評卷參考 *

Marking Scheme

香港考試及評核局

Hong Kong Examinations and Assessment Authority

2007年香港高級程度會考

Hong Kong Advanced Level Examination 2007

數學及統計學　高級補充程度

Mathematics and Statistics　AS-Level

* 此部分只設英文版本

AS Mathematics and Statistics

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for poor presentation (*pp*). The symbol (*pp-1*) should be used to denote 1 mark deducted for *pp*. At most deducted 1 mark from Section A and 1 mark from Section B for *pp*. In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

7. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol (*a-1*) should be used to denote 1 mark deducted for *a*. At most deducted 1 mark from Section A and 1 mark from Section B for *a*. In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.

8. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with [rectangles].

Solution	Marks
1. (a) (i) $\left(1+\frac{x}{a}\right)^r$	
$=1+\frac{r}{a}x+\frac{r(r-1)}{2!}(\frac{x}{a})^2+\cdots$	1M for any two terms correct
$=1+\frac{rx}{a}+\frac{r(r-1)}{2a^2}x^2+\cdots$	
So, we have $\frac{r}{a}=\frac{2r}{3}$ and $\frac{r(r-1)}{2a^2}=\frac{-1}{18}$.	
Solving, we have $a=\frac{3}{2}$ and $r=\frac{1}{2}$.	1A for both correct
(ii) The binomial expansion is valid for $\left\|\frac{2x}{3}\right\|<1$.	1M can be absorbed
Thus, the range of values of x is $\frac{-3}{2}<x<\frac{3}{2}$.	1A accept $\|x\|<\frac{3}{2}$
(b) (i) $\left(1-\frac{x}{a}\right)^r=1-\frac{1}{3}x-\frac{1}{18}x^2+\cdots$	1M for replacing x by $-x$
(ii) The binomial expansion is valid for $\left\|\frac{-2x}{3}\right\|<1$.	
Thus, the range of values of x is $\frac{-3}{2}<x<\frac{3}{2}$.	1M accept $\|x\|<\frac{3}{2}$ ----------(6)
2. (a) Let $u=2t^2+50$.	
Then, we have $\frac{du}{dt}=4t$.	1A
N	
$=\int\frac{800t}{(2t^2+50)^2}\,dt$	
$=\int\frac{200}{u^2}\,du$	1M
So, we have $N=\frac{-200}{u}+C$, where C is a constant.	1A
Therefore, we have $N=\frac{-200}{2t^2+50}+C$.	
Using the condition that $N=4$ when $t=0$, we have $4=-4+C$.	1M for finding C
Hence, we have $C=8$.	1A
Thus, we have $N=8-\frac{200}{2t^2+50}$.	

Solution	Marks
Alternative Solution	
Let $u = 2v^2 + 50$.	
Then, we have $\mathrm{d}u = 4v\mathrm{d}v$.	1A
$[N]_0^t = \int_0^t \frac{800v}{(2v^2+50)^2}\mathrm{d}v$	
$= \int_{50}^{2t^2+50} \frac{200}{u^2}\mathrm{d}u$	1M For substitution
$= \left[\frac{-200}{u}\right]_{50}^{2t^2+50}$	1A For $\frac{-200}{u}$
$\therefore N - 4 = \frac{-200}{2t^2+50} - \frac{-200}{50}$	1M For using $N(0) = 4$
i.e. $N = 8 - \frac{200}{2t^2+50}$.	1A
(b) When $N = 6$, we have $8 - \frac{200}{2t^2+50} = 6$.	1M
So, we have $t = 5$.	1A
The number of bacteria will be 6 million 5 days after the start of the research.	----------(7)
3. (a) $y = \frac{1-e^{4x}}{1+e^{8x}}$	
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(1+e^{8x})(-4e^{4x}) - (1-e^{4x})(8e^{8x})}{(1+e^{8x})^2}$	1M for quotient rule or product rule
When $x = 0$, we have $\frac{\mathrm{d}y}{\mathrm{d}x} = -2$.	1A
Alternative Solution	
$y = \frac{1-e^{4x}}{1+e^{8x}}$	
$\ln y = \ln(1-e^{4x}) - \ln(1+e^{8x})$	
$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4e^{4x}}{1-e^{4x}} - \frac{8e^{8x}}{1+e^{8x}}$	1M For log differentiation
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1-e^{4x}}{1+e^{8x}}\left(\frac{-4e^{4x}}{1-e^{4x}} - \frac{8e^{8x}}{1+e^{8x}}\right)$	
$= \frac{-4e^{4x}}{1+e^{8x}} - \frac{8(1-e^{4x})e^{8x}}{(1+e^{8x})^2}$	
When $x = 0$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4}{1+1} - 0 = -2$	1A

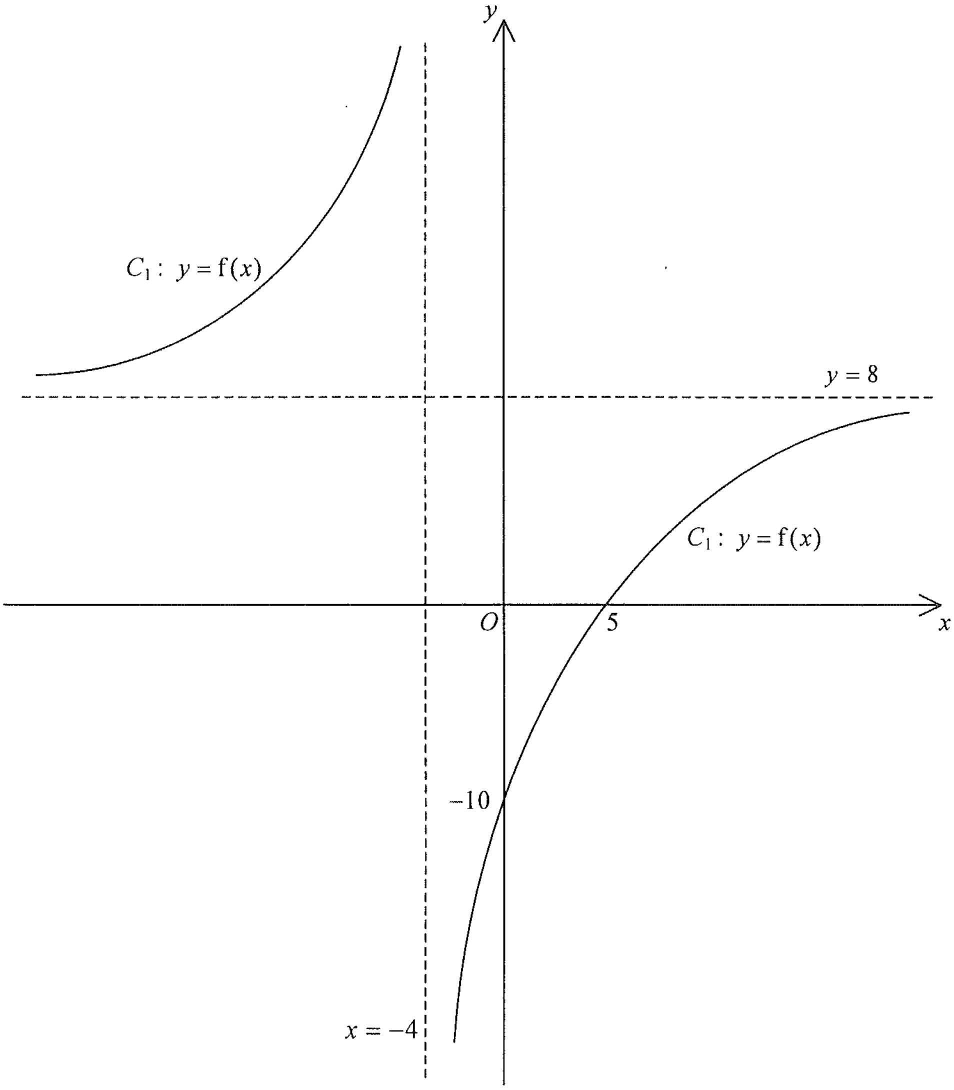
Solution	Marks
(b) (i) Since $(z^2+1)e^{3z}=e^{\alpha+\beta x}$, we have $\ln(z^2+1)+3z=\alpha+\beta x$.	1A
(ii) Since the graph of the linear function passes through the origin and the slope of the graph is 2 , we have $\alpha=0$ and $\beta=2$.	1A for both correct
(iii) $\ln(z^2+1)+3z=2x$	
$\frac{2z}{z^2+1}+3=2\frac{dx}{dz}$	
Therefore, we have $\left.\frac{dx}{dz}\right\|_{z=0}=\frac{3}{2}$.	1A
Note that $x=0$ when $z=0$.	
Also note that $\left.\frac{dy}{dx}\right\|_{x=0}=-2$.	
$\left.\frac{dy}{dz}\right\|_{z=0}$	
$=\left(\left.\frac{dy}{dx}\right\|_{x=0}\right)\left(\left.\frac{dx}{dz}\right\|_{z=0}\right)$	
$=(-2)\left(\frac{3}{2}\right)$	1M for chain rule
$=-3$	1A
$y=\frac{1-e^{6z+2\ln(z^2+1)}}{1+e^{12z+4\ln(z^2+1)}}$	1A
$y=\frac{1-(z^2+1)^2e^{6z}}{1+(z^2+1)^4e^{12z}}$	
$\frac{dy}{dz}=\frac{\left(1+(z^2+1)^4e^{12z}\right)\left(-6(z^2+1)^2e^{6z}-2(z^2+1)(2z)e^{6z}\right)-\left(1-(z^2+1)^2e^{6z}\right)\left(12(z^2+1)^4e^{12z}+4(z^2+1)^3(2z)e^{12z}\right)}{\left(1+(z^2+1)^4e^{12z}\right)^2}$	1M for quotient rule or product rule
$\left.\frac{dy}{dz}\right\|_{z=0}=-3$	1A
	----------(7)

Solution	Marks
4. (a) (i) Note that $5.1 < 5.3$ and $k \geq 0$.	
Hence, we have $5.3 = k - 1.2$.	
Thus, we have $k = 6.5$.	1A
$5.3 = k - 1.2$ or $5.3 = 5.1 - k$	
So, we have $k = 6.5$ or $k = -0.2$ (rejected as $k \geq 0$) .	
Thus, we have $k = 6.5$.	1A
(ii) Stem (units) | Leaf (tenths)	
1 | 2 8 9	
2 | 1 1 2 3 4 4 9	1M + 1A
3 | 6 7 9	
4 | 7	
5 | 1	
6 | 5	
(iii) The mean	
$= 3.05$ hours	1A (accept 3.0500 hours)
The median	
$= 2.4$ hours	1A (accept 2.4000 hours)
(b) The revised mean is greater than the mean obtained in (a)(iii).	1A
The revised median is the same as the median obtained in (a)(iii).	1A
The change in the mean is positive.	1A
There is no change in the median.	1A
	----------(7)

Solution	Marks
5. (a) $P(A' \cap B)$	
$= P(B \mid A')P(A')$	
$= 0.3(1-a)$	1M (either one)
$P(A' \cap B)$	
$= P(A' \mid B)P(B)$	
$= 0.6b$	
Hence, we have $0.6b = 0.3(1-a)$.	
Thus, we have $a + 2b = 1$.	1
(b) $P(A \cap B')$	
$= P(B' \mid A)P(A)$	
$= 0.7a$	
$P(A \cup B')$	
$= 1 - P(A' \cap B)$	
$= 1 - 0.6b$	1M for complementary events
Note that $P(A \cup B') = P(A) + P(B') - P(A \cap B')$.	
Hence, we have $1 - 0.6b = a + (1-b) - 0.7a$	1M
So, we have $3a = 4b$.	
Solving $a + 2b = 1$ and $3a = 4b$, we have $a = 0.4$ and $b = 0.3$.	1A for both correct
$P(A \cap B')$	
$= P(B' \mid A)P(A)$	
$= 0.7a$	
$P(A \cup B')$	
$= 1 - P(A' \cap B)$	
$= 1 - 0.3(1-a)$	1M for complementary events
$= 0.7 + 0.3a$	
Note that $P(A \cup B') = P(A) + P(B') - P(A \cap B')$.	
Hence, we have $0.7 + 0.3a = a + 1 - b - 0.7a$	1M
So, we have $b = 0.3$.	
By (a), we have $a + 2(0.3) = 1$.	
Thus, we have $a = 0.4$.	1A (both correct)
(c) Since $P(A \cap B) = P(A) - P(A \cap B')$, $P(A) = 0.4$ and $P(A \cap B') = 0.28$,	
we have $P(A \cap B) = 0.12 = (0.4)(0.3) = P(A)P(B)$.	1M for relating $P(A \cap B)$ and $P(A)P(B)$
Thus, A and B are independent events.	1A f.t.
Since $P(A) = a$, we have $P(A \cap B) = P(A) - P(A \cap B') = a - 0.7a = 0.3a$.	
With the help of $P(B) = 0.3$, we have $P(A \cap B) = P(A)P(B)$.	1M for relating $P(A \cap B)$ and $P(A)P(B)$
Thus, A and B are independent events.	1A f.t.
Since $P(A' \mid B) = 0.6$, we have $P(A \mid B) = 1 - P(A' \mid B) = 1 - 0.6 = 0.4$.	
With the help of $P(A) = 0.4$, we have $P(A \mid B) = P(A)$.	1M for relating $P(A \mid B)$ and $P(A)$
Thus, A and B are independent events.	1A f.t.
	-----------(7)

Solution	Marks
6. (a) The required probability	
$=(\frac{1}{10})(\frac{4}{9})(\frac{3}{8})$	1A for numerator or denominator + 1A for all
$=\frac{1}{60}$	1A
≈ 0.016666667	
≈ 0.0167	$a-1$ for r.t. 0.017
The required probability	
$=\left(\frac{C_1^1 C_2^4}{C_3^{10}}\right)\left(\frac{1}{C_1^3}\right)$	1A for 1st blacket + 1A for all
$=\frac{1}{60}$	1A
≈ 0.016666667	
≈ 0.0167	$a-1$ for r.t. 0.017
The required probability	
$=\left(\frac{C_1^1}{C_1^{10}}\right)\left(\frac{C_2^4}{C_2^9}\right)$	1A for 2nd blacket + 1A for all
$=\frac{1}{60}$	1A
≈ 0.016666667	
≈ 0.0167	$a-1$ for r.t. 0.017
The required probability	
$=\frac{P_1^1 P_2^4}{P_3^{10}}$	1A for numerator or denominator + 1A for all
$=\frac{1}{60}$	1A
≈ 0.016666667	
≈ 0.0167	$a-1$ for r.t. 0.017
(b) The required probability	
$=\frac{1}{60}+(\frac{5}{10})(\frac{5}{9})(\frac{4}{8})$	1M for $(p+q+r+s)$ + 1M for using (a)
$=\frac{7}{45}$	1A
≈ 0.155555556	
≈ 0.1556	$a-1$ for r.t. 0.156
The required probability	
$=\frac{1}{60}+\frac{C_1^5 C_2^5}{C_1^{10} C_2^9}$	1M for $(p+q+r+s)$ + 1M for using (a)
$=\frac{7}{45}$	1A
≈ 0.155555556	
≈ 0.1556	$a-1$ for r.t. 0.156

Solution	Marks
The required probability $=\frac{1}{60}+\frac{P_1^5P_2^5}{P_3^{10}}$ $=\frac{7}{45}$ ≈ 0.155555556 ≈ 0.1556	1M for $(p+q)$ + 1M for using (a) 1A a–1 for r.t. 0.156
The required probability $=(\frac{4}{10})(\frac{3}{9})(\frac{6}{8})+(\frac{2}{10})(\frac{4}{9})(\frac{5}{8})$ $=\frac{7}{45}$ ≈ 0.155555556 ≈ 0.1556	1M for $(p+q+r+s)$ 2A a–1 for r.t. 0.156
The required probability $=\frac{C_2^4C_1^6}{C_2^{10}C_1^8}+\frac{C_1^4C_1^1C_1^5}{C_2^{10}C_1^8}$ $=\frac{7}{45}$ ≈ 0.155555556 ≈ 0.1556	1M for $(p+q+r+s)$ 2A a–1 for r.t. 0.156
The required probability $=\frac{P_2^4P_1^6}{P_3^{10}}+\frac{P_1^2P_1^4P_1^5}{P_3^{10}}$ $=\frac{7}{45}$ ≈ 0.155555556 ≈ 0.1556	1M for $(p+q+r+s)$ 2A a–1 for r.t. 0.156
	----------(6)

Solution	Marks
7. (a) $\because \lim_{x\to\pm\infty}\frac{8x-40}{x+4}=\lim_{x\to\pm\infty}\frac{8-\frac{40}{x}}{1+\frac{4}{x}}=8$	
$\therefore$ the equation of the horizontal asymptote to C_1 is $y=8$.	
$\because \lim_{x\to -4^-} f(x)=+\infty$ and $\lim_{x\to -4^+} f(x)=-\infty$	
$\therefore$ the equation of the vertical asymptote to C_1 is $x=-4$.	
The x-intercept of C_1 is 5 .	
The y-intercept of C_1 is -10 .	
	1A for all the asymptotes of C_1 1A for all the intercepts of C_1 1A for the shape of C_1 ----------(3)

Solution	Marks

(b) (i) $g(x)=\dfrac{(x+4)^2(x+5)}{8}$

$g'(x)=\dfrac{3(x+4)(x-2)}{8}$

$g'(x)=0$ when $x\neq -4$ or $x=2$

$g''(x)=\dfrac{3(x+1)}{4}$

$g''(x)=0$ when $x=-1$

x	$x<-4$	-4	$-4<x<-1$	-1	$-1<x<2$	2	$x>2$
$g'(x)$	+	0	−	−	−	0	+
$g''(x)$	−	−	−	0	+	+	+
$g(x)$	↗	0	↘	$\frac{-27}{4}$	↘	$\frac{-27}{2}$	↗

Since $g'(2)=0$ and $g''(2)>0$, — 1M for justification (either one)

the coordinates of the relative minimum point are $(2,\frac{-27}{2})$. — 1A

Since $g'(-4)=0$ and $g''(-4)<0$,

the coordinates of the relative maximum point are $(-4,0)$. — 1A

Since $g''(x)\begin{cases}<0 & \text{if } x<-1\\ =0 & \text{if } x=-1\\ >0 & \text{if } x>-1\end{cases}$, — 1M for justification

the coordinates of the point of inflexion are $(-1,\frac{-27}{4})$. — 1A

(ii) $C_1: y=f(x)$, where $f(x)=\dfrac{8x-40}{x+4}$.

$C_2: y=g(x)$, where $g(x)=\dfrac{(x+4)^2(x-5)}{8}$.

Note that $f(x)=g(x)$

$\Leftrightarrow \dfrac{8x-40}{x+4}=\dfrac{(x+4)^2(x-5)}{8}$

$\Leftrightarrow 64(x-5)=(x+4)^3(x-5)$

$\Leftrightarrow (x-5)\left((x+4)^3-64\right)=0$

$\Leftrightarrow x=0$ or $x=5$

So, the coordinates of the points of intersection are $(0,-10)$ and $(5,0)$.

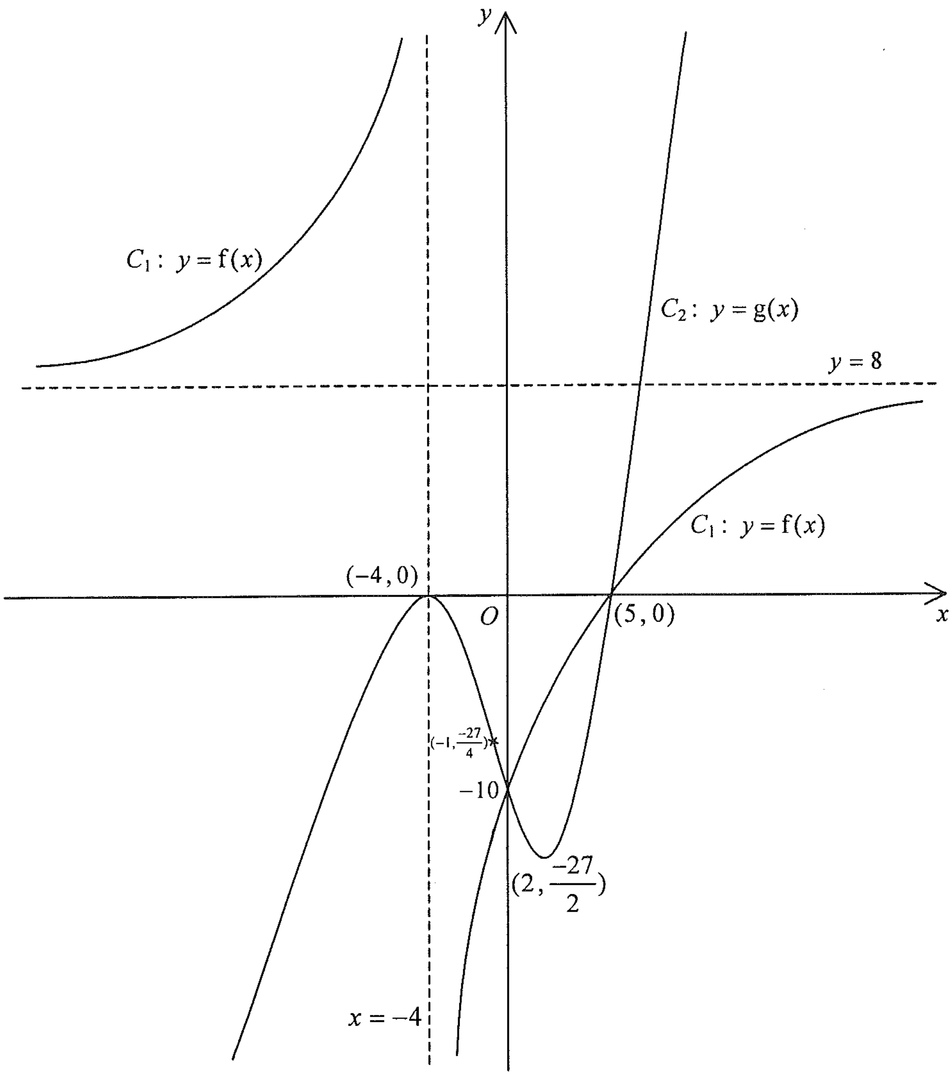
Also, the y-intercepts of C_1 and C_2 are -10.

When $f(x)=0$, we have $x=5$.

When $g(x)=0$, we have $x\neq -4$ or $x=5$.

So, the x-intercept of C_1 is 5.

Also, the x-intercepts of C_2 are -4 and 5.

Solution	Marks
	1A for the shape of C_2 1A for all the extreme points and the point of inflexion 1A for all the points of intersection ----------(8)
(c) The required area	
$= \int_0^5 (f(x) - g(x))\,dx$	1M accept $\int_5^0 (g(x) - f(x))\,dx$
$= \int_0^5 \left(\frac{8x-40}{x+4} - \frac{(x+4)^2(x-5)}{8} \right) dx$	
$= \int_0^5 \left[\left(8 - \frac{72}{x+4} \right) - \left(\frac{x^3}{8} + \frac{3x^2}{8} - 3x - 10 \right) \right] dx$	1M for division
$= \left[8x - 72\ln(x+4) - \left(\frac{x^4}{32} + \frac{x^3}{8} - \frac{3x^2}{2} - 10x \right) \right]_0^5$	1A for correct integration
$= \frac{2955}{32} - 72\ln\left(\frac{9}{4}\right)$	1A
$= \frac{2955}{32} - 144\ln\left(\frac{3}{2}\right)$	
≈ 33.95677443	
≈ 33.9568	a–1 for r.t. 33.957 ----------(4)

Solution	Marks
8. (a) (i) The total profit made by company A	
$= \int_0^6 \mathrm{f}(t)\,\mathrm{d}t$	1A withhold 1A for omitting this step
$\approx \frac{1}{2}\left(\mathrm{f}(0)+\mathrm{f}(6)+2(\mathrm{f}(1)+\mathrm{f}(2)+\mathrm{f}(3)+\mathrm{f}(4)+\mathrm{f}(5))\right)$	1M for trapezoidal rule
≈ 37.48705341	
≈ 37.4871 billion dollars	1A $a-1$ for r.t. 37.487
(ii) $\mathrm{f}(t)=\ln(e^t+2)+3$	
$\frac{\mathrm{d}\mathrm{f}(t)}{\mathrm{d}t}=\frac{e^t}{e^t+2}$	1A
$\frac{\mathrm{d}^2\mathrm{f}(t)}{\mathrm{d}t^2}$	
$=\frac{(e^t+2)e^t-e^t(e^t)}{(e^t+2)^2}$	
$=\frac{2e^t}{(e^t+2)^2}$	1A
Since $\frac{\mathrm{d}^2\mathrm{f}(t)}{\mathrm{d}t^2}>0$, $\mathrm{f}(t)$ is concave upward for $0\le t\le 6$.	1M
Thus, the estimate in (a)(i) is an over-estimate.	1A f.t. ----------(7)
(b) (i) $\frac{1}{40-t^2}=\frac{1}{40}\left(1+\frac{t^2}{40}+\frac{t^4}{1600}+\cdots\right)=\frac{1}{40}+\frac{1}{1600}t^2+\frac{1}{64000}t^4+\cdots$	1A pp−1 for omitting '$\cdots$'
(ii) Note that $e^t=1+t+\frac{1}{2}t^2+\frac{1}{6}t^3+\frac{1}{24}t^4+\cdots$. Hence, we have	1M for any four terms correct
$\frac{8e^t}{40-t^2}$	
$=8\left(\frac{1}{40}+\frac{1}{1600}t^2+\frac{1}{64000}t^4+\cdots\right)\left(1+t+\frac{1}{2}t^2+\frac{1}{6}t^3+\frac{1}{24}t^4+\cdots\right)$	
$=\frac{1}{5}+\frac{1}{5}t+\frac{21}{200}t^2+\frac{23}{600}t^3+\frac{263}{24000}t^4+\cdots$	1A pp−1 for omitting '$\cdots$'
(iii) The total profit made by company B	
$=\int_0^6 \mathrm{g}(t)\,\mathrm{d}t$	
$\approx \int_0^6\left(\frac{1}{5}+\frac{1}{5}t+\frac{21}{200}t^2+\frac{23}{600}t^3+\frac{263}{24000}t^4\right)\mathrm{d}t$	1M
$=\left[\frac{1}{5}t+\frac{1}{10}t^2+\frac{7}{200}t^3+\frac{23}{2400}t^4+\frac{263}{120000}t^5\right]_0^6$	1A for correct integration
$=41.8224$ billion dollars	1A $a-1$ for r.t. 41.822 ----------(6)
(c) Since the estimate in (b)(iii) is an under-estimate, we have	1A
$\int_0^6 \mathrm{f}(t)\,\mathrm{d}t<37.4871<41.8224<\int_0^6 \mathrm{g}(t)\,\mathrm{d}t$.	
Thus, Mary's claim is correct.	1A f.t. ----------(2)

Solution	Marks
9. (a) $A(t)=(-t^2+5t+a)e^{kt}+7$	
Since $A(0)=3$, we have $a+7=3$.	
Thus, we have $a=-4$.	1A
The required amount of water stored	
$=(-1^2+5-4)e^k+7$	
$=7$ million cubic metres	1A
	----------(2)
(b) $A(t)=(-t^2+5t-4)e^{kt}+7$	
$\frac{dA(t)}{dt}$	
$=(-2t+5)e^{kt}+(-t^2+5t-4)(ke^{kt})$	1M for product rule
$=\left(-kt^2+(5k-2)t+5-4k\right)e^{kt}$	
Note that when $t=2$, $\frac{dA(t)}{dt}=0$.	
So, we have $2k+1=0$.	1M
Thus, we have $k=\frac{-1}{2}$.	1A
	----------(3)
(c) (i) When $A(t)\geq 7$, we have	
$-t^2+5t-4\geq 0$	1M accept setting quadratic equation
$t^2-5t+4\leq 0$	
$1\leq t\leq 4$	1A (accept $t=1\to t=4$)
Thus, the *adequate* period lasts for 3 months.	
(ii) Note that $A(t)=(-t^2+5t-4)e^{\frac{-t}{2}}+7$.	
So, $\frac{dA(t)}{dt}=\left(\frac{t^2}{2}-\frac{9t}{2}+7\right)e^{\frac{-t}{2}}$	
and $\frac{dA(t)}{dt}=0$ when $t=2$ (rejected since $t>4$) or $t=7$.	1A for t = 2 or 7
$\frac{dA(t)}{dt}\begin{cases}<0 & \text{if } 4<t<7\\ =0 & \text{if } t=7\\ >0 & \text{if } 7<t\leq 12\end{cases}$	1M for testing + 1A
So, $A(t)$ attains its least value when $t=7$.	
The least amount of water stored	
$=A(7)$	
≈ 6.456447098	
≈ 6.4564 million cubic metres	1A $a-1$ for r.t. 6.456 million cubic metres

Solution	Marks
Note that $A(t)=(-t^2+5t-4)e^{\frac{-t}{2}}+7$.	
So, $\frac{dA(t)}{dt}=\left(\frac{t^2}{2}-\frac{9t}{2}+7\right)e^{\frac{-t}{2}}$	
and $\frac{dA(t)}{dt}=0$ when $t=2$ (rejected since $t>4$) or $t=7$.	1A for t = 2 or 7
$\frac{d^2A(t)}{dt^2}$ $=\left(\frac{-t^2}{4}+\frac{9t}{4}-\frac{7}{2}\right)e^{\frac{-t}{2}}+\left(t-\frac{9}{2}\right)e^{\frac{-t}{2}}$	
$=\left(\frac{-t^2}{4}+\frac{13t}{4}-8\right)e^{\frac{-t}{2}}$	1A
Therefore, we have $\left.\frac{d^2A(t)}{dt^2}\right\|_{t=7}=\frac{5}{2}e^{\frac{-7}{2}}>0$.	1M for testing + 1A
Note that there is only one local minimum after the *adequate* period. So, $A(t)$ attains its least value when $t=7$.	
The least amount of water stored $=A(7)$ ≈ 6.456447098 ≈ 6.4564 million cubic metres	1A $a-1$ for r.t. 6.456 million cubic metres
	either one
(iii) $\frac{d^2A(t)}{dt^2}$ $=\left(\frac{-t^2}{4}+\frac{9t}{4}-\frac{7}{2}\right)e^{\frac{-t}{2}}+\left(t-\frac{9}{2}\right)e^{\frac{-t}{2}}$ $=\left(\frac{-t^2}{4}+\frac{13t}{4}-8\right)e^{\frac{-t}{2}}$	
(iv) Since $\frac{dA(t)}{dt}=\frac{1}{2}\left((t-\frac{9}{2})^2-\frac{25}{4}\right)e^{\frac{-t}{2}}>0$ for $10<t<12$,	1M for considering the sign
$A(t)$ increases, within that year, after the *adequate* period has ended for 6 months.	1A f.t. — either one
Since $\frac{d^2A(t)}{dt^2}=\frac{-1}{4}\left((t-\frac{13}{2})^2-\frac{41}{4}\right)e^{\frac{-t}{2}}<0$ for $10<t<12$,	
$\frac{dA(t)}{dt}$ decreases, within that year, after the *adequate* period has ended for 6 months.	1A f.t.
	----------(10)

Solution	Marks
10. (a) The required probability	
$= 1 - \left(\frac{2.4^0 e^{-2.4}}{0!} + \frac{2.4^1 e^{-2.4}}{1!} + \frac{2.4^2 e^{-2.4}}{2!} \right)$	1M for complemeantary events + 1M for Poisson probability
≈ 0.430291254	
≈ 0.4303	1A a–1 for r.t. 0.430 ----------(3)
(b) Let $\$X$ be the expense of a customer.	
Then, $X \sim \mathrm{N}(375, 125^2)$.	
The required probability	
$= \mathrm{P}(300 < X < 600)$	
$= \mathrm{P}(\frac{300-375}{125} < Z < \frac{600-375}{125})$	1M (accept $\mathrm{P}(\frac{300-375}{125} \le Z \le \frac{600-375}{125})$)
$= \mathrm{P}(-0.6 < Z < 1.8)$	
$= 0.2257 + 0.4641$	
$= 0.6898$	1A a–1 for r.t. 0.690 ----------(2)
(c) The required probability	
$= (0.25)(0.6898) + (0.8)(0.5 - 0.4641)$	1M for $0.25(\mathrm{b}) + 0.8p$, $0 < p < 0.5$
$= 0.20117$	
≈ 0.2012	1A a–1 for r.t. 0.201 ----------(2)
(d) The required probability	
$\approx \frac{2.4^3 e^{-2.4}}{3!}(0.20117)^3$	1M for $\frac{2.4^3 e^{-2.4}}{3!}(\mathrm{c})^3$
≈ 0.00170163	
≈ 0.0017	1A a–1 for r.t. 0.002 ----------(2)
(e) The required probability	
$\approx \frac{0.00170163 + (0.20117)^4 \left(\frac{2.4^4 e^{-2.4}}{4!} \right)}{0.430291254}$	1M for numerator using (c) and (d) + 1M for denominator using (a)
≈ 0.004431931	
≈ 0.0044	1A a–1 for r.t. 0.004 ----------(3)
(f) Suppose that the revised least expense is $\$x$.	
Then, we have $\mathrm{P}(X \ge x) = 0.33$.	1A
So, we have $\mathrm{P}(Z \ge \frac{x-375}{125}) = 0.33$.	
Therefore, we have $\frac{x-375}{125} = 0.44$.	1A
Hence, we have $x = 430$.	1A
Thus, the revised least expense is $\$430$.	----------(3)

Solution	Marks
11. Let X g be the net weight of a can of brand D coffee beans. Then, $X \sim \mathrm{N}(300, 7.5^2)$.	
(a) The required probability	
$= \mathrm{P}(X < 283.5 \text{ or } X > 316.5)$	
$= \mathrm{P}(Z < \frac{283.5-300}{7.5} \text{ or } Z > \frac{316.5-300}{7.5})$	1M (accept $\mathrm{P}(Z \le \frac{283.5-300}{7.5} \text{ or } Z \ge \frac{316.5-300}{7.5})$)
$= \mathrm{P}(Z < -2.2 \text{ or } Z > 2.2)$	
$= 2(0.0139)$	
$= 0.0278$	1A a−1 for r.t. 0.028
	----------(2)
(b) (i) The required probability	
$= (1-0.0278)^{11}(0.0278)$	1M for $(1-p)^{11}p$
≈ 0.020387152	
≈ 0.0204	1A a−1 for r.t. 0.020
(ii) The required probability	1M for $p = $ (a) (either one)
$= C_1^{30}(1-0.0278)^{29}(0.0278)$	1M for $C_1^{30}(1-p)^{29}p$
≈ 0.368195889	
≈ 0.3682	1A a−1 for r.t. 0.368
(iii) The required probability	
$\approx (1-0.0278)^{30} + 0.368195889$	1M for $\left((1-p)^{30}+q\right)$ + 1M for $q = $ (b)(ii)
≈ 0.797404575	
≈ 0.7974	1A a−1 for r.t. 0.797
	----------(8)
(c) (i) The required probability	
$\approx \frac{1}{2}(0.368195889)$	1M for $\frac{1}{2}$((b)(ii))
≈ 0.184097944	
≈ 0.1841	1A a−1 for r.t. 0.184
(ii) The required probability	
$\approx \frac{0.184097944}{0.797404575}$	1M for numerator using (c)(i) + 1M for denominator using (b)(iii)
≈ 0.230871443	
≈ 0.2309	1A a−1 for r.t. 0.231
	----------(5)

Solution	Marks
12. (a) The required probability	
$=\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)$	1M for geometric probability
$=\frac{5124}{15625}$	1A
$=0.327936$	
≈ 0.3279	$a-1$ for r.t. 0.328 ----------(2)
(b) The required probability	
$=\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)+\cdots$	1M must indicate infinite series and have at least 3 terms
$=\frac{\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)}{1-\left(\frac{4}{5}\right)^2}$	1M for summing geometric sequence
$=\frac{4}{9}$	1A
≈ 0.4444444444	
≈ 0.4444	$a-1$ for r.t. 0.444 ----------(3)
(c) The required probability	
$=\frac{\frac{4}{9}-\frac{5124}{15625}}{\frac{4}{9}}$	1M for numerator $=(b)-(a)$ + 1M for denominator using (b)
$=\frac{4096}{15625}$	1A
$=0.262144$	
≈ 0.2621	$a-1$ for r.t. 0.262
The required probability	
$=1-\frac{\frac{5124}{15625}}{\frac{4}{9}}$	1M for complementary probability + 1M for denominator using (b)
$=\frac{4096}{15625}$	1A
$=0.262144$	
≈ 0.2621	$a-1$ for r.t. 0.262
	----------(3)

Solution	Marks
(d) (i) The required probability	
$=(\frac{1}{2})(\frac{3}{7})+(\frac{1}{2})(1)$	1M for either case
$=\frac{5}{7}$	1A
≈ 0.714285714	
≈ 0.7143	$a-1$ for r.t. 0.714
The required probability	
$=1-(\frac{1}{2})(\frac{4}{7})$	1M for complementary probability
$=\frac{5}{7}$	1A
≈ 0.714285714	
≈ 0.7143	$a-1$ for r.t. 0.714
(ii) The required probability	
$=1-\frac{5}{7}$	1M for $1-$(d)(i)
$=\frac{2}{7}$	1A
≈ 0.285714286	
≈ 0.2857	$a-1$ for r.t. 0.286
The required probability	
$=(\frac{1}{2})(\frac{4}{7})(1)(1)$	1M for denominator $=(2)(7)$
$=\frac{2}{7}$	1A
≈ 0.285714286	
≈ 0.2857	$a-1$ for r.t. 0.286
(iii) The required probability	
$=\dfrac{(\frac{4}{9})(\frac{2}{7})}{(\frac{4}{9})(\frac{2}{7})+(1-\frac{4}{9})(1-\frac{2}{7})}$	1M for $\dfrac{pq}{pq+(1-p)(1-q)}$ + 1M for $\begin{cases} p=\text{(b)} \\ q=\text{(d)(ii)} \end{cases}$ or $\begin{cases} p=\text{(d)(ii)} \\ q=\text{(b)} \end{cases}$
$=\frac{8}{33}$	1A
≈ 0.2424242424	
≈ 0.2424	$a-1$ for r.t. 0.242
	----------(7)