the last mark if
	Hence N is always increasing.		this argument is missing
	$\lim_{t \to \infty} N = \lim_{t \to \infty} \left \frac{3}{2(1 + e^{-0.2t})} - \frac{1}{4} \right $		OR by arguing that
		11.00+	$e^{-0.2t} > 0 \Rightarrow N < \frac{3}{2} - \frac{1}{4}$
	= 1.25 Hence the number of members will never reach 1300.	1A 1	OR by arguing that
	Hence the number of members will hever reach 1300.		$\frac{3}{2(1+e^{-0.2t})} - \frac{1}{4} = 1.3$
		(7)	
			has no real solution
(a)	Let X be the number of defective packs in a day.		
	$P(X \ge 1) = 1 - \frac{e^{-\lambda} \lambda^0}{0!}$	1A	e in District School (19)
		IA.	n-1n
	$\therefore 1 - e^{-2} = 1 - e^{-\lambda}$	7.0	
	i.e. $\lambda = 2$	1A	
(b)	P(the company will have to inspect the production line in a given day)	104	
(0)	$= P(X \ge 4)$		
	$e^{-2}e^{-2}2^1 e^{-2}2^2 e^{-2}2^3$	13.6	
	$=1-e^{-2}-\frac{e^{-2}2^{1}}{1!}-\frac{e^{-2}2^{2}}{2!}-\frac{e^{-2}2^{3}}{3!}$	1M	
	≈ 0.142876539	- o Tipo i to	-1.77
	≈ 0.1429	1A	
(c)	$(1-0.142876539)^n > 0.5$	1M	4 -nn
(0)	$n\ln(1-0.142876539) > \ln 0.5$		
	n < 4.495896098		
	i.e. the greatest integral value of n is 4.	1A	196
		(6)	
		(0)	<u>-</u>

	Solution	Marks	Remarks
. (a)	$P(A \cap B) = P(A)P(B \mid A)$		the series of the
	$=\frac{3a}{8}$	1A	45 - 45 0 - H BL
	- 8	1111	4.00
(1-)	$\mathbf{p}(A - \mathbf{p}) = \mathbf{p}(\mathbf{p})\mathbf{p}(A \mid \mathbf{p})$		
(b)	$P(A \cap B) = P(B)P(A \mid B)$	arent or d	
	$\frac{3a}{8} = \frac{3}{4} P(B)$	1M	
		H	
	$P(B) = \frac{a}{2}$	1A	
(c)	(i) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$		
	$1 - \frac{7}{16} = a + \frac{a}{2} - \frac{3a}{8}$	1M	de A tre t mater.
	$1 - \frac{1}{16} = a + \frac{1}{2} - \frac{1}{8}$	TIVI	
	$a=\frac{1}{2}$	1A	
	u - 2		
	$\mathbf{p}(A - \mathbf{p}')$		
	(ii) $P(A B') = \frac{P(A \cap B')}{P(B')}$		
	$=\frac{P(A)-P(A\cap B)}{1-P(B)}$		
	$\frac{1}{2} - \frac{3}{8} \times \frac{1}{2}$		LOT THE DECLARATION OF THE
	$=\frac{2 \cdot 6 \cdot 2}{1 \cdot 1}$	1M	For numerator
	$=\frac{\frac{1}{2} - \frac{3}{8} \times \frac{1}{2}}{1 - \frac{1}{2} \times \frac{1}{2}}$		
		1A	
	$=\frac{5}{12}$	IA	
		(7)	
()	$\frac{(30+a)+52+\dots+(90+b)}{30} = 71 \text{ and } (90+b)-(30+a) = 56$	13/113/	
(a)	= 71 and $(90+b)-(30+a)=36$	1M+1M	The second secon
	30	STATES THE PROPERTY.	the trademing part of the
	a+b=10 and $b-a=-4$	total flat distance	to todania adi vicisi
		1A	For both
	a+b=10 and $b-a=-4Solving, a=7 and b=3$		
	a + b = 10 and $b - a = -4$	1A 1A	For both OR 12.6728
	a+b=10 and $b-a=-4Solving, a=7 and b=3\sigma = \sqrt{\frac{803}{5}}$		
(b)	a+b=10 and $b-a=-4Solving, a=7 and b=3\sigma = \sqrt{\frac{803}{5}}P(3 of the excessive students will have scores higher than 80)$		OR 12.6728
(b)	a+b=10 and $b-a=-4Solving, a=7 and b=3\sigma = \sqrt{\frac{803}{5}}P(3 of the excessive students will have scores higher than 80)$	1A	OR 12.6728
(b)	a+b=10 and $b-a=-4Solving, a=7 and b=3\sigma = \sqrt{\frac{803}{5}}$		
(b)	a+b=10 and $b-a=-4Solving, a=7 and b=3\sigma = \sqrt{\frac{803}{5}} P(3 of the excessive students will have scores higher than 80) = \frac{C_3^7 C_1^6}{C_4^{13}}$	1A 1M	OR 12.6728 OR $4 \cdot \frac{7}{13} \cdot \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{6}{10}$
(b)	a+b=10 and $b-a=-4Solving, a=7 and b=3\sigma = \sqrt{\frac{803}{5}}P(3 of the excessive students will have scores higher than 80)$	1A	OR 12.6728
	a+b=10 and $b-a=-4Solving, a=7 and b=3\sigma = \sqrt{\frac{803}{5}} P(3 of the excessive students will have scores higher than 80) = \frac{C_3^7 C_1^6}{C_4^{13}} = \frac{42}{143}$	1A 1M	OR 12.6728 OR $4 \cdot \frac{7}{13} \cdot \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{6}{10}$
	a+b=10 and $b-a=-4Solving, a=7 and b=3\sigma = \sqrt{\frac{803}{5}} P(3 of the excessive students will have scores higher than 80) = \frac{C_3^7 C_1^6}{C_4^{13}} = \frac{42}{143} Let x_i (i=1,2,,30) be the original data.$	1A 1M	OR 12.6728 OR $4 \cdot \frac{7}{13} \cdot \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{6}{10}$
	a+b=10 and $b-a=-4Solving, a=7 and b=3\sigma = \sqrt{\frac{803}{5}} P(3 of the excessive students will have scores higher than 80) = \frac{C_3^7 C_1^6}{C_4^{13}} = \frac{42}{143} Let x_i (i=1,2,,30) be the original data.The new standard deviation$	1A 1M	OR 12.6728 OR $4 \cdot \frac{7}{13} \cdot \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{6}{10}$
	a+b=10 and $b-a=-4Solving, a=7 and b=3\sigma = \sqrt{\frac{803}{5}} P(3 of the excessive students will have scores higher than 80) = \frac{C_3^7 C_1^6}{C_4^{13}} = \frac{42}{143} Let x_i (i=1,2,,30) be the original data.The new standard deviation$	1A 1M 1A	OR $4 \cdot \frac{7}{13} \cdot \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{6}{10}$
	a+b=10 and $b-a=-4Solving, a=7 and b=3\sigma = \sqrt{\frac{803}{5}} P(3 of the excessive students will have scores higher than 80) = \frac{C_3^7 C_1^6}{C_4^{13}} = \frac{42}{143} Let x_i (i=1,2,,30) be the original data.$	1A 1M	OR $4 \cdot \frac{7}{13} \cdot \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{6}{10}$
	$a+b=10 \text{ and } b-a=-4$ Solving, $a=7$ and $b=3$ $\sigma = \sqrt{\frac{803}{5}}$ P(3 of the excessive students will have scores higher than 80) $= \frac{C_3^7 C_1^6}{C_4^{13}}$ $= \frac{42}{143}$ Let x_i $(i=1,2,,30)$ be the original data. The new standard deviation $= \sqrt{\frac{1}{30+2n} \left[\sum_{i=1}^{30} (x_i - \mu)^2 + n(\mu - \sigma - \mu)^2 + n(\mu + \sigma - \mu)^2 \right]}$	1A 1M 1A	OR $4 \cdot \frac{7}{13} \cdot \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{6}{10}$
	$a+b=10 \text{ and } b-a=-4$ Solving, $a=7$ and $b=3$ $\sigma = \sqrt{\frac{803}{5}}$ P(3 of the excessive students will have scores higher than 80) $= \frac{C_3^7 C_1^6}{C_4^{13}}$ $= \frac{42}{143}$ Let x_i $(i=1,2,,30)$ be the original data. The new standard deviation $= \sqrt{\frac{1}{30+2n} \left[\sum_{i=1}^{30} (x_i - \mu)^2 + n(\mu - \sigma - \mu)^2 + n(\mu + \sigma - \mu)^2 \right]}$	1M 1A	OR $4 \cdot \frac{7}{13} \cdot \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{6}{10}$
	$a+b=10 \text{ and } b-a=-4$ Solving, $a=7$ and $b=3$ $\sigma = \sqrt{\frac{803}{5}}$ P(3 of the excessive students will have scores higher than 80) $= \frac{C_3^7 C_1^6}{C_4^{13}}$ $= \frac{42}{143}$ Let x_i $(i=1,2,,30)$ be the original data. The new standard deviation $= \sqrt{\frac{1}{30+2n}} \left[\sum_{i=1}^{30} (x_i - \mu)^2 + n(\mu - \sigma - \mu)^2 + n(\mu + \sigma - \mu)^2 \right]$ $= \sqrt{\frac{1}{30+2n}} (30\sigma^2 + n\sigma^2 + n\sigma^2)$	1M 1A	OR $4 \cdot \frac{7}{13} \cdot \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{6}{10}$
	$a+b=10 \text{ and } b-a=-4$ Solving, $a=7$ and $b=3$ $\sigma = \sqrt{\frac{803}{5}}$ P(3 of the excessive students will have scores higher than 80) $= \frac{C_3^7 C_1^6}{C_4^{13}}$ $= \frac{42}{143}$ Let x_i $(i=1,2,,30)$ be the original data. The new standard deviation $= \sqrt{\frac{1}{30+2n}} \left[\sum_{i=1}^{30} (x_i - \mu)^2 + n(\mu - \sigma - \mu)^2 + n(\mu + \sigma - \mu)^2 \right]$ $= \sqrt{\frac{1}{30+2n}} (30\sigma^2 + n\sigma^2 + n\sigma^2)$ $= \sigma$	1M 1A	OR 12.6728 OR $4 \cdot \frac{7}{13} \cdot \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{6}{10}$ OR 0.2937
	$a+b=10 \text{ and } b-a=-4$ Solving, $a=7$ and $b=3$ $\sigma = \sqrt{\frac{803}{5}}$ P(3 of the excessive students will have scores higher than 80) $= \frac{C_3^7 C_1^6}{C_4^{13}}$ $= \frac{42}{143}$ Let x_i $(i=1,2,,30)$ be the original data. The new standard deviation $= \sqrt{\frac{1}{30+2n}} \left[\sum_{i=1}^{30} (x_i - \mu)^2 + n(\mu - \sigma - \mu)^2 + n(\mu + \sigma - \mu)^2 \right]$ $= \sqrt{\frac{1}{30+2n}} (30\sigma^2 + n\sigma^2 + n\sigma^2)$	1M 1A	OR $4 \cdot \frac{7}{13} \cdot \frac{6}{12} \cdot \frac{5}{11} \cdot \frac{6}{10}$

Solution	Marks	Remarks
(a) $f(x) = \frac{ax+b}{c-x}$ gives $g(x) = \frac{-ax+b}{c+x}$		Selection of the
• • •		1 7 - 7 1
Since the vertical asymptotes of $C_2: y = g(x)$ is $x = -3$,	1A	Srn C+ DISV
c=3	IA	The angular ball
Since the y-intercept of C_1 : $y = f(x)$ is $\frac{4}{3}$,		
$\frac{b}{c} = \frac{4}{3} \tag{1}$	1M	For either (1) or (2)
b = 4		
Since the x-intercept of $C_2: y = g(x)$ is 2,	ab Ta	$-a \cdot 2 + b$
$\frac{b}{a} = 2$ (2)		$OR 0 = \frac{-a \cdot 2 + b}{c + 2}$
a = 2	1A	For both b and a
	(2)	A CONTRACTOR
	(3)	arenati en
2x+4		
(b) (i) $C_1: y = \frac{2x+4}{3-x}$	wine busine	League lace ou
The vertical asymptote is $x = 3$.	1A 1A	
The horizontal asymptote is $y = -2$.	IA.	
(ii) \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \		
	- 101	Andread Towns
C_2 C_1	nie akurūk	12-46-51
4 /	Charles of the Co.	100
3 /		THE SERVE
2 4	TOTAL TO SO	Man Islan adv
$1 + (0, \frac{\pi}{3})$		Calle in application for
-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8	$\stackrel{x}{\longrightarrow}$	1+12
-8 -7 -6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 -1	1A	For shape of C_1
-2	1A	For $x = 3$ and $y = -2$
y = -2	1A	For shape of C_2
$x = -3 \qquad x = -4 \qquad x = 3$ $C_2 \qquad C_1$	1A	For $x = -3$ and $y = -2$
C_2 C_1 C_1	1A	For $(\pm 2,0)$ and $\left(0,\frac{4}{3}\right)$
	77 F	(3)
-6	(7)	Mary and the first of the first
	(1)	$ a = \{Y_1 - A(p_1)\}$
(c) The area = $\int_{-k}^{0} \left(\frac{-2x+4}{3+x} - \frac{2x+4}{3-x} \right) dx$	1M	
		0 10
$= \int_{-k}^{0} \left(-2 + \frac{10}{3+x} + 2 - \frac{10}{3-x} \right) dx$	1M	OR $\int_{x=-k}^{0} \frac{10}{9-x^2} d(9-x^2)$
	0.0 - 5 bas 1	
$= \left[10 \ln 3 + x + 10 \ln 3 - x \right]_{k}^{0}$	1A	For primitive function
$= 10[\ln 3 + \ln 3 - \ln(3-k) - \ln(3+k)]$	MESTER PRO	OR $10[\ln 9 - \ln(9 - k^2)]$
$\frac{9}{10 \ln \frac{9}{10}} = 10 \ln \frac{3}{10}$	1A	21 C 180 M
$10 \ln \frac{9}{9 - k^2} = 10 \ln \frac{3}{2}$	0.000	
$6 = 9 - k^2$	ion 5 (ch. 21%)	District and the
$k = \sqrt{3}$ or $-\sqrt{3}$ (rejected)	1A	The second second
terreserves and the second sec		

		Solution	Marks	Remarks
(a)		u=1+6t .	1A	AND THE REST OF
		= 6dt		
		en $t = 0$, $u = 1$; when $t = 12$, $u = 73$		
	\int_0^{12}	$\left[4.5 + 2t(1+6t)^{\frac{-2}{3}}\right] dt$		gentaling all mold
	= \int_{1}	$\int_{1}^{73} \left(4.5 + \frac{u - 1}{3} u^{\frac{-2}{3}} \right) \frac{\mathrm{d}u}{6}$	1M	For integrand
	= \int_{\int}	$\int_{1}^{73} \left(\frac{3}{4} + \frac{1}{18} u^{\frac{1}{3}} - \frac{1}{18} u^{\frac{-2}{3}} \right) du$	7 3 30	
	=	$\frac{3u}{4} + \frac{1}{24}u^{\frac{4}{3}} - \frac{1}{6}u^{\frac{1}{3}}\bigg]_{1}^{73}$	1A	For primitive function
		5.14060019 ne total amount of sewage emitted by machine $P \approx 66.1406$ tonnes.	1A	OR $\frac{433 + 23\sqrt[3]{73}}{8}$ tonnes
			si sdolyta	au tealitov aa T
			(4)	ARTICLE OF THE
(b)	(i)	$\int_0^{12} [3 + \ln(2t + 1)] dt$		The consists λ
		$= \frac{12 - 0}{2(5)} [3 + \ln 1 + 3 + \ln 25 + 2(3 + \ln 5.8 + 3 + \ln 10.6 + 3 + \ln 15.4 + 3 + \ln 20.2)]$	1M	
		≈ 63.52367987 ∴ the total amount of sewage emitted by machine $Q \approx 63.5237$ tonnes.	1A	
	(ii)	$q''(t) = \frac{2}{2t+1}$		
		$q'''(t) = \frac{-4}{(2t+1)^2} < 0$ for all $t \ge 0$	1A	For $\frac{-4}{(2t+1)^2}$
		Hence the estimate in (b)(i) is an under-estimate. Therefore we cannot conclude that the amount of sewage emitted by Q will be less than that by P and so the manager cannot be agreed with.	1M } 1	11.000
			(5)	
(c)	(i)	$R = 16 - ae^{-bx}$	1.4	
		$\ln(16 - R) = \ln a - bx$	1A	
	(ii)	$\begin{cases} 1 = \ln a + 10b \\ 0 = \ln a - 90b \end{cases}$	1M	
		등하는 모든 10 THE STATE CONTROL OF STATE CO	1A+1A	OR $a \approx 2.4596$
		Solving, $a = e^{0.9}$ and $b = 0.01$	IATIA	OK 4 ~ 2.4390
	(iii)	Total amount of sewage ≈ 80 + 66.14060019 + 63.52367987 = 209.6642801	1M	- Cri (A) = Cri (A) =
		Hence $R = 16 - e^{0.9}e^{-0.01(209.6642801)}$		1 =
		≈ 15.69779292	1A	100
		i.e. the tax paid is 15.6978 million dollars.	IA	
	, -		(6)	

Solution	Marks	Remarks
a) $r(t) = 20 - 40e^{-at} + be^{-2at}$	Market goods	
$r(0) = 20 - 40e^0 + be^0 = 30$		
b = 50	1A	*354
	(1)	
	(1)	ea-mev
b) $r'(t) < 0$ for 9 days		74494
$40ae^{-at} - 100ae^{-2at} < 0 \text{ for } t < 9$	1M	
$20ae^{-2at}(2e^{at}-5)<0$		NAME OF THE PARTY
$e^{at} < 2.5$		10. 1 s =
	1A	Paul Comment
$t < \frac{\ln 2.5}{a}$	IA.	
$\therefore \frac{\ln 2.5}{\ln 2.5} = 9$	Jan Ber (R.C	
a	1A	
i.e. $a \approx 0.1$ (correct to 1 decimal place)		
	(3)	
The rate of change of the rate of selling of handbags is $r'(t) = 4e^{-0.1t} - 10e^{-0.2t}$	e de hitti	e y halves Filt
$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{r}'(t) = -0.4e^{-0.1t} + 2e^{-0.2t}$		RRUSE) TOUR
$\frac{d}{dt}$ r'(t) = 0 when $0.4e^{-0.1t} = 2e^{-0.2t}$	1M	
$\frac{1}{dt} r'(t) = 0 \text{when} 0.4e^{-t} = 2e^{-t}$	APOT G	
$e^{0.1t}=5$	1.4	OR 16.0944
$t = 10 \ln 5$	1A	OK 10.0944
$\frac{d^2}{dt^2}r'(t) = 0.04e^{-0.1t} - 0.4e^{-0.2t}$		
	} 1M	OR by using sign test
When $t = 10 \ln 5$, $\frac{d^2}{dt^2} r'(t) = -0.008 < 0$	J	1+06e
Hence $r'(t)$ is maximum when $t = 10 \ln 5$		Laurence Contra
$r(10\ln 5) = 20 - 40e^{-0.1(10\ln 5)} + 50e^{-0.2(10\ln 5)} = 14$		OR 14000 1
The rate of selling $= 14$ thousand per day	1A	OR 14000 per day
	(4)	
	- 01	1000
(d) (i) $r(t) = 20 - 40e^{-0.1t} + 50e^{-0.2t} < 18$	1M	
$25e^{-0.2t} - 20e^{-0.1t} + 1 < 0$	101	
$0.053589838 < e^{-0.1t} < 0.746410161$	1A	
2.924800155 < t < 29.26395809 29.26395809 - 2.924800155 = 26.33915794	d'intrins	selmon a fight (i)
∴ the 'sales warning' will last for 26 days.	1A	101119
		Mar Care
(ii) Number of handbags sold (in thousand) during the 'sales warning' period		000000
$= \int_{2.92480155}^{29.26395809} (20 - 40e^{-0.1t} + 50e^{-0.2t}) dt$	1M	beauter and paid
$= [20t + 400e^{-0.1t} - 250e^{-0.2t}]_{2.924800155}^{29.26395809}$	1A	Accept 388.2191
≈ 388.2190941 388.2190941	1M	
$\frac{388.2190941}{26.33915794} \approx 14.7392$	10	OP 15000
Hence the average number of handbags sold per day is 15 thousand.	1A	OR 15000
	(7)	

	Solution	Marks	Remarks
0. Let	A and B be the operation time of a randomly chosen battery A and B respective	ly.	
(a)	(i) $P(A < 152 \text{ or } A > 184)$		
(a)			
	$= P\left(Z < \frac{152 - 168}{32} \text{ or } Z > \frac{184 - 168}{32}\right)$		
	= P(Z < -0.5 or Z > 0.5)	1.4	
	= 0.617	1A	
	(ii) $P(A > k) = 0.05$		
		1M	Accept 1.64 or 1.65
	$\frac{k-168}{32} = 1.645$		Accept 220.48 or 220
	k = 220.64	1A	Accept 220.40 of 220
	(iii) $P(B > 188) = 0.33$ and $P(B < 213.2) = 0.877$		
	$P\left(Z > \frac{188 - \mu}{\sigma}\right) = 0.33 \text{ and } P\left(Z < \frac{213.2 - \mu}{\sigma}\right) = 0.877$		
	$P(Z > \frac{1}{\sigma}) = 0.33$ and $P(Z < \frac{1}{\sigma}) = 0.077$	vi se farb	alico) Lileae au
	$\frac{188 - \mu}{\sigma} = 0.44$ and $\frac{213.2 - \mu}{\sigma} = 1.16$	1M	
	Solving, $\mu = 172.6$ and $\sigma = 35$.	1A+1A	
	Solving, $\mu = 172.0$ and $\theta = 20$.	Admir kr	(a) The rate of chatter
	(iv) $P(B < 146) = P\left(Z < \frac{146 - 172.6}{35}\right)$		$a = (0)^{\frac{1}{2}}$
	= P(Z < -0.76)	1A	(1) (1) (1) (1) (1) (1) (1) (1) (1) (1)
	= 0.2236		1 = 1%
		(7)	\$610 × 1
4.	(2) 1 1500 1 vP(4 < 104)	1M €	5500 - (1)7 - P
(b)	(i) $\lambda_A = 1500 \times \frac{1}{3} \times P(A < 104)$	16	3 74 2
	$=500 \times P\left(Z < \frac{104 - 168}{32}\right)$	140.00	
	$= 500 \times P(Z < -2)$	May to the state.	Either one
	$= 300 \times 1 (2 \times -2)$ $= 11.4 \text{(correct to 1 d.p.)}$	1A	
	$\lambda_B = 1500 \times \frac{2}{3} \times P(B < 104)$	<	Haller 14 State 861
	$=1000 \times P\left(Z < \frac{104 - 172.6}{35}\right)$		
	$= 1000 \times P(Z < -1.96)$		ns - 121 - 11
	= 25.0 (correct to 1 d.p.)	1A	Accept 25
		4110	100845 P.5
	(ii) $P(4 \le \text{number of 'faulty' batteries } A \text{ produced } \le 6)$	9 FISH 9028	
	$=\frac{e^{-11.4}11.4^4}{4!} + \frac{e^{-11.4}11.4^5}{5!} + \frac{e^{-11.4}11.4^6}{6!}$	1M	
	≈ 0.0600	1A	a lo estareix (iii)
	(iii) The required probability	144 - 657	
	(iii) The required probability $e^{-11.4}11.4^4 e^{-25}25^6 e^{-11.4}11.4^5 e^{-25}25^5$		AUROCAV.,
	$= \frac{\frac{e^{-11.4}11.4^4}{4!} \times \frac{e^{-25}25^6}{6!} + \frac{e^{-11.4}11.4^5}{5!} \times \frac{e^{-25}25^5}{5!}}{\frac{e^{-11.4}11.4^4}{4!} \times \frac{e^{-25}25^6}{6!} + \frac{e^{-11.4}11.4^5}{5!} \times \frac{e^{-25}25^5}{5!} + \frac{e^{-11.4}11.4^6}{6!} \times \frac{e^{-25}25^4}{4!}}$	1M+1M	1M for numerator 1M for denominator
	$= \frac{11.4}{e^{-11.4}11.4^4} e^{-25}25^6 + \frac{e^{-11.4}11.4^5}{e^{-25}25^5} + \frac{e^{-11.4}11.4^6}{e^{-25}25^4}$		Tivi for denominator
	4: 0.	1A	WEIGHT BE
	≈ 0.8815	IA	es estreamold
		(8)	

	Eĥ	Solution	Marks	Remarks
. (a) ((i)	$\frac{e^{-\lambda}\lambda^8}{8!} = \frac{12.39}{120}$ and $\frac{e^{-\lambda}\lambda^9}{9!} = \frac{8.26}{120}$	1M	
(4)				
		Dividing, we have $\frac{e^{-\lambda} \lambda^9}{9!} \cdot \frac{8!}{e^{-\lambda} \lambda^8} = \frac{8.26}{120} \cdot \frac{120}{12.39}$		
		i.e. $\lambda = 6$	1A	OR 6.0
			1M	
		$C_7^{10} p^7 (1-p)^3 = \frac{25.80}{120}$ and $C_8^{10} p^8 (1-p)^2 = \frac{14.51}{120}$	11V1	
		Dividing, we have $\frac{C_8^{10} p^8 (1-p)^2}{C_7^{10} p^7 (1-p)^3} = \frac{14.51}{120} \cdot \frac{120}{25.80}$		
		Dividing, we have $\frac{1}{C_7^{10}p^7(1-p)^3} - \frac{1}{120} = 25.80$	100	
		$\frac{3p}{8(1-p)} = \frac{1451}{2580}$		
		8(1-p) 2580		2902
		$p \approx 0.6$ (correct to 1 decimal place)	1A	$\frac{2902}{4837}$ not acceptable
		$a^{-6}6^{7}$)	
	(ii)	$a = \frac{e^{-6}6^7}{7!} \cdot 120 \approx 16.52$		
		$b = \frac{e^{-6} 6^{10}}{10!} \cdot 120 \approx 4.96$	J 1A	1A for any two correct
		10:	1A	1A for the other two correct
		$c = C_9^{10} (0.6)^9 (0.4) \cdot 120 \approx 4.84$		No. of the Control of
		$d = (0.6)^{10} \cdot 120 \approx 0.73$)	2.0
	(iii)	For the number of new born babies diagnosed with congenital diseases	or over on	Bellevices etterni
		greater than 10, the expected frequency by Po (6) is $120-72.76-16.52-12.39-8.26-4.96 \approx 5.11$	1M	
		The sum of errors for model fitted by Po (6) is		
		$E_1 = 74 - 72.76 + 20 - 16.52 + 14 - 12.39 + 8 - 8.26 + 4 - 4.96 + 0 - 5.11 $		
		=12.66	1A	a signor daine medil
		The sum of errors for model fitted by B (10, 0.6) is		
		$E_2 = 74 - 74.13 + 20 - 25.80 + 14 - 14.51 + 8 - 4.84 + 4 - 0.73 $		
		= 12.87	1A	Accept 12.88
		Since $E_1 < E_2$, Po (6) fits the observed data better.	1	STASSAGE STATE
			(10)	
(b)	(i)	P(a new born baby has congenital diseases)	id si fino	ato dili alipi 185
(0)	(1)	$= 0.45 \times 0.025 + 0.55 \times 0.01$	1.4	Accept 0.0168
		= 0.01675	1A	Accept 0.0108
	(ii)	P(a baby is born to a non-local mother the baby has congenital diseases)		eren le 5 Aug
		$=\frac{0.55\times0.01}{0.01675}$		
			1A	OR 0.3284
		$=\frac{22}{67}$	of claim	The second second
	(iii)	P(7 babies are born to non-local mothers 8 to 10 babies have congenital diseases)		264
	()	$e^{-6}6^{8}$ $_{6}^{8}(22)^{7}(45)$ $e^{-6}6^{9}$ $_{6}^{9}(22)^{7}(45)^{2}$ $e^{-6}6^{10}$ $_{6}^{10}(22)^{7}(45)^{2}$)3	
		$\frac{1}{8!}C_7^{\circ}\left(\frac{1}{67}\right)\left(\frac{1}{67}\right)^{+}\frac{1}{9!}C_7^{\circ}\left(\frac{1}{67}\right)\left(\frac{1}{67}\right)^{-}\frac{1}{10!}C_7^{\circ}\left(\frac{1}{67}\right)\left(\frac{1}{67}\right)$) 1M+1M	1M for numerator
		$= \frac{e^{-6} \frac{6^8}{8!} C_7^8 \left(\frac{22}{67}\right)^7 \left(\frac{45}{67}\right) + \frac{e^{-6} \frac{6^9}{9!} C_7^9 \left(\frac{22}{67}\right)^7 \left(\frac{45}{67}\right)^2 + \frac{e^{-6} \frac{6^{10}}{10!} C_7^{10} \left(\frac{22}{67}\right)^7 \left(\frac{45}{67}\right)^2}{\frac{e^{-6} \frac{6^8}{6^9}}{6^9} + \frac{e^{-6} \frac{6^{10}}{10!}}{\frac{10!}{10!}} C_7^{10} \left(\frac{22}{67}\right)^7 \left(\frac{45}{67}\right)^7 \left(45$		1M for denominator
		8! 9! 10!	A 1777 M	Beerla on the
		≈ 0.0061	1A	1881.0 H
			(5)	

	Solution	Marks	Remarks
(a)	(i) P(the centre needs to give out 2 or 3 coupons) = P(10 or 11 customers show up)	La Lac	
	$=C_{10}^{12} \left(\frac{2}{3}\right)^{10} \left(\frac{1}{3}\right)^2 + C_{11}^{12} \left(\frac{2}{3}\right)^{11} \left(\frac{1}{3}\right)$	1M	
	$=\frac{10240}{59049}$	1A	OR 0.1734
	 (ii) P(every customer with booking who shows up can be assigned a trainer) = P(at most 8 customers show up) 		
	$= 1 - C_9^{12} \left(\frac{2}{3}\right)^9 \left(\frac{1}{3}\right)^3 - C_{10}^{12} \left(\frac{2}{3}\right)^{10} \left(\frac{1}{3}\right)^2 - C_{11}^{12} \left(\frac{2}{3}\right)^{11} \left(\frac{1}{3}\right) - \left(\frac{2}{3}\right)^{12}$	1M	
	$=\frac{107515}{177147}$	1A	OR 0.6069
	177147	(4)	restroined
(b)	If the centre accepts 10 bookings, then P(every customer who have made a booking can be assigned a trainer)	3C.01 x 4	$1 = \frac{e^{-2\pi i k}}{2\pi} = 0 (4)$
	$=1-C_9^{10} \left(\frac{2}{3}\right)^9 \left(\frac{1}{3}\right) - \left(\frac{2}{3}\right)^{10}$	- 1 a m 1 2	(1) (1) (1) (1) (1) (1) (1) (1) (1) (1)
	≈ 0.8960 > 0.8	1A	(a.0) (3 = 5 (a.0) (3 = 5
	If the centre accepts 11 bookings, then P(every customer who have made a booking can be assigned a trainer)	e englere	deade sittled (iii)
	$=1-C_9^{11}\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)^2-C_{10}^{11}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)-\left(\frac{2}{3}\right)^{11}$		COLD TRANSPORT
	≈ 0.7659 < 0.8	1A	r to quartit
	Hence the centre can accepts 10 bookings at most.	(3)	, då SI se Floritus (d)
(c)	(i) The expected income in that evening		
(0)	$= \$ (0.5 \times 3800 + 0.3 \times 2800 + 0.2 \times 1800) \times 8$ $= \$ 24800$	1M 1A	- ₁ 3
	(ii) P(the 8th customer is the first one to select Jade programs)		
	$=(0.8)^7(0.2)$		000 1000 27 (1) (1) (1) (2) (2)
	$=\frac{16384}{390625}$	1A	OR 0.0419
	(iii) P(all programs are selected and exactly 3 are Diamond programs)		OR
	$= \frac{8!}{3! \cdot 4! \cdot 1!} (0.5)^3 (0.3)^4 (0.2)^1 + \frac{8!}{3! \cdot 3! \cdot 2!} (0.5)^3 (0.3)^3 (0.2)^2$		$C_3^8 (0.5)^3 [(0.5)^5 - (0.3)^5 - (0.2)^5]$
	$+\frac{8!}{3!2!3!}(0.5)^3(0.3)^2(0.2)^3 + \frac{8!}{3!1!4!}(0.5)^3(0.3)^1(0.2)^4$	1M+1A 1A	ela sardos 127 (m)
	= 0.1995 (iv) The required probability		
	$= \frac{1}{0.1995} \left[\frac{8!}{3! 4! 1!} (0.5)^3 (0.3)^4 (0.2)^1 + \frac{8!}{3! 3! 2!} (0.5)^3 (0.3)^3 (0.2)^2 \right]$	1M	OR $C_3^8 (0.5)^3 [C_1^5 (0.3)^4 (0.2) + C_3^5 (0.3)^3 (0.3)^4 (0.2) + C_3^5 (0.3)^3 (0.3)^4 $
	0.1995 [3! 4! 1! 5: 5: 2:] ≈ 0.6632	1A	0.1995
		(8)	