AS Mathematics and Statistics

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Makers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for poor presentation (*pp*). The symbol *pp-1* should be used to denote 1 mark deducted for *pp* . At most deducted 1 mark from Section A and 1 mark from Section B for *pp* . In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

7. Marks may be deducted for numerical answers with inappropriate degree of accuracy (a). The symbol *a-1* should be used to denote 1 mark deducted for a . At most deducted 1 mark from Section A and 1 mark from Section B for a . In any case, do not deduct any marks for a in those steps where candidates could not score any marks.

8. Marks entered in the Page Total Box should be the NET total scored on that page.

9. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles .

Solution	Marks
1. (a) (i) $(1+ax)^{-3}=1-3ax+\frac{(-3)(-4)}{2!}(ax)^2+\frac{(-3)(-4)(-5)}{3!}(ax)^3+\cdots$	1M for any two terms correct
$=1-3ax+6a^2x^2-10a^3x^3+\cdots$	1A
(ii) $-10a^3=\frac{80}{27}$	
$a=\frac{-2}{3}$	1A accept $a\approx-0.6667$
(b) (i) $(3-2x)^{-3}=\frac{1}{27}\left(1-\frac{2}{3}x\right)^{-3}$	1A
$=\frac{1}{27}\left[1+3\left(\frac{2}{3}\right)x+6\left(\frac{2}{3}\right)^2x^2+\frac{80}{27}x^3+\cdots\right]$	
$=\frac{1}{27}\left(1+2x+\frac{8}{3}x^2+\frac{80}{27}x^3+\cdots\right)$	1A
$=\frac{1}{27}+\frac{2}{27}x+\frac{8}{81}x^2+\frac{80}{729}x^3+\cdots$	
(ii) The range of values of x is $\left\lvert\frac{-2}{3}x\right\rvert<1$	1M can be absorbed
i.e. $\frac{-3}{2}<x<\frac{3}{2}$	1A accept $\lvert x\rvert<\frac{3}{2}$
	----------(7)
2. (a) (i) Let $u=1+3te^{\frac{-t}{100}}$. Then,	
$\frac{du}{dt}=3e^{\frac{-t}{100}}-\frac{3t}{100}e^{\frac{-t}{100}}$	1M for Product Rule + 1A
$=\frac{3}{100}(100-t)e^{\frac{-t}{100}}$	
(ii) $\theta=\int\frac{\frac{12}{25}(100-t)e^{\frac{-t}{100}}}{(1+3te^{\frac{-t}{100}})}dt$	
$=\int\frac{16}{u}du$	1A
$=16\ln(1+3te^{\frac{-t}{100}})+C$	
When $t=0$, $\theta=16$, we have $C=16$	1M for finding C
$\therefore\quad\theta=16\ln(1+3te^{\frac{-t}{100}})+16$	1A

Solution	Marks
(b) $\dfrac{d\theta}{dt}=\dfrac{12(100-t)e^{\frac{-t}{100}}}{25(1+3te^{\frac{-t}{100}})}$	
$\begin{cases} >0 & \text{if} \quad 0\le t<100 \\ =0 & \text{if} \quad t=100 \\ <0 & \text{if} \quad t>100 \end{cases}$	1M for testing
$\therefore$ θ attains its greatest value when $t=100$	
Note that $\theta(100)=16\ln(1+300e^{-1})+16$ ≈ 91.40484176 ≤ 95	
Thus, the temperature of the surface of the vessel will not get higher than 95°C .	1A ----------(7)
3. (a) $\dfrac{dy}{dx}=\dfrac{4(4x^2+3x+9)^{\frac{1}{2}}-\frac{1}{2}(4x^2+3x+9)^{\frac{-1}{2}}(8x+3)(4x-9)}{4x^2+3x+9}$	1M for quotient rule + 1M for chain rule + 1A
$=\dfrac{8(4x^2+3x+9)-(8x+3)(4x-9)}{2(4x^2+3x+9)^{\frac{3}{2}}}$	
$=\dfrac{84x+99}{2(4x^2+3x+9)^{\frac{3}{2}}}$	
When $x=10$, $\dfrac{dy}{dx}=\dfrac{84(10)+99}{2(4(10)^2+3(10)+9)^{\frac{3}{2}}}$ ≈ 0.051043308 ≈ 0.0510	1A $a-1$ for r.t. 0.051
(b) The required amount	
$=\lim\limits_{x\to\infty}\left(3+\dfrac{4x-9}{\sqrt{4x^2+3x+9}}\right)$	
$=3+\lim\limits_{x\to\infty}\dfrac{4-\frac{9}{x}}{\sqrt{4+\frac{3}{x}+\frac{9}{x^2}}}$	1M can be absorbed
$=3+2$ $=5$ grams	1A ----------(6)

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Solution	Marks
4. (a) $P(A\cap B) = P(A)\,P(B\|A)$	
$P(A\cap B) = 0.4P(A)$	1M can be absorbed
$P(A\cap B) = 0.4a$	1A
	either one
$P(A\cap B) = P(B)\,P(A\|B) = 0.5P(B)$	
$0.4a = 0.5P(B)$	
$P(B) = \dfrac{0.4}{0.5}a$	
$P(B) = 0.8a$	1A
$P(A\cap B) = P(A)\,P(B\|A)$	
$P(A\cap B) = 0.4P(A)$	1M can be absorbed
$P(A\cap B) = 0.4a$	1A
$P(A\cup B) = P(A) + P(B) - P(A\cap B)$	
$0.84 = a + P(B) - 0.4a$	1M can be absorbed
$P(B) = 0.84 - 0.6a$	1A
$P(A\cap B) = P(B)\,P(A\|B)$	
$P(B) = 2\,P(A\cap B)$	1M can be absorbed
$P(A\cup B) = P(A) + P(B) - P(A\cap B)$	
$0.84 = a + 2\,P(A\cap B) - P(A\cap B)$	1M can be absorbed
$P(A\cap B) = 0.84 - a$	1A accept $P(A\cap B) = 1.8a - 0.84$
$P(B) = 1.68 - 2a$	1A accept $P(B) = 0.8a$
(b) $P(A\cup B) = P(A) + P(B) - P(A\cap B)$	
$0.84 = a + 0.8a - 0.4a$ (by (a))	1M can be absorbed
$0.84 = 1.4a$	
$a = 0.6$	1A
$P(A\cap B) = P(B)\,P(A\|B) = 0.5P(B)$	
$0.4a = 0.5P(B)$	
$P(B) = \dfrac{0.4}{0.5}a$	
$P(B) = 0.8a$	
$0.8a = 0.84 - 0.6a$ (by (a))	
$1.4a = 0.84$	
$a = 0.6$	1A
$P(A\cap B) = P(A)\,P(B\|A)$	
$P(A\cap B) = 0.4\,P(A)$	
$0.84 - a = 0.4a$	
$a = 0.6$	1A
(c) $\because$ $P(A)\,P(B) = (0.6)(0.8)(0.6) = 0.288 \neq 0.24 = (0.4)(0.6) = P(A\cap B)$	1M
$\therefore$ A and B are not independent.	1A
$\because$ $P(A\|B) = 0.5 \neq 0.6 = P(A)$	1M
$\therefore$ A and B are not independent.	1A
	----------(7)

Solution	Marks
5. (a)	
<table><tr><th>Stem (tens)</th><th>Leaf (units)</th></tr><tr><td>1</td><td>2 3 4 5 5</td></tr><tr><td>2</td><td>1 5 9</td></tr><tr><td>3</td><td>6 7 8</td></tr><tr><td>4</td><td>1 7 9 9 9</td></tr><tr><td>5</td><td>2 4 7</td></tr></table>	1M+1A
(b) (i) Revised median	
= 49 minutes	1A do not accept 48 minutes
Revised Q_1	
= 25 minutes	accept 24 , 26 or 27
Revised Q_3	
= 60 minutes	
Revised interquartile range	
= 60 − 25	
= 35 minutes	1A accept 36 , 34 or 33
(ii) The revised mean is larger than the original mean.	1A
The revised range is larger than the original range.	1A
$\boxed{\text{The change in the revised mean is positive.}}$	1A
$\boxed{\text{The change in the revised range is positive.}}$	1A
	----------(6)
6. Let \$$X$ be the amount of a business transaction. Then, $X \sim N(215, 50^2)$.	
(a) The required probability	
$= \mathrm{P}(X > 300)$	
$= \mathrm{P}\left(Z > \frac{300-215}{50}\right)$	1A accept $\mathrm{P}\left(Z \geq \frac{300-215}{50}\right)$
$= \mathrm{P}(Z > 1.7)$	
$= 0.0446$	1A $a-1$ for r.t. 0.045
(b) The required probability	
$= C_2^7 (0.0446)^2 (1-0.0446)^5$	1M for Binomial probability
≈ 0.033251802	
≈ 0.0333	1A $a-1$ for r.t. 0.033
(c) The required probability	
$\approx (0.033251802)(0.0446)$ (by (b))	
≈ 0.001483030369	
≈ 0.0015	1A $a-1$ for r.t. 0.001
(d) $\mathrm{P}(X > K) = 64.8\%$	
$\mathrm{P}\left(Z > \frac{K-215}{50}\right) = 0.648$	
$\frac{K-215}{50} = -0.38$	1M accept $\frac{K-215}{50} = 0.38$
$K = 196$	1A
	----------(7)

Solution	Marks

7. (a) $\because \lim_{x\to\frac{7}{2}^-}\frac{20-4x}{7-2x}=+\infty$ and $\lim_{x\to\frac{7}{2}^+}\frac{20-4x}{7-2x}=-\infty$

$\therefore$ the equation of the vertical asymptote to C_1 is $x=\frac{7}{2}$. — 1A

$\because \lim_{x\to\pm\infty}\frac{20-4x}{7-2x}=\lim_{x\to\pm\infty}\frac{\frac{20}{x}-4}{\frac{7}{x}-2}=2$

$\therefore$ the equation of the horizontal asymptote to C_1 is $y=2$. — 1A

----------(2)

(b) $\because$ $g(-3)=0$

$\therefore$ $a-3b=0$

$\because$ $g(0)=4$

$\therefore$ $\frac{a}{3}=4$

So, $a=12$ and $b=4$ — 1A for both correct

Also, $\lim_{x\to\pm\infty}\frac{12+4x}{3+cx}=2$ — 1M can be absorbed

$\therefore \lim_{x\to\pm\infty}\frac{\frac{12}{x}+4}{\frac{3}{x}+c}=2$

Thus, $c=2$ — 1A

Therefore, $a=12$, $b=4$ and $c=2$.

----------(3)

(c) $C_1: y=f(x)$, where $f(x)=\frac{20-4x}{7-2x}$

$C_2: y=g(x)$, where $g(x)=\frac{12+4x}{3+2x}$

When $x=0$, $f(x)=\frac{20}{7}$

When $f(x)=0$, $x=5$

So, the x-intercept of C_1 is 5 and the y-intercept of C_1 is $\frac{20}{7}$.

$\because \lim_{x\to\frac{-3}{2}^-}g(x)=\lim_{x\to\frac{-3}{2}^-}\frac{12+4x}{3+2x}=-\infty$ and $\lim_{x\to\frac{-3}{2}^+}g(x)=\lim_{x\to\frac{-3}{2}^+}\frac{12+4x}{3+2x}=+\infty$

$\therefore$ the equation of the vertical asymptote to C_2 is $x=\frac{-3}{2}$.

Also, $f(x)=g(x)$

$$\frac{20-4x}{7-2x}=\frac{12+4x}{3+2x}$$

$$\frac{5-x}{7-2x}=\frac{3+x}{3+2x}$$

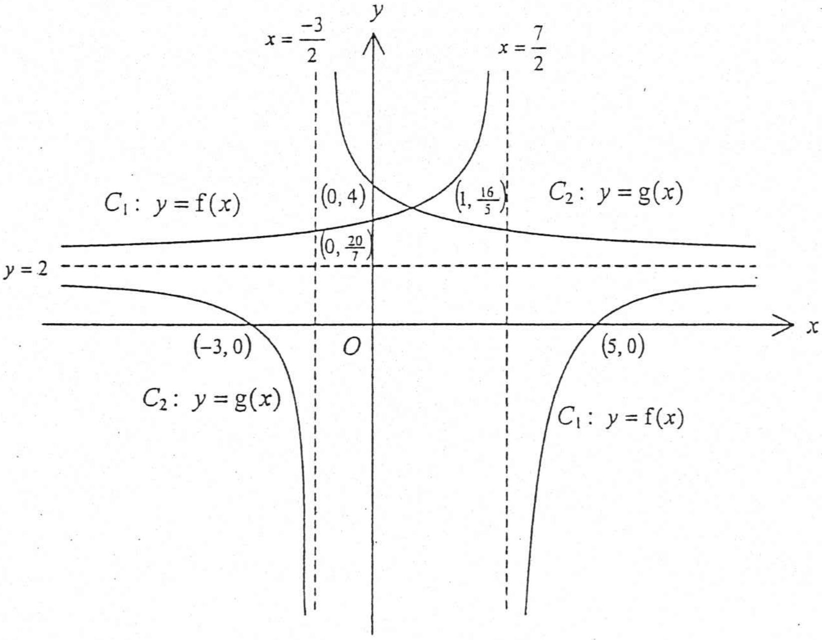
$$(x-5)(2x+3)=(x+3)(2x-7)$$

$$-7x-15=-x-21$$

$$-6x=-6$$

$$x=1$$

Thus, the point of intersection is $(1,\frac{16}{5})$.

Solution	Marks
	1A all asymptotes 1A shape of C_1 1A shape of C_2 1A intercepts of C_1 1A intersection point ----------(5)
(d) $\int_1^{\lambda}(\mathrm{f}(x)-\mathrm{g}(x))\,\mathrm{d}x = 3\ln 3$	1A accept $\int_{\lambda}^{1}(\mathrm{g}(x)-\mathrm{f}(x))\,\mathrm{d}x = 3\ln 3$
$\int_1^{\lambda}\left(\frac{20-4x}{7-2x}-\frac{12+4x}{3+2x}\right)\mathrm{d}x = 3\ln 3$	
$\int_1^{\lambda}\left((2+\frac{6}{7-2x})-(2+\frac{6}{3+2x})\right)\mathrm{d}x = 3\ln 3$	1M for either one correct
$6\int_1^{\lambda}\left(\frac{1}{7-2x}-\frac{1}{3+2x}\right)\mathrm{d}x = 3\ln 3$	
$\frac{-6}{2}\left[\ln(7-2x)+\ln(3+2x)\right]_1^{\lambda} = 3\ln 3$	1A for integration
$\left[\ln((7-2x)(3+2x))\right]_1^{\lambda} = -\ln 3$	
$\ln[(7-2\lambda)(3+2\lambda)]-\ln 25 = -\ln 3$	
$(7-2\lambda)(3+2\lambda) = \frac{25}{3}$	1M
$3(2\lambda+3)(2\lambda-7)+25=0$	
$3(4\lambda^2-8\lambda-21)+25=0$	
$12\lambda^2-24\lambda-38=0$	
$6\lambda^2-12\lambda-19=0$	
$\lambda = \frac{12\pm\sqrt{600}}{12}$	
$\lambda = 1\pm\frac{5}{6}\sqrt{6}$	
$\lambda = 1+\frac{5}{6}\sqrt{6}$ $\left(\because\ 0<\lambda<\frac{7}{2}\right)$	1A
	----------(5)

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Solution	Marks
8. (a) $f(t)=5+2^{-kt+h}$	
$\ln(f(t)-5)=-(k\ln 2)t+h\ln 2$	1A do not accept $-k\ln 2t+h\ln 2$ ----------(1)
(b) $-k\ln 2=-0.35$	
$k\approx 0.504943264$	
$k\approx 0.5$ (correct to 1 decimal place)	1A
$h\ln 2=1.39$	
$h\approx 2.005346107$	
$h\approx 2.0$ (correct to 1 decimal place)	1A ----------(2)
(c) The total amount	
$=\int_2^{12} g(t)\mathrm{d}t$	
$\approx \frac{12-2}{10}\left(g(2)+g(12)+2(g(4)+g(6)+g(8)+g(10))\right)$	1M for trapezoidal rule
≈ 75.77699747	
≈ 75.7770 thousand barrels	1A ----------(2)
(d) (i) $2^t=e^{at}$ for all $t\geq 0$	
$t\ln 2=at$ for all $t\geq 0$	
$a=\ln 2$	1A accept $a\approx 0.6931$
(ii) $g(t)=5+\ln(t+1)+2^{\frac{-t}{2}+2}$	
$=5+\ln(t+1)+4e^{\left(\frac{-\ln 2}{2}\right)t}$	
$\frac{\mathrm{d}g(t)}{\mathrm{d}t}=\frac{1}{t+1}+(4)\left(\frac{-\ln 2}{2}\right)e^{\left(\frac{-\ln 2}{2}\right)t}$	1A for the first term + 1M for Chain Rule
$=\frac{1}{t+1}-2(\ln 2)e^{\left(\frac{-\ln 2}{2}\right)t}$	1M for the second term
$\frac{\mathrm{d}^2g(t)}{\mathrm{d}t^2}=\frac{-1}{(t+1)^2}-(2\ln 2)\left(\frac{-\ln 2}{2}\right)e^{\left(\frac{-\ln 2}{2}\right)t}$	
$=(\ln 2)^2e^{\left(\frac{-\ln 2}{2}\right)t}-\frac{1}{(t+1)^2}$	1A for all being correct
$=(\ln 2)^2 2^{\frac{-t}{2}}-\frac{1}{(t+1)^2}$	
$\therefore\ p(t)=(\ln 2)^2 2^{\frac{-t}{2}}$	1A accept $p(t)=(\ln 2)^2e^{\left(\frac{-\ln 2}{2}\right)t}$

Solution	Marks
(iii) $\because$ $p(2) = (\ln 2)^2 2^{-1} \approx 0.240226506 \approx 0.2402$	1M for testing
$q(2) = \frac{1}{9} \approx 0.111111111 \approx 0.1111$	
$\therefore$ $p(2) > q(2)$	1A
It is known that $y = p(t)$ and $y = q(t)$ have no intersection, where $2 \le t \le 12$.	
So, we have $p(t) > q(t)$ for all $2 \le t \le 12$.	
$\therefore$ $\frac{d^2 g(t)}{dt^2} > 0$ on $[2, 12]$	1M
Thus, the estimate is an over-estimate of I .	1M
	----------(10)

Solution	Marks
9. (a) $\because$ $\mathrm{P}(0)=5.9$	
$\therefore$ $a+\frac{1}{5}(0-0-8)=5.9$	
So, $a=7.5$	1A
$\mathrm{P}(t)=7.5+\frac{1}{5}(t^2-8t-8)e^{-kt}$	
$\because$ $\mathrm{P}(8)-\mathrm{P}(4)=1.83$	
$\therefore$ $-1.6e^{-8k}+4.8e^{-4k}=1.83$	1M+1A
$160(e^{-4k})^2-480e^{-4k}+183=0$	1M can be absorbed
$e^{-4k}\approx 2.551784198$ or $e^{-4k}\approx 0.448215801$	
$k\approx -0.2341982$ or $k\approx 0.200620116$	
$\because$ $k>0$	
$\therefore$ $k\approx 0.2$ (correct to 1 decimal place)	1A
	----------(5)
(b) $\mathrm{P}(t)=\frac{15}{2}+\frac{1}{5}(t^2-8t-8)e^{-0.2t}$	
(i) $\frac{\mathrm{dP}(t)}{\mathrm{d}t}=\frac{-1}{25}\left[(t^2-8t-8)-5(2t-8)\right]e^{-0.2t}$	1M for Product Rule or Chain Rule
$=\frac{-1}{25}(t^2-18t+32)e^{-0.2t}$	1A independent of the obtained value of a
$=\frac{-1}{25}(t-2)(t-16)e^{-0.2t}$	
For $\frac{\mathrm{dP}(t)}{\mathrm{d}t}=0$, we have $t=2$ or $t=16$.	
$\frac{\mathrm{dP}(t)}{\mathrm{d}t}\begin{cases}<0 & \text{if } 0\le t<2\\ =0 & \text{if } t=2\\ >0 & \text{if } 2<t<16\end{cases}$	
So, the minimum pH value occurred at $t=2$.	1M+1A
$\frac{\mathrm{dP}(t)}{\mathrm{d}t}\begin{cases}>0 & \text{if } 2<t<16\\ =0 & \text{if } t=16\\ <0 & \text{if } t>16\end{cases}$	
So, the maximum pH value occurred at $t=16$.	1M+1A accept max at $t=0$ and at $t=16$

Solution	Marks
$\dfrac{dP(t)}{dt} = \dfrac{-1}{25}\left[(t^2-8t-8)-5(2t-8)\right]e^{-0.2t}$	1M for Product Rule or Chain Rule
$= \dfrac{-1}{25}(t^2-18t+32)\,e^{-0.2t}$	1A independent of the obtained value of a
$= \dfrac{-1}{25}(t-2)(t-16)\,e^{-0.2t}$	
For $\dfrac{dP(t)}{dt}=0$, we have $t=2$ or $t=16$.	
$\dfrac{d^2P(t)}{dt^2} = \dfrac{1}{125}[t^2-18t+32-5(2t-18)]\,e^{-0.2t}$	
$= \dfrac{1}{125}(t^2-28t+122)\,e^{-0.2t}$	
$\left.\dfrac{d^2P(t)}{dt^2}\right\vert_{t=2} \approx 0.375379225 > 0$	
So, the minimum pH value occurred at $t=2$.	1M+1A
$\left.\dfrac{d^2P(t)}{dt^2}\right\vert_{t=16} \approx -0.022826834 < 0$	
So, the maximum pH value occurred at $t=16$.	1M+1A accept max at $t=0$ and at $t=16$
(ii) $\dfrac{d^2P}{dt^2} = \dfrac{1}{125}[t^2-18t+32-5(2t-18)]\,e^{-0.2t}$	
$= \dfrac{1}{125}(t^2-28t+122)\,e^{-0.2t}$	1A
$\because \dfrac{d^2P}{dt^2} = \dfrac{1}{125}\left(t-(14-\sqrt{74})\right)\left(t-(14+\sqrt{74})\right)e^{-0.2t}$	
$5 < 14-\sqrt{74} < 6$ and $22 < 14+\sqrt{74} < 23$	
$\therefore \dfrac{d^2P}{dt^2} > 0$ for all $t \ge 23$.	1
	----------(8)
(c) The required pH value	
$= \lim\limits_{t\to\infty}\left(7.5+\dfrac{1}{5}(t^2-8t-8)\,e^{-0.2t}\right)$	
$= 7.5+\dfrac{1}{5}\lim\limits_{t\to\infty}\left(t^2e^{-0.2t}\right)-\dfrac{8}{5}\lim\limits_{t\to\infty}\left(te^{-0.2t}\right)-\dfrac{8}{5}\lim\limits_{t\to\infty}e^{-0.2t}$	
$= 7.5+\dfrac{1}{5}(0)-\dfrac{8}{5}(0)-\dfrac{8}{5}(0)$ $\left(\because \lim\limits_{t\to\infty}(te^{-0.2t}) = \left(\lim\limits_{t\to\infty}\dfrac{1}{t}\right)\left(\lim\limits_{t\to\infty}t^2e^{-0.2t}\right) = (0)(0) = 0\right)$	1A for $\lim\limits_{t\to\infty}(te^{-0.2t}) = 0$ (can be absorbed)
$= 7.5$	1M accept the required pH value $= a$
	----------(2)

Solution	Marks
10. (a) Sample mean	
$= \dfrac{12(1)+14(2)+10(3)+6(4)+2(5)+1(6)}{5+12+14+10+6+2+1}$	
$= 2.2$	1A
Sample Standard deviation	
$= \sqrt{\dfrac{12(1^2)+14(2^2)+10(3^2)+6(4^2)+2(5^2)+1(6)^2-(50)(2.2)^2}{5+12+14+10+6+2+1-1}}$	
$= \sqrt{2}$	1A
≈ 1.414213562	
≈ 1.4142	$a-1$ for r.t. 1.414
	----------(2)
(b) (i) The required probability	
$= \dfrac{2.2^0 e^{-2.2}}{0!}+\dfrac{2.2^1 e^{-2.2}}{1!}+\dfrac{2.2^2 e^{-2.2}}{2!}+\dfrac{2.2^3 e^{-2.2}}{3!}$	1M for the 4 cases + 1M for Poisson probability
≈ 0.819352421	
≈ 0.8194	1A (accept 0.8193) $a-1$ for r.t. 0.819
(ii) The required probability	
$\approx C_2^6(0.819352421)^2(1-0.819352421)^4$	1M for Binomial probability + 1M for using (b)(i)
≈ 0.010724111	
≈ 0.0107	1A $a-1$ for r.t. 0.011
	----------(6)
(c) (i) The required probability	
$= C_2^3(0.55)^2(0.45)$	1M for $(0.55)^2(0.45)$
$= 0.408375$	1A
≈ 0.4084	$a-1$ for r.t. 0.408
(ii) The required probability	
$= \dfrac{2.2^2 e^{-2.2}}{2}(0.55^2)$	1M
≈ 0.081113452	
≈ 0.0811	1A $a-1$ for r.t. 0.081
(iii) The required probability	
$\approx \dfrac{\left(\dfrac{2.2^2 e^{-2.2}}{2!}\right)(0.55)^2+\left(\dfrac{2.2^3 e^{-2.2}}{3!}\right)(0.55)^3}{0.819352421}$	1M for numerator + 1M for denominator using (b)(i)
≈ 0.138925825	
≈ 0.1389	1A $a-1$ for r.t. 0.139
	----------(7)

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Solution	Marks
11. (a) The required probability	
$= (1)\left(\frac{1}{n}\right)$	
$= \frac{1}{n}$	1A
The required probability	
$= (n)\left(\frac{1}{n}\right)\left(\frac{1}{n}\right)$	
$= \frac{1}{n}$	1A
	----------(1)
(b) (i) The required probability	
$= p$	1A
(ii) $p + p + \frac{1}{n} = 1$	1M
$p = \frac{1}{2}(1 - \frac{1}{n})$	1A
(iii) $p \ge 0.46$	1A can be absorbed
$\frac{1}{2}(1 - \frac{1}{n}) \ge 0.46$	1M
$n \ge 12.5$	
$\because$ n is a positive integer.	
$\therefore$ the least value of n is 13 .	1A
	----------(6)
(c) (i) The required probability	
$= \left(\frac{5}{6}\right)^4 \frac{1}{6}$	1M for $\left(\frac{5}{6}\right)^k \frac{1}{6}$
$= \frac{625}{7776}$	1A
≈ 0.080375514	
≈ 0.0804	a–1 for r.t. 0.080
(ii) The required probability	
$= \frac{1}{6} + \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^6\left(\frac{1}{6}\right) + \cdots$	1A must indicate infinite series and have at least 3 terms
$= \dfrac{\frac{1}{6}}{1 - \frac{25}{36}}$	1M for sum of GP
$= \frac{6}{11}$	1A
≈ 0.545454545	
≈ 0.5455	a–1 for r.t. 0.545

Solution	Marks
(iii) The required probability	
$= \dfrac{\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^9\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^{11}\left(\frac{1}{6}\right)+\cdots}{1-\frac{6}{11}}$	1M for denominator using 1−(c)(ii) + 1A for numerator
$= \dfrac{\dfrac{\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^5}{1-\frac{25}{36}}}{\frac{5}{11}}$	
$= \dfrac{625}{1296}$	1A
≈ 0.482253086	
≈ 0.4823	a−1 for r.t. 0.482
The required probability	
$= \dfrac{\frac{5}{11}-\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)-\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)}{1-\frac{6}{11}}$	1M for denominator using 1−(c)(ii) + 1A for numerator
$= \dfrac{\frac{5}{11}-\frac{5}{36}-\frac{125}{1296}}{\frac{5}{11}}$	
$= \dfrac{625}{1296}$	1A
≈ 0.482253086	
≈ 0.4823	a−1 for r.t. 0.482
	----------(8)

Solution	Marks
12. (a) The required probability	
$= 1 - \frac{C_7^{17} + C_7^{13}}{C_7^{30}}$	1M for counting cases + 1A for correctness of probability
$= \frac{38743}{39150}$	1A
≈ 0.989604086	
≈ 0.9896	$a-1$ for r.t. 0.990
	----------(3)
(b) The required probability	
$= \frac{\frac{C_4^{17}C_3^{13} + C_5^{17}C_2^{13} + C_6^{17}C_1^{13}}{C_7^{30}}}{\frac{38743}{39150}}$	1M for denominator using (a) + 1A for numerator
$= \frac{1498}{2279}$	1A
≈ 0.657305835	
≈ 0.6573	$a-1$ for r.t. 0.657
The required probability	
$= \frac{C_4^{17}C_3^{13} + C_5^{17}C_2^{13} + C_6^{17}C_1^{13}}{C_7^{30} - C_7^{17} - C_7^{13}}$	1M for denominator using (a) + 1A for numerator
$= \frac{1498}{2279}$	1A
≈ 0.657305835	
≈ 0.6573	$a-1$ for r.t. 0.657
	----------(3)
(c) Let $\$X$ be the amount of money collected by a boy and $\$Y$ be the amount of money collected by a girl. Then, $X \sim N(673, 100^2)$ and $Y \sim N(708, 100^2)$.	
(i) The required probability	
$= \mathrm{P}(X > 800)$	
$= \mathrm{P}\left(Z > \frac{800-673}{100}\right)$	1M accept $Z \geq \frac{800-708}{100}$
$= \mathrm{P}(Z > 1.27)$	
$= 0.102$	1A accept 0.1020

Solution	Marks
(ii) $\mathrm{P}(Y > 800)$	
$= \mathrm{P}\left(Z > \dfrac{800-708}{100}\right)$	
$= \mathrm{P}(Z > 0.92)$	
$= 0.1788$	1A
The required probability	
$= \left(C_1^3(0.102)(0.898)^2\right)\left(C_1^4(0.1788)(0.8212)^3\right)$	1M for Binomial probability + 1M for Binomial × Binomial
≈ 0.097734619	
≈ 0.0977	1A $a-1$ for r.t. 0.098
(iii) The required probability	
$\approx \dfrac{0.097734619}{0.097734619 + C_2^3(0.102)^2(0.898)(0.8212)^4 + (0.898)^3 C_2^4(0.1788)^2(0.8212)^2}$	1M for numerator +1M for denominator
≈ 0.478730045	
≈ 0.4787	1A (accept 0.4786) $a-1$ for r.t. 0.479
	----------(9)

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