香 港 考 試 及 評 核 局
HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

2 0 1 1 年 香 港 高 級 程 度 會 考
HONG KONG ADVANCED LEVEL EXAMINATION 2011

數學及統計學　高級補充程度

MATHEMATICS AND STATISTICS AS-LEVEL

評 卷 參 考

MARKING SCHEME

本評卷參考乃香港考試及評核局專爲今年本科考試而編寫，供閱卷員參
考之用。閱卷員在完成閱卷工作後，若將本評卷參考提供其任教會考班
的本科同事參閱，本局不表反對，但須切記，在任何情況下均不得容許
本評卷參考落入學生手中。學生若索閱或求取此等文件，閱卷員／教師
應嚴詞拒絕，因學生極可能將評卷參考視爲標準答案，以致但知硬背死
記，活剝生吞。這種落伍的學習態度，既不符現代教育原則，亦有違考
試着重理解能力與運用技巧之旨。因此，本局籲請各閱卷員／教師通力合
作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and
Assessment Authority for markers' reference. The Authority has no objection to
markers sharing it, after the completion of marking, with colleagues who are teaching
the subject. However, under no circumstances should it be given to students because
they are likely to regard it as a set of model answers. Markers/teachers should
therefore firmly resist students' requests for access to this document. Our
examinations emphasise the testing of understanding, the practical application of
knowledge and the use of processing skills. Hence the use of model answers, or
anything else which encourages rote memorisation, should be considered outmoded
and pedagogically unsound. The Authority is counting on the co-operation of
markers/teachers in this regard.

General Instructions To Markers

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct alternative solution merits <u>all the marks</u> allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept / technique had been used.

3. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

4. Unless the form of the answer is specified in the question, alternative simplified forms of answers different from those in the marking scheme should be accepted if they are correct.

5. Unless otherwise specified in the question, use of notations different from those in the marking scheme should not be penalised.

6. In the marking scheme, marks are classified into the following three categories:

   'M' marks   –   awarded for applying correct methods
   'A' marks   –   awarded for the accuracy of the answers
   Marks without 'M' or 'A' – awarded for correctly completing a proof or arriving at an answer given in the question.
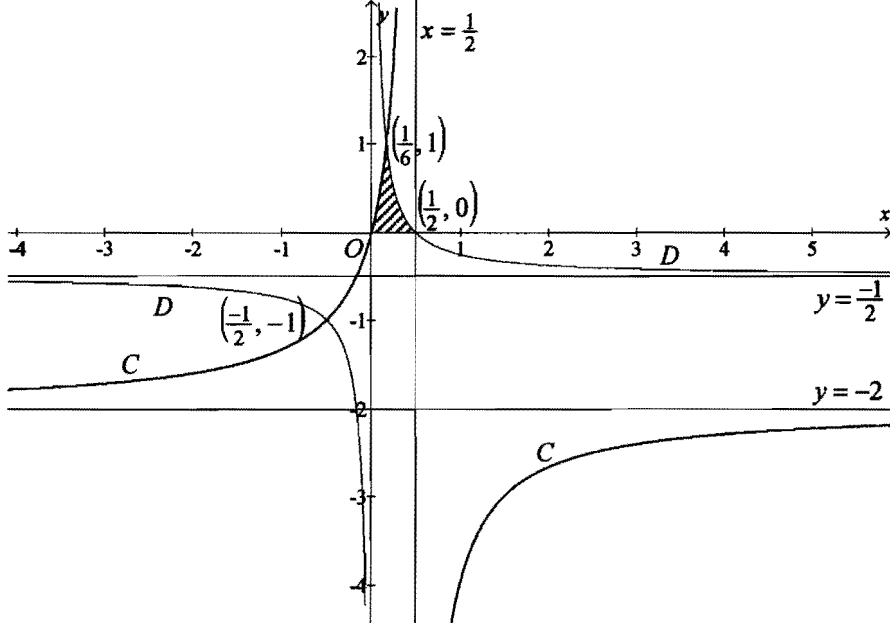
   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. (I.e. Markers should follow through candidates' work in awarding 'M' marks.) However, 'A' marks for the corresponding answers should <u>NOT</u> be awarded, unless otherwise specified.
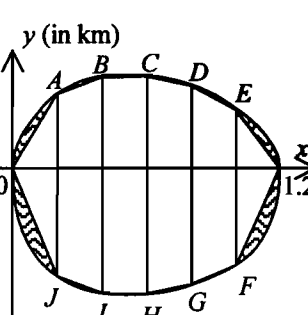
7. In the marking scheme, steps which can be skipped are enclosed by ⌐dotted rectangles⌐, whereas alternative answers are enclosed by solid rectangles .

8. Marks may be deducted for poor presentation (*pp*). The symbol (*pp*–1) should be used to denote 1 mark deducted for *pp*.
   (a) At most deduct 1 mark for *pp* in each section.
   (b) In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

9. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol (*a*–1) should be used to denote 1 mark deducted for *a*.
   (a) At most deduct 1 mark for *a* in each section.
   (b) In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.

10. Marks entered in the Page Total Box should be the NET total scored on that page.

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Solution	Marks	Remarks

1. (a) $$\frac{1}{1+\sqrt{1-x}} = \frac{1}{1+\sqrt{1-x}} \cdot \frac{1-\sqrt{1-x}}{1-\sqrt{1-x}}$$

          $$= \frac{1}{x}(1-\sqrt{1-x})$$ **1M**

                                     1

  (b) $$\therefore \quad \frac{1}{1+\sqrt{1-x}} = \frac{1}{x}\left[1 - \left(1 - \frac{1}{2}x + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}x^2 - \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}x^3 + \cdots\right)\right]$$ **1M**

                 $$= \frac{1}{2} + \frac{1}{8}x + \frac{1}{16}x^2 + \cdots \qquad \text{(where } -1 < x < 1 \text{)}$$ **1A**

        $$\therefore \quad I = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{1}{x}(1-\sqrt{1-x})\,dx$$

              $$\approx \int_{\frac{1}{4}}^{\frac{1}{2}} \left(\frac{1}{2} + \frac{1}{8}x + \frac{1}{16}x^2\right)dx$$ **1M**

              $$= \left[\frac{x}{2} + \frac{x^2}{16} + \frac{x^3}{48}\right]_{\frac{1}{4}}^{\frac{1}{2}}$$

              $$= \frac{427}{3072}$$ **1A**   OR 0.1390

  (c) Since the expansion of $\sqrt{1-x}$ is only valid for $-1 < x < 1$, we cannot use the same method in (b) to estimate the value of J.   **1A**

                                       (7)

2. (a) $$\frac{dX}{dt} = 6\left(\frac{t}{0.2t^3 + 1}\right)^2$$

        $$X = 6\int \frac{t^2}{(0.2t^3 + 1)^2}\,dt$$

        Let $u = 0.2t^3 + 1$, and therefore $du = 0.6t^2\,dt$.   **1M**   For $u = 0.2t^3 + 1$ or $u = (0.2t^3 + 1)^2$

        $$\therefore \quad X = 6\int \frac{1}{0.6u^2}\,du$$ **1M**

             $$= \frac{-10}{u} + C$$ **1A**

             $$= \frac{-10}{0.2t^3 + 1} + C$$

        When $t = 0$, $X = 4$ and hence $C = 14$.

        i.e. $X = \dfrac{-10}{0.2t^3 + 1} + 14$ **1A**

  (b) $$13 = \frac{-10}{0.2t^3 + 1} + 14$$

        $t = \sqrt[3]{45}$ months   **1A**   OR 3.5569 months

  (c) $X = 14 - \dfrac{10}{0.2t^3 + 1} < 14$ for any value of t.   **1M**

        Hence the plan can be run for a long time.   **1A**

                                         (7)

Solution	Marks	Remarks
3. $f(x)=x+\dfrac{3}{x}+\ln(x^2)$		
$f'(x)=1-\dfrac{3}{x^2}+\dfrac{2}{x}$	1A	
$f'(x)=0$ gives $\dfrac{x^2+2x-3}{x^2}=0$	1M	For $f'(x)=0$
i.e. $x=1$ or -3	1A	For both
$f(1)=4$ and $f(-3)=-4+\ln 9$		
$f''(x)=\dfrac{6}{x^3}-\dfrac{2}{x^2}$		
$f''(1)=4>0$ and $f''(-3)=\dfrac{-4}{9}<0$	1M	OR using sign test
Hence $(1,4)$ is a minimum point and $(-3,-4+\ln 9)$ is a maximum point.	1A	For both max pt also: $(-3,-1.8028)$
$f''(x)=0$ gives $\dfrac{6-2x}{x^3}=0$		
i.e. $x=3$		
$f(3)=4+\ln 9$		
<table><tr><td>x</td><td>$x<3$ and $x\neq 0$</td><td>$x=3$</td><td>$x>3$</td></tr><tr><td>$f''(x)$</td><td>+ve</td><td>0</td><td>-ve</td></tr></table>	1M	
Hence $(3,4+\ln 9)$ is a point of inflexion.	1A	OR $(3,6.1972)$
	(7)	
4. (a) P(Susan wins) $=0.75^2(1-0.55^2)+C_1^2 0.75(1-0.75)(1-0.55)^2$	1M+1M	1M for the (2,0) and (2,1) cases 1M for the (1,0) case
$=\dfrac{2997}{6400}$	1A	OR 0.4683
(b) P(Susan wins | Peter scores at least 1 point) $=\dfrac{0.75^2\cdot C_1^2 0.55(1-0.55)}{1-(1-0.55)^2}$	1M+1M	1M for conditional prob 1M for numerator
$=\dfrac{81}{232}$	1A	OR 0.3491
	(6)	
5. (a) Total number of ways $=C_2^7=21$	1M+1A	OR $=C_3^8-C_3^7=21$
(b) Total number of ways $=C_3^8-C_2^5\times C_1^3=26$	1M+1A	
(c) P(A to B | traps at T_1 & T_2) $=\dfrac{C_2^7-C_1^4\times C_1^3}{C_3^8}=\dfrac{9}{56}$	1M+1A	OR 0.1607
	(6)	

Solution	Marks	Remarks

6.   (a) $\text{Mean} = \dfrac{743 + (30 + a) + (40 + b)}{19} = 40 + b$ **1M**

          $a = 18b - 53$ **1A**

          Since a and b are integers where $0 \le a \le 2$ and $0 \le b \le 3$,   **1M**

          b can only be 3 and hence $a = 1$.   **1A**   For both

    (b) $\sigma = 12.1915$ minutes   **1A**

    (c)   Since the mean does not change, the two new data will be "equidistance" from the mean. Hence the two data should be 43, 43 for minimum σ and 26, 60 for maximum σ since the range does not change.   **1M**

          Minimum $\sigma = 11.5964$ and maximum $\sigma = 12.7279$.

          i.e. $11.5964 \le \sigma \le 12.7279$ **1A**

                                                              (7)

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7.   (a)   The equation of the vertical asymptote of C is $x = \dfrac{-d}{c}$.

          Since it is given that the vertical asymptote of C is $x = \dfrac{1}{2}$, $\dfrac{-d}{c} = \dfrac{1}{2}$.

          i.e. $d = \dfrac{-c}{2}$ --- (1)   **1**

          Similarly, the horizontal asymptote is $y = \dfrac{a}{c}$ which is the same as $y = -2$.   **1M**

          $\therefore \dfrac{a}{c} = -2$ which gives $a = -2c$ -------------------------- (2)

          Since C passes through the origin, $0 = \dfrac{a(0) + b}{c(0) + d}$.   **1M**

          i.e. $b = 0$ --- (3)

          Substituting (1), (2) and (3) into $y = \dfrac{ax + b}{cx + d}$, we get the equation of C as

          $y = \dfrac{-2cx + 0}{cx - \frac{c}{2}}$ where $c \neq 0$

          i.e. $y = \dfrac{-4x}{2x - 1}$ **1A**

                                                           (4)

    (b)   The equation of curve D is $y = \dfrac{2x - 1}{-4x}$.

          Solving C and D, we get $\dfrac{-4x}{2x - 1} = \dfrac{2x - 1}{-4x}$.   **1M**

          $16x^2 = 4x^2 - 4x + 1$

          $12x^2 + 4x - 1 = 0$

          $x = \dfrac{1}{6}$ or $\dfrac{-1}{2}$ **1A**   For both

          When $x = \dfrac{1}{6}$, $y = 1$; when $x = \dfrac{-1}{2}$, $y = -1$.

          Hence the coordinates of all the intersecting points are $\left(\dfrac{1}{6}, 1\right)$ and $\left(\dfrac{-1}{2}, -1\right)$.   **1A**

                                                              (3)

Solution	Marks	Remarks

(c)

	1A	For the shape of C
	1A	For the shape of D
	1A	For the asymptotes of D
	1A	For the intercepts of C & D
	(4)	

(d) The area bounded by C, D and the positive x-axis

$$= \int_0^{\frac{1}{6}} \frac{-4x}{2x-1}\,dx + \int_{\frac{1}{6}}^{\frac{1}{2}} \frac{2x-1}{-4x}\,dx \qquad \text{1M}$$

$$= \int_0^{\frac{1}{6}} \left(-2 - \frac{2}{2x-1}\right)dx + \int_{\frac{1}{6}}^{\frac{1}{2}} \left(\frac{-1}{2} + \frac{1}{4x}\right)dx \qquad \text{1M}$$

$$= \left[-2x - \ln|2x-1|\right]_0^{\frac{1}{6}} + \left[\frac{-x}{2} + \frac{1}{4}\ln|x|\right]_{\frac{1}{6}}^{\frac{1}{2}} \qquad \text{1A} \quad \text{For primitive functions}$$

$$= \frac{-1}{3} - \ln\frac{2}{3} - \frac{1}{4} + \frac{1}{4}\ln\frac{1}{2} + \frac{1}{12} - \frac{1}{4}\ln\frac{1}{6}$$

$$= \frac{-1}{2} - \ln 2 + \ln 3 - \frac{1}{4}\ln 2 + \frac{1}{4}(\ln 2 + \ln 3)$$

$$= \frac{5}{4}\ln 3 - \ln 2 - \frac{1}{2} \qquad \text{1A} \quad \text{OR } \frac{-1}{2} - \ln\frac{2}{3} + \frac{1}{4}\ln\frac{1}{2} - \frac{1}{4}\ln\frac{1}{6}$$

| | (4) | |

Solution	Marks	Remarks

8. (a) $e^{t^2+t} = 1 + (t^2+t) + \dfrac{(t^2+t)^2}{2} + \dfrac{(t^2+t)^3}{3!} + \cdots$ **1M**

$= = 1 + t^2 + t + \dfrac{2t^3 + t^2 + \cdots}{2} + \dfrac{t^3 + \cdots}{6} + \cdots$

$= 1 + t + \dfrac{3t^2}{2} + \dfrac{7t^3}{6} + \cdots$ **1A**

$V = \displaystyle\int_0^{\frac{1}{2}} \dfrac{1}{25} e^{t^2+t+2}\, dt$

$\approx \dfrac{e^2}{25} \displaystyle\int_0^{\frac{1}{2}} \left(1 + t + \dfrac{3t^2}{2} + \dfrac{7t^3}{6}\right) dt$ **1M**

$= \dfrac{e^2}{25}\left[t + \dfrac{t^2}{2} + \dfrac{t^3}{2} + \dfrac{7t^4}{24} \right]_0^{\frac{1}{2}}$

$= \dfrac{271}{9600} e^2 \text{ hundred thousand } m^3$ **1A**   OR 0.2086 hund. th. m^3
OR 20858.6896 m^3

Since for $t > 0$, $e^{t^2+t} = 1 + t + \dfrac{3t^2}{2} + \dfrac{7t^3}{6} + \text{positive terms}$,   **1M**

$e^{t^2+t} > 1 + t + \dfrac{3t^2}{2} + \dfrac{7t^3}{6}$

Hence the estimation is an under-estimate.   **1A**

(6)

(b) (i) Area $\approx \dfrac{0.2}{2}[0 + 2(3.8 + 4.2 + 4.3 + 4.1 + 3.4) + 0]$ **1M**

$= 3.96 \text{ km}^2$ **1A**

Since the upper half of the curve is concave downwards and the lower half is concave upwards, the estimation is an under-estimate.   **1A**

(ii) Thickness $\approx \dfrac{20858.6896}{3.96 \times 1000^2}$ m   OR 5.2673×10^{-3} m
OR 5.2673 mm

$\approx 0.0053 \text{ m}$ **1A**

Since both the numerator and denominator are under-estimates, we cannot determine whether the thickness is an over- or under-estimate.   **1A**

(5)

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Solution	Marks	Remarks

(c) $\dfrac{\mathrm{d}W}{\mathrm{d}t}=\dfrac{-(W+1)^{\frac{1}{3}}}{40}$

$\dfrac{\mathrm{d}t}{\mathrm{d}W}=-40(W+1)^{\frac{-1}{3}}$ 1M

$t=-40\displaystyle\int (W+1)^{\frac{-1}{3}}\,\mathrm{d}W$

$\qquad =-60(W+1)^{\frac{2}{3}}+C$ 1A

When $t=0$, $W=\dfrac{271}{9600}e^{2}$.   1M

$\therefore\ \ 0=-60\left(\dfrac{271}{9600}e^{2}+1\right)^{\frac{2}{3}}+C$

$C=68.07743296$

Hence $t=-60(W+1)^{\frac{2}{3}}+68.07743296$

When $W=0$, $t\approx 8.0774$.   1A

Thus all the oil spread will be cleaned up after 8.0774 days.

| | (4) | |

9. (a) (i) $P=\dfrac{ke^{-\lambda t}}{t^{2}}$

$\ln P+2\ln t=\ln k-\lambda t$ 1A

(ii) Intercept of vertical axis $=2.3=\ln k$

$\therefore\ k\approx 10$ correct to the nearest integer   1A

slope $=\dfrac{2.3-0}{0-(-1.15)}=-\lambda$

$\therefore\ \lambda=-2$ 1A

$P=\dfrac{10e^{2t}}{t^{2}}$

$\dfrac{\mathrm{d}P}{\mathrm{d}t}=10\dfrac{t^{2}2e^{2t}-e^{2t}2t}{t^{4}}$ 1A

$\qquad =\dfrac{20e^{2t}(t-1)}{t^{3}}$

t	$0<t<1$	$t=1$	$t>1$
$\dfrac{\mathrm{d}P}{\mathrm{d}t}$	-ve	0	+ ve

1M

Hence the minimum population size is attained when $t=1$.

$P=\dfrac{10e^{2(1)}}{(1)^{2}}$

$\quad =74$ 1A

Hence the minimum population size is 74 hundred thousands.

| | (6) | |

	Solution	Marks	Remarks

(b) (i) $\dfrac{dE}{dt} = hte^{ht} - 1.2e^{ht} + 4.214$

$\dfrac{d^2E}{dt^2} = he^{ht} + h^2te^{ht} - 1.2he^{ht}$ 1A

$\quad\quad = he^{ht}(ht - 0.2)$

When $t = 1$, $\dfrac{dE}{dt}$ is minimum and hence $\dfrac{d^2E}{dt^2} = 0$.

Thus, $h = 0.2$.   1A

(ii) $\dfrac{d}{dt}(te^{0.2t}) = 0.2te^{0.2t} + e^{0.2t}$ 1A

$\therefore \ te^{0.2t} = 5\dfrac{d}{dt}(te^{0.2t}) - 5e^{0.2t}$

$\int te^{0.2t}\,dt = 5te^{0.2t} - 5\int e^{0.2t}\,dt$ 1M

$\quad\quad = 5te^{0.2t} - 25e^{0.2t} + C_1$ 1A

$\dfrac{dE}{dt} = 0.2te^{0.2t} - 1.2e^{0.2t} + 4.214$

$E = 0.2\int te^{0.2t}\,dt - 1.2\int e^{0.2t}\,dt + \int 4.214\,dt$

$\quad = te^{0.2t} - 5e^{0.2t} - 6e^{0.2t} + 4.214t + C$ 1M

$\quad = te^{0.2t} - 11e^{0.2t} + 4.214t + C$

When $t = 0$, $E = 1$.

Hence $1 = 0 - 11 + 0 + C$ which gives $C = 12$.

i.e. $E = te^{0.2t} - 11e^{0.2t} + 4.214t + 12$

When $t = 1$, $E = e^{0.2} - 11e^{0.2} + 4.214 + 12$

$\quad\quad\quad \approx 4$ 1A

Thus the annual electricity consumption is 4 thousand terajoules per year.

(iii) $F = \dfrac{6}{1 - 5e^{rt} + 3e^{2rt}} + 2$

$\dfrac{6}{1 - 5e^{r} + 3e^{2r}} + 2 \approx 4$ 1M

$3e^{2r} - 5e^r - 2 \approx 0$

$e^r \approx 2$ or $\dfrac{-1}{3}$ (rejected)

$r \approx \ln 2$ 1A   OR 0.6931

     (9)

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Solution	Marks	Remarks

10. (a) $P(160 \le Y < K) = 78.88\%$

$$P\left(\frac{160-165}{4} \le Z < \frac{K-165}{4}\right) = 0.7888$$

$$0.3944 + P\left(0 \le Z < \frac{K-165}{4}\right) = 0.7888 \qquad \text{1M}$$

$$\frac{K-165}{4} = 1.25$$

$$K = 170 \qquad \text{1A}$$

(2)

(b) $P(\text{score } 30) = P(170 \le Y < 174)$

$\qquad = P(1.25 \le Z < 2.25)$ $\qquad$ 1M

$\qquad = 0.4878 - 0.3944$

$\qquad = 0.0934$ $\qquad$ 1A

(2)

(c) $P(\text{6th game is the 3rd Bingo}) = C_2^5 (0.2112)^3 (0.7888)^3$ $\qquad$ 1M

$\qquad\qquad \approx 0.0462$ $\qquad$ 1A

(2)

(d) The number of "Bingo" in n games $\sim B(n, 0.7888)$.

$\therefore \quad n(0.7888)(0.2112) \le 2.3$ $\qquad$ 1M

$n \le 13.80597302$

Thus the largest value of n is 13. $\qquad$ 1A

(2)

(e) (i) $P(\text{score } 20) = P(-2.75 \le Z < -1.25) = 0.1026$ $\qquad$ 1A

$\therefore P(\text{win a prize})$

$= P(\text{total score in 4 games} \ge 160)$

$= (0.7888)^4 + C_1^4 (0.7888)^3 (0.0934 + 0.1026) + C_2^4 (0.7888)^2 (0.0934)^2$ $\qquad$ 1M

≈ 0.804490478

≈ 0.8045 $\qquad$ 1A

(ii) $P(\text{win a prize and average score in the first 2 games} \ge 40)$

$= P(\text{total score in 4 games} \ge 160 \text{ and total score in first 2 games} \ge 80)$

$= (0.7888)^4 + C_1^4 (0.7888)^3 (0.0934) + C_1^2 (0.7888)^3 (0.1026)$

$\quad + (C_2^4 - 1)(0.7888)^2 (0.0934)^2$ $\qquad$ 1M

> **Alternative Solution**
> $= P(\text{total score in 4 games} \ge 160) - P(\text{total score in 4 games} \ge 160 \text{ and}$
> $\quad \text{total score in first 2 games} < 80)$
> $\approx 0.804490478 - (0.7888)^2 (0.0934)^2 - C_1^2 (0.7888)^3 (0.1026)$ $\qquad$ 1M

≈ 0.698351364

≈ 0.6984 $\qquad$ 1A

(iii) $P(\text{average score in the first 2 games} < 40 \mid \text{win a prize})$

$\approx \dfrac{0.804490478 - 0.698351364}{0.804490478}$ $\qquad$ 1M

≈ 0.1319 $\qquad$ 1A

(7)

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Solution	Marks	Remarks

11. (a) (i) P(lift is full at G/F)

$$=1-e^{-4}\left(1+4+\frac{4^2}{2!}+\frac{4^3}{3!}+\frac{4^4}{4!}+\frac{4^5}{5!}\right)$$ **1M**

≈ 0.214869613

≈ 0.2149 **1A**

    (ii) P(4 persons gets into the lift and it stops at each floor)

$$=\frac{e^{-4}4^4}{4!}\cdot 4!\left(\frac{1}{4}\right)^4$$ **1M**

≈ 0.0183 **1A**

    (iii) P(lift stops at each floor)

$$=\frac{e^{-4}4^4}{4!}\cdot\frac{4!}{4^4}+\frac{e^{-4}4^5}{5!}\cdot\frac{C_2^5\cdot 4!}{4^5}+0.214869613\cdot\frac{C_3^6\cdot 4!+C_2^6 C_2^4\cdot\frac{4!}{2}}{4^6}$$ **1M+1M** · 1M for using (i) and (ii) 1M for correct cases

≈ 0.1368 **1A**

    (7)

(b) (i) P(3 persons from different floor waits for the lift)

$$=C_3^4(e^{-3}3)^3(e^{-3})$$ **1M**

≈ 0.0007 **1A**

    (ii) P(2 persons waits for the lift)

$$=C_1^4\left(\frac{e^{-3}3^2}{2!}\right)(e^{-3})^3+C_2^4(e^{-3}3)^2(e^{-3})^2$$ **1M** · OR $\dfrac{e^{-12}12^2}{2!}$

≈ 0.0004 **1A**

    (iii) Let the number of persons waiting <u>above</u> 62/F be X.

P(3 persons get into the lift at the 62/F | 3 persons wait at the 62/F)

$=P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$$=(e^{-3})^2+C_1^2(e^{-3}3)(e^{-3})+\left[C_1^2\left(\frac{e^{-3}3^2}{2!}\right)(e^{-3})+(e^{-3})(e^{-3}3)\right]$$

$$+\left[C_1^2\left(\frac{e^{-3}3^3}{3!}\right)(e^{-3})+C_1^2\left(\frac{e^{-3}3^2}{2!}\right)(e^{-3}3)\right]$$ **1M+1M+1M**

1M for 4 cases
1M for the case $X=2$
1M for the case $X=3$

OR $\displaystyle\sum_{k=0}^{3}\frac{e^{-6}6^k}{k!}$

≈ 0.1512 **1A**

    (8)

12. (a) $P(Y<100)=0.121$

$$P\left(\frac{100-\mu}{\sigma}\le Z<0\right)=0.379$$

$$\frac{100-\mu}{\sigma}=-1.17 \text{ ------------------------------------ (1)}$$ **1A**

$P(Y\ge 200)=0.0918$

$$P\left(0<Z<\frac{200-\mu}{\sigma}\right)=0.4082$$

$$\frac{200-\mu}{\sigma}=1.33 \text{ -------------------------------------- (2)}$$ **1A**

Solving (1) and (2), we get $\mu=146.8$ and $\sigma=40$ **1A**

    (3)

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Solution	Marks	Remarks
(b) P(High level) $= P(150 \le Y < 200)$ $= P(0.08 \le Z < 1.33)$ $\approx 0.4082 - 0.0319$ $= 0.3763$	1A	
	(1)	
(c) P(High | rainfall exceeds 100 mm) $= \dfrac{0.3763}{1-0.121}$ ≈ 0.428100113 ≈ 0.4281	1M 1A	For conditional probability
	(2)	
(d) **(i)** P(Severe | rainfall exceeds 100 mm) $= \dfrac{0.0918}{1-0.121}$ ≈ 0.10443686 P(Medium | rainfall exceeds 100 mm) $= \dfrac{1-0.121-0.0918-0.3763}{1-0.121}$ ≈ 0.467463026 P(job will NOT be postponed | rainfall exceeds 100 mm) $= (0.467463026)e^{-1} + (0.428100113)e^{-3} + (0.10443686)e^{-6}$ ≈ 0.193542759 ≈ 0.1935	1A 1A 1M 1A	
(ii) P(job will be postponed for 1 day | rainfall exceeds 100 mm) $= (0.467463026) \cdot e^{-1}1 + (0.428100113) \cdot e^{-3}3 + (0.10443686) \cdot e^{-6}6$ ≈ 0.237464824 ≈ 0.2375	1M 1A	
(iii) P(job will be postponed for 2 days | rainfall exceeds 100 mm) $= (0.467463026) \cdot \dfrac{e^{-1}1^2}{2!} + (0.428100113) \cdot \dfrac{e^{-3}3^2}{2!} + (0.10443686) \cdot \dfrac{e^{-6}6^2}{2!}$ ≈ 0.186557057 P(High level | job will be postponed for at least 3 days) $= \dfrac{0.428100113\left(1 - e^{-3} - e^{-3}3 - \dfrac{e^{-3}3^2}{2!}\right)}{1 - 0.193542759 - 0.237464824 - 0.186557057}$ ≈ 0.6457	1A 1M 1A	
	(9)	

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