AS Mathematics and Statistics

General Marking Instructions

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits ***all the marks*** allocated to that part, unless a particular method has been specified in the question. Makers should be patient in marking alternative solutions not specified in the marking scheme.

2. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).

3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates' work. In general, marks for a certain step should be awarded if candidates' solution indicated that the relevant concept/technique had been used.

4. Use of notation different from those in the marking scheme should not be penalized.

5. In marking candidates' work, the benefit of doubt should be given in the candidates' favour.

6. Marks may be deducted for poor presentation (*pp*). The symbol (*pp-1*) should be used to denote 1 mark deducted for *pp* . At most deducted 1 mark from Section A and 1 mark from Section B for *pp* . In any case, do not deduct any marks for *pp* in those steps where candidates could not score any marks.

7. Marks may be deducted for numerical answers with inappropriate degree of accuracy (*a*). The symbol (*a-1*) should be used to denote 1 mark deducted for *a* . At most deducted 1 mark from Section A and 1 mark from Section B for *a* . In any case, do not deduct any marks for *a* in those steps where candidates could not score any marks.

8. Marks entered in the Page Total Box should be the NET total scored on that page.

9. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are **shaded** whereas alternative answers are enclosed with $\boxed{\text{rectangles}}$.

Solution	Marks
1. $\frac{dx}{dt}=\frac{10}{t^3}-6e^{-3t}$ $\frac{dy}{dt}=-\frac{20}{t^3}+2e^{2t}$	1M+1A (1M for $(e^{at})'=ae^{at}$)
$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}=\left(\frac{dy}{dt}\right)\left(\frac{1}{\frac{dx}{dt}}\right)=\frac{-\frac{20}{t^3}+2e^{2t}}{\frac{10}{t^3}-6e^{-3t}}$	1M for Chain Rule and Inverse Function Rule
For $\frac{dy}{dx}=-2$	
$\frac{-\frac{20}{t^3}+2e^{2t}}{\frac{10}{t^3}-6e^{-3t}}=-2$	1M
$e^{5t}=6$	
$t=\frac{1}{5}\ln 6(\approx 0.3584)$	1A $a-1$ for r.t. 0.358
	----------(5)
2. (a) At $t=2$, $r(2)=5.5035$(m) $\frac{dV}{dr}=4\pi r^2$	
$\frac{dr}{dt}=\frac{18\times 2e^{-t}}{(3+2e^{-t})^2}=\frac{36e^{-t}}{(3+2e^{-t})^2}$	1A
At $t=2$, $\frac{dV}{dr}=380.6109$ $\frac{dr}{dt}=0.45545$ $\therefore \frac{dV}{dt}=\frac{dV}{dr}\frac{dr}{dt}$	
At $t=2$, $\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$	1M
$=380.6109\times 0.45545$ $=173.35$ (m^3/h)	1A (Accept : 173.31−173.39) $a-1$ for more than 2 d.p.
$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$	1M
$\frac{dr}{dt}=\frac{36e^{-t}}{(3+2e^{-t})^2}$	1A
$\frac{dV}{dt}=4\pi r^2\cdot\frac{36e^{-t}}{(3+2e^{-t})^2}$ $=\frac{144\pi r^2e^{-t}}{(3+2e^{-t})^2}$	(Accept : $\frac{dV}{dt}=\frac{46656\pi e^{-t}}{(3+2e^{-t})^4}$)
At $t=2$, $r\approx 5.50346$ $\therefore \frac{dV}{dt}\approx 173.35$ (m^3 / h)	1A (Accept : 173.31−173.39) $a-1$ for more than 2 d.p.

Solution

Marks

(b) $\lim_{t\to\infty} r(t) = \lim_{t\to\infty} \frac{18}{3+2e^{-t}} = 6$ (m)

1M

$\therefore$ the volume of the balloon will be

$$V = \frac{4}{3}\pi(6)^3 + 5\pi$$

$$= 293\,\pi$$

$$= 920.49\ (\text{m}^3)$$

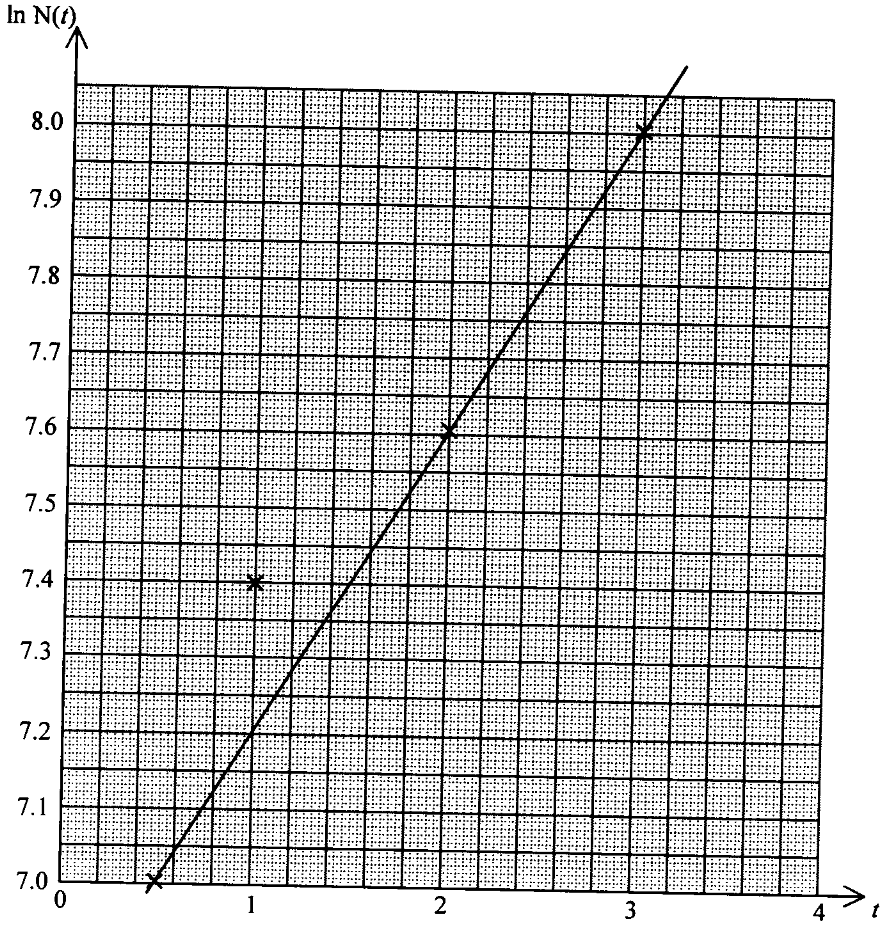
1A a−1 for more than 2 d.p.
----------(5)

3. $N(t) = 900\,a^{kt}$

$\ln N(t) = (k \ln a)t + \ln 900$

1A

t	0.5	1.0	2.0	3.0
$N(t)$	1100	1630	2010	2980
$\ln N(t)$	7.0031	7.3963	7.6059	7.9997



1M

At $t = 1.0$, $N(t) = 1630$ is incorrect,

1A

$\ln N(2.5) \approx 7.8$

$\therefore$ $N(2.5) \approx 2440$

1A a−1 for more than 3 s.f.
(Accept : $N(2.5) \in [2420, 2470]$)
----------(4)

Solution	Marks
4. (a) $S=\int_0^{10}\frac{8100t}{(3t+10)^3}\mathrm{d}t$.	1A
Let $u=3t+10$. $\mathrm{d}u=3\mathrm{d}t$. When $t=0$, $u=10$. When $t=10$, $u=40$. $S=\int_{10}^{40}\frac{8100\left(\frac{u-10}{3}\right)\cdot\frac{1}{3}\mathrm{d}u}{u^3}$	
$=900\int_{10}^{40}(\frac{1}{u^2}-\frac{10}{u^3})\mathrm{d}u$	1M change of variable
$=900\left[-\frac{1}{u}+\frac{5}{u^2}\right]_{10}^{40}$	
$=\frac{405}{16}=25.3125$	1A
The percentage of smoke removed is 25.3125% .	
$S=\int_0^{10}\frac{8100t}{(3t+10)^3}\mathrm{d}t$	1A
$=900\int_0^{10}\left[\frac{1}{(3t+10)^2}-\frac{10}{(3t+10)^3}\right]\mathrm{d}(3t+10)$	1M change of variable
$=900\left[-\frac{1}{3t+10}+\frac{5}{(3t+10)^2}\right]_0^{10}$	
$=25.3125$	1A
$S=\int\frac{8100t}{(3t+10)^3}\mathrm{d}t=900\left[-\frac{1}{3t+10}+\frac{5}{(3t+10)^2}\right]+C$	1M+1M for change of variable
When $t=0$, $S=0$. Hence, we have $C=45$. So, $S=900\left[-\frac{1}{3T+10}+\frac{5}{(3T+10)^2}\right]+45$.	
When $t=10$, $S=25.3125$.	1A
(b) $S=\int_0^{T}\frac{8100t}{(3t+10)^3}\mathrm{d}t$	
$=900\left[-\frac{1}{u}+\frac{5}{u^2}\right]_{10}^{3T+10}$	
$=900\left[-\frac{1}{3T+10}+\frac{5}{(3T+10)^2}+0.05\right]$	
$\lim_{T\to\infty}S=\lim_{T\to\infty}900\left[-\frac{1}{3T+10}+\frac{5}{(3T+10)^2}+0.05\right]$	1M taking limit and in terms of T
$=45$	1A
∴ 45% of smoke will be removed.	
	----------(5)

Solution	Marks
5. (a) The required probability	
$= \frac{6}{8}\times\frac{1}{3}+\frac{4}{7}\times\frac{1}{3}+\frac{2}{7}\times\frac{1}{3}$	1A
$= \frac{15}{28}$ (≈ 0.5357)	1A *a*−1 for r.t. 0.536
(b) P(the boy is selected from group A $\mid$ a boy is selected)	
$= \frac{\frac{6}{8}\times\frac{1}{3}}{\frac{15}{28}}$	1M + 1A (1A for numerator)
$= \frac{7}{15}$ (≈ 0.4667)	1A *a*−1 for r.t. 0.467
	----------(5)
6. Let N be the number of passengers arriving the bus stop in an hour and M be the number of male passengers.	
(a) $P(N=4)=\frac{5^4}{4!}e^{-5}$	1A
$\approx 0.17547 \approx 0.1755$	1A *a*−1 for r.t. 0.175
(b) $P(M=2$ and $N=4)$	
$= C_2^4(0.65)^2(1-0.65)^2\cdot 0.17547$	1M for binomial distribution
≈ 0.0545	1M for multiplication rule 1A *a*−1 for r.t. 0.055
	----------(5)
7. (a) Mean $= 61$	1A
(b) After deleting two marks, there are two modes. One deleted mark must be 54.	1A
Let the other deleted mark be x. $54+x = 22\times 61-20\times(61+1.2)\ (=98)$	
$x = 44$	1A
(c) There are 5 students with marks more that 75. The required probability is	
$\frac{C_2^5}{C_2^{20}}=\frac{1}{19}(\approx 0.0526)$	1M for numerator
	1A *a*−1 for r.t. 0.053
There are 5 students with marks more that 75. The required probability is	
$\frac{5}{20}\times\frac{4}{19}=\frac{1}{19}(\approx 0.0526)$	1M for multiplication rule
	1A *a*−1 for r.t. 0.053
	-------------(5)

Solution	Marks
8. (a) The sample space is $\{1R2W, 1R1W1Y, 1R2Y, 2R1W, 2R1Y, 1W2Y, 2W1Y, 3W, 3Y\}$	1A withhold this mark if not given in set notation
(b) The probability is $\frac{C_1^2 \cdot C_2^{11}}{C_3^{13}} = \frac{5}{13} \approx 0.3846$	1M+1A $a-1$ for r.t. 0.385
The probability is $C_1^3\left(\frac{2}{13}\right)\left(\frac{11}{12}\right)\left(\frac{10}{11}\right) = \frac{5}{13} \approx 0.3846$	1M+1A $a-1$ for r.t. 0.385
(c) P(one of the others is white | one is red) $= \frac{\text{P(one is red and one is white)}}{\text{P(one is red)}}$	1M for conditional probability
$= \frac{\frac{C_1^2 \cdot C_1^5 \cdot C_1^6}{C_3^{13}}}{\frac{C_1^2 \cdot C_2^{11}}{C_3^{13}}}$	1M for numerator
$= \frac{6}{11} \approx 0.5455$	1A $a-1$ for r.t. 0.545
P(one of the others is white | one is red) $= C_1^2\left(\frac{5}{11}\right)\left(\frac{6}{10}\right)$	1M for C_1^2 + 1M for $\left(\frac{5}{11}\right)\left(\frac{6}{10}\right)$
$= \frac{6}{11}$ ≈ 0.5455	1A $a-1$ for r.t. 0.545
	-------------(6)

Solution | Marks

9. (a) (i)

t	0	0.5	1.0	1.5	2	2.5
$\frac{dM}{dt}$	4	4.78496	5.84320	7.24875	9.10480	11.55161

$$M = \int_0^{2.5} \frac{12e^{\frac{2}{3}t}}{3+t}\,dt \approx \frac{0.5}{2}[4+11.55161$$
$$+2(4.78496+5.8432+7.24875+9.1048)]$$
1M

$$= 17.3788 \text{ (m mol/L)}$$
1A a−1 for r.t. 17.379

(ii) $\because \quad \frac{dM}{dt} = \frac{12e^{\frac{2}{3}t}}{3+t}$,

$$\frac{d}{dt}\left(\frac{12e^{\frac{2}{3}t}}{3+t}\right) = 12\left[\frac{2}{3}\cdot\frac{e^{\frac{2}{3}t}}{3+t} - \frac{e^{\frac{2}{3}t}}{(3+t)^2}\right] = \frac{4(3+2t)e^{\frac{2}{3}t}}{(3+t)^2}$$
1A need not simplify

and $$\frac{d^2}{dt^2}\left(\frac{12e^{\frac{2}{3}t}}{3+t}\right) = \frac{8(9+6t+2t^2)}{3(3+t)^3}e^{\frac{2}{3}t}$$
1A need simplification

$$\therefore \quad \frac{d^2}{dt^2}\left(\frac{dM}{dt}\right) > 0 \quad (\text{for } 0 \le t \le 2.5\ )$$

So, $\frac{dM}{dt}$ is concave upward on $[0, 2.5]$.

Hence it is over-estimate.
1

----------(5)

(b) (i)
$$\frac{1}{3+t} = \frac{1}{3}(1-\frac{1}{3}t+\frac{1}{9}t^2-\frac{1}{27}t^3+\cdots)$$
1A
$$= \frac{1}{3}-\frac{1}{9}t+\frac{1}{27}t^2-\frac{1}{81}t^3+\cdots$$

$$e^{\frac{2}{3}t} = 1+\frac{2}{3}t+\frac{1}{2!}(\frac{2}{3}t)^2+\frac{1}{3!}(\frac{2}{3}t)^3+\cdots$$
1M any three terms
$$= 1+\frac{2}{3}t+\frac{2}{9}t^2+\frac{4}{81}t^3+\cdots$$
1A

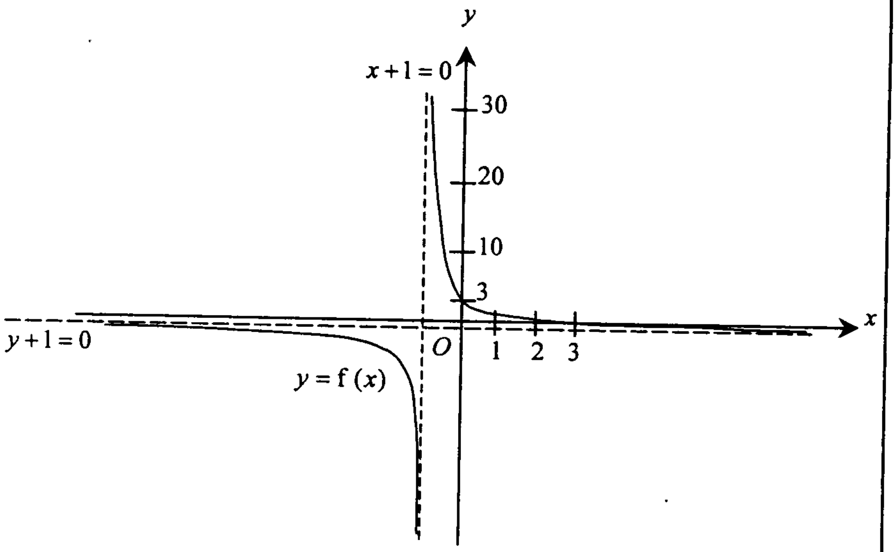
$$\frac{12e^{\frac{2}{3}t}}{3+t} = 12(\frac{1}{3}-\frac{1}{9}t+\frac{1}{27}t^2-\frac{1}{81}t^3+\cdots)(1+\frac{2}{3}t+\frac{2}{9}t^2+\frac{4}{81}t^3+\cdots)$$
$$= 4+\frac{4}{3}t+\frac{4}{9}t^2+\frac{4}{81}t^3+\cdots$$
1A for the first three terms or the term t^3

1A for all being correct

(ii)
$$\int_0^{2.5}\frac{12e^{\frac{2}{3}t}}{3+t}\,dt \approx \int_0^{2.5}(4+\frac{4}{3}t+\frac{4}{9}t^2+\frac{4}{81}t^3)\,dt$$
$$= \left[4t+\frac{2}{3}t^2+\frac{4}{27}t^3+\frac{1}{81}t^4\right]_0^{2.5}$$
1M
$$= 16.9637 \text{ (m mol/L)}$$
1A a−1 for r.t. 16.964

----------(7)

Solution	Marks
(c) The expansion is valid only when $-1 < \frac{t}{3} < 1$ $-3 < t < 3$	2A
Hence $0 \leq t < 3$ (as $t \geq 0$) $\therefore$ this method is not valid to estimate the amount of lactic acid for $t \geq 3$.	1A ----------(3)

Solution	Marks
10. (a) (i) $f(0) = g(0)$ $\Rightarrow \quad b = 3$ ………… (1) $f(3) = g(3)$ $\Rightarrow \quad \frac{3a+b}{3c+1} = 0$ $\Rightarrow \quad 3a + b = 0$ ………… (2) $f(-2) = g(-2)$ $\Rightarrow \quad \frac{-2a+b}{-2c+1} = -5$ $\Rightarrow \quad 2a - b + 10c = 5$ ………… (3)	1M for using any two of the conditions $f(0) = g(0)$, $f(3) = g(3)$ and $f(-2) = g(-2)$
Using (1) and (2), $a = -1$ Using (3), $c = 1$	1A for all correct values of a, b and c
(ii) $f(x) = \frac{-x+3}{x+1}$ $\because \quad \lim_{x \to \pm\infty} f(x) = \lim_{x \to \pm\infty} \frac{-1 + \frac{3}{x}}{1 + \frac{1}{x}} = -1$ The horizontal asymptote is $y + 1 = 0$	1A
The vertical asymptote is $x + 1 = 0$	1A
(iii)  	1A Shape, asymptotes and y-intercept
	----------(5)

Solution	Marks

(b) (i) $g'(x) = -(x+1)^3 - 3(x-3)(x+1)^2$

$= -4(x+1)^2(x-2)$

$g''(x) = -8(x+1)(x-2) - 4(x+1)^2$

$= -12(x+1)(x-1)$

$g'(x) = 0 \quad \Rightarrow \quad x = -1$ or 2

$g''(2) = -36 < 0$

1M

$g'(x) = -(x+1)^3 - 3(x-3)(x+1)^2$

$= -4(x+1)^2(x-2)$

$g'(x) = 0 \quad \Rightarrow \quad x = -1$ or 2

x	$x < -1$	$-1 < x < 2$	$x > 2$
$g'(x)$	+	+	−
$g(x)$	↗	↗	↘

1M

∴ $g(x)$ is maximum when $x = 2$ and $g(2) = 27$

∴ the maximum point is at $(2, 27)$.

1A

$g''(x) = 0 \quad \Rightarrow \quad x = -1$ or 1

x	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$x > 1$
$x + 1$	−	0	+	+	+
$x - 1$	−	−	−	0	+
$g''(x)$	−	0	+	0	−

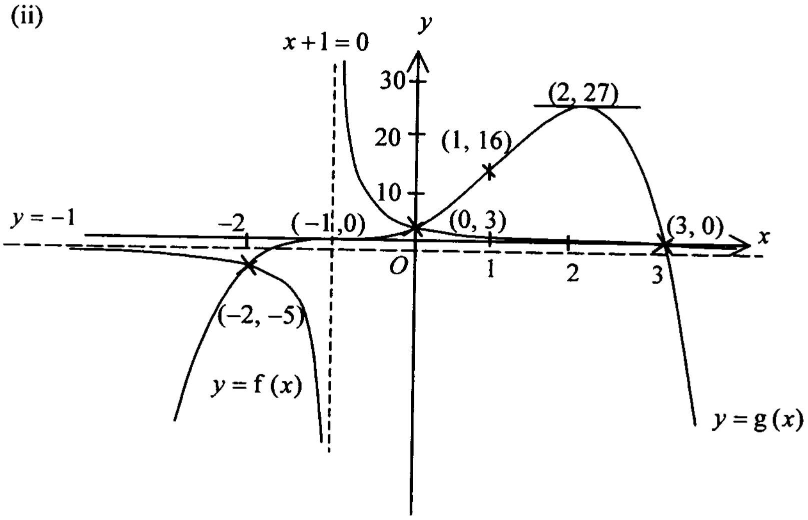
1M for testing any one point

$g(-1) = 0$, $g(1) = 16$

∴ $(-1, 0)$ and $(1, 16)$ are points of inflexion of $g(x)$.

1A for both points

(ii)



1M for the shape

1A for all being correct

----------(6)

Solution	Marks
(c) Using the graphs, the area is	
$\int_0^3 \left[-(x-3)(x+1)^3 - \frac{-x+3}{x+1}\right] dx$	1M for $\int_0^3 [f(x)-g(x)]\,dx$ or $\int_0^3 [g(x)-f(x)]\,dx$
$= \int_0^3 \left\{-(x^4-6x^2-8x-3)-\left[-1+\frac{4}{x+1}\right]\right\} dx$	
$= \int_0^3 \left(-x^4+6x^2+8x+4-\frac{4}{x+1}\right) dx$	
$= \left[-\frac{1}{5}x^5+2x^3+4x^2+4x-4\ln(x+1)\right]_0^3$	1A + 1A for each of $\int f(x)dx$ and $\int g(x)dx$
$= \frac{267}{5}-4\ln 4$	1A
$= 47.8548$	$a-1$ for r.t. 47.855
$\int \frac{-x+3}{x+1}dx = -x+4\ln(x+1)+C$	1A
$\int (x-3)(x+1)^3 dx = \frac{x^5}{5}-2x^3-4x^2-3x+C$	1A
$\int_0^3 [-(x-3)(x+1)^3\,dx = \frac{252}{5} = 50.4$	
$\int_0^3 \frac{-x+3}{x+1}dx = -3+4\ln 4 \approx 2.5452$	
The required area = $\frac{267}{5}-4\ln 4$	1M+1A (≈ 47.8548) $a-1$ for r.t. 47.855
	----------(4)

Solution	Marks
11. (a) (i) $G=\int \frac{2t-8}{t^2-8t+20}\,\mathrm{d}t$	1A
$=\ln(t^2-8t+20)+C$	1A
When $t=0$, $G=50$. $C=50-\ln 20$	1A
$G=\ln(t^2-8t+20)+50-\ln 20$	
(ii) For $G=50$, $\ln(t^2-8t+20)+50-\ln 20=50$	1M
$t^2-8t+20=20$ $t^2-8t=0$ $t=0$ or $t=8$.	
At the end of the 8th week, the weekly sale is the same as at the start of the promotion plan.	1A
	----------(5)
(b) (i) $\because \frac{\mathrm{d}G}{\mathrm{d}t}=\frac{2t-8}{t^2-8t+20}=\frac{2(t-4)}{[(t-4)^2+4]}$ $\therefore \frac{\mathrm{d}G}{\mathrm{d}t}=0$ when $t=4$ Since $\frac{\mathrm{d}G}{\mathrm{d}t}<0$ when $t<4$ and $\frac{\mathrm{d}G}{\mathrm{d}t}>0$ when $t>4$, $\boxed{\begin{aligned}G&=\ln(t^2-8t+20)+C\\&=\ln[(t-4)^2+4]+C\end{aligned}}$	1M
$\therefore$ G is least at $t=4$. At the end of the 4th week, the weekly sale is least.	1A
(ii) $G(6)-G(5)=(\ln 8+50-\ln 20)-(\ln 5+50-\ln 20)$	
$=\ln\frac{8}{5}\approx 0.4700$ (thousand dollars)	1A $a-1$ for more than 4 d.p. or r.t. 0.470
(iii) $G(t+1)-G(t)<0.2$	
$\{\ln[(t+1)^2-8(t+1)+20]+50-\ln 20\}$ $-\{\ln(t^2-8t+20)+50-\ln 20\}<0.2$	1M
$\ln\frac{t^2-6t+13}{t^2-8t+20}<0.2$	1A
$(e^{0.2}-1)t^2-(8e^{0.2}-6)t+(20e^{0.2}-13)>0$	$0.22140t^2-3.77122t+11.42806>0$
t < 3.94316 or t > 13.09015	t < 3.94315 or t >13.09037
$\because \frac{\mathrm{d}G}{\mathrm{d}t}<0$ when $0<t<4$, G is decreasing $\therefore$ $t<3.94316$ is rejected. $\therefore$ $t=14$.	
Thus the promotion plan will be terminated at the end of the 15th week.	1A must show reasons ----------(6)

Solution | Marks

$G(t)-G(t-1)<0.2$

$\{\ln(t^2-8t+20)+50-\ln 20\}-\{\ln[(t-1)^2-8(t-1)+20]+50-\ln 20\}<0.2$ 1M

$$\ln\frac{t^2-8t+20}{t^2-10t+29}<0.2$$ 1A

$$(e^{0.2}-1)t^2-(10e^{0.2}-8)t+(29e^{0.2}-20)>0$$

$$0.22140t^2-4.21403t+15.42068>0$$

$$t<4.94316 \text{ or } t>14.09015$$

(c) $\dfrac{\mathrm{d}G}{\mathrm{d}t}=\dfrac{2t-8}{t^2-8t+20}$

$$\frac{\mathrm{d}^2G}{\mathrm{d}t^2}=\frac{2(t^2-8t+20)-(2t-8)(2t-8)}{(t^2-8t+20)^2}$$ 1A

$$=-\frac{2(t-2)(t-6)}{(t^2-8t+20)^2}$$

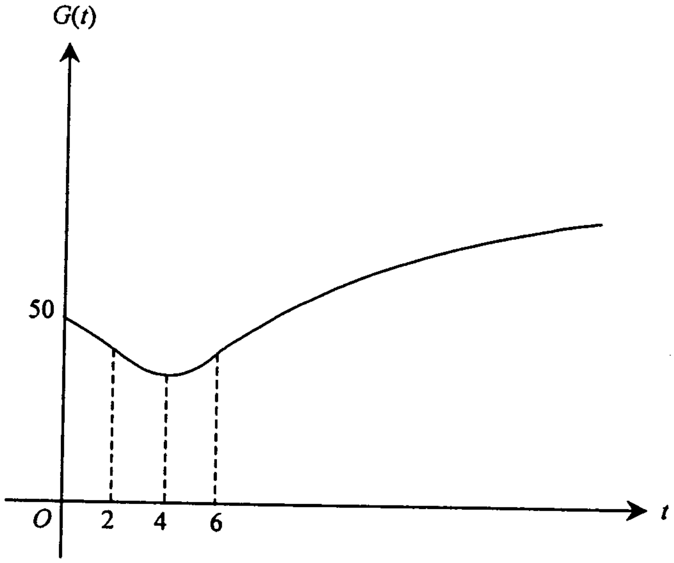
$\dfrac{\mathrm{d}^2G}{\mathrm{d}t^2}=0$ when $t=2$ or $t=6$. $\therefore$ $t_2=6$ 1A

Although G keeps increasing, 1A

$\left.\begin{array}{l}\dfrac{\mathrm{d}G}{\mathrm{d}t} \text{ increases immediately before } t=6\text{,}\\ \dfrac{\mathrm{d}G}{\mathrm{d}t} \text{ decreases immediately after } t=6\text{.}\end{array}\right\}$ 1A

-----------(4)

For reference only



	$G(t)$	$\Delta G(t)$
0	50.0	
1.0	49.5692	− 0.4308
2.0	49.0837	− 0.4855
3.0	48.6137	− 0.47
4.0	48.3906	− 0.2231
5.0	48.6137	+ 0.2231
6.0	49.0837	+ 0.47
7.0	49.5692	+ 0.4855
8.0	50.0	+ 0.4308
9.0	50.3716	+ 0.3716
10.0	50.6931	+ 0.3215
11.0	50.9746	+ 0.2815
12.0	51.2238	+ 0.2492
13.0	51.4469	+ 0.2231
14.0	51.6487	+ 0.2018
15.0	51.8326	+ 0.1839
16.0	52.0015	+ 0.1689
17.0	52.1576	+ 0.1561
18.0	52.3026	+ 0.145
19.0	52.438	+ 0.1354
20.0	52.5649	+ 0.1269

Solution	Marks
12. (a) Let λ_1 be the sample mean of car accidents at the road junction in a month. $\lambda_1 = \frac{0\times12+1\times15+2\times9+3\times4}{40} = 1.125$	1A (Accept $\lambda_1 = 1.1247$)
Let X be the number of car accidents at the road junction in a month. For researcher A , $a = 40\cdot \text{P}(X=3)$ $= 40\times\frac{1.125^3}{3!}e^{-1.125}$	1M
≈ 3.08	1A (≈ 3.08166) $a-1$ for more than 2 d.p. -----------(3)
(b) For researcher B , let the mean be λ_2 . Then (i) $12.05 = 40\cdot \text{P}(X=0)$ $12.05 = 40e^{-\lambda_2}$ $\lambda_2 = -\ln\frac{12.05}{40}$ ≈ 1.1998	1A $a-1$ for more than 4 d.p. (accept $\lambda_2 \approx 1.1999$, 1.2000 or 1.2)
(ii) $b = 40\cdot P(X=2) \approx 40\times\frac{1.1998^2}{2!}e^{-1.1998} (\approx 8.6732) \approx 8.67$	1A $a-1$ for more than 2 d.p. (accept $b = 8.68$)
Equivalent forms: $\frac{b}{12.05} = \frac{\lambda^2}{2}$, $\frac{b}{14.46} = \frac{\lambda}{2}$, $\frac{b}{3.47} = \frac{3}{\lambda}$	Accept $b = 8.68$
	----------(2)
(c) For the number of car accidents is 4 or more, the expected number of months observed by researcher A is $40-(12.99+14.61+8.22+3.08) \approx 1.10$	1M (either A or B)
Let TSE_1 = Total sum of errors for model fitted by researcher A $= \sum \lvert f_0 - f_{E_1} \rvert$ $= \lvert 12-12.99\rvert+\lvert 15-14.61\rvert+\lvert 9-8.22\rvert+\lvert 4-3.08\rvert+\lvert 0-1.10\rvert$	1M + 1M (1M for the first 4 terms 1M for the last term) (either A or B)
≈ 4.18	1A Sum of the first 4 terms 3.08 (for both A and B)
For the number of car accidents is 4 or more, the expected number of months observed by researcher B is $40-(12.05+14.46+8.67+3.47) \approx 1.35$ TSE_2 = Total sum of errors for model fitted by researcher B $= \sum \lvert f_0 - f_{E_2} \rvert$ $\approx \lvert 12-12.05\rvert+\lvert 15-14.46\rvert+\lvert 9-8.67\rvert+\lvert 4-3.47\rvert+\lvert 0-1.35\rvert$ $= 2.8$	Sum of the first 4 terms 1.45
As $\text{TSE}_2 < \text{TSE}_1$, researcher B fits the data of car accidents better than researcher A does.	1M (accept using the first 4 terms) ----------(5)

Solution	Marks
(d) (i) P(the number of car accidents at the road junction in a month is 3 and one of which involves a bus)	
= P($X = 3$ and one of which involves a bus)	
= P(one accident involves a bus $\mid X = 3$) P($X = 3$)	1M for multiplication rule (for (i) or (ii))
$= C_1^3\ 0.3\times(1-0.3)^2\times\frac{\lambda_2^3}{3!}e^{-\lambda_2}$	1M for $C_1^3\ 0.3\times(1-0.3)^2$
$= 3\times0.3\times0.7^2\times\frac{1.1998^3}{3!}e^{-1.1998}$ ($\approx$ 0.038254)	0.0383 if $\lambda_2\approx1.200$
$\approx$ 0.0382	1A (Accept 0.0383) $a-1$ for r.t. 0.038
P(the number of car accidents at the road junction in a month is 3 and one of which involves a bus) = P($X = 3$ and one of which involves a bus) = P(one accident involves a bus $\mid X = 3$) P($X = 3$)	1M for multiplication rule
$= C_1^3\ 0.3\times(1-0.3)^2\times\frac{3.47}{40}$	1M for $C_1^3\ 0.3\times(1-0.3)^2$
$\approx$ 0.0383	1A $a-1$ for r.t. 0.038
(ii) P(the number of car acidents at the road junction in a month is 3 and only the third car accident involves a bus) = P($X = 3$ and only the third car accident involves a bus) $= \frac{1}{3}$ P($X = 3$ and one of which involves a bus)	
$\approx$ 0.0127	1A $a-1$ for r.t. 0.013 (Accept 0.0128)
P(the number of car accidents at the road junction in a month is 3 and only the third car accident involves a bus) = P($X = 3$ and only the third car accident involves a bus) = P(only the third car accident involves a bus $\mid X = 3$) P($X = 3$) $= (1-0.3)^2\times0.3\times\frac{\lambda_2^3}{3!}e^{-\lambda_2}$ $= 0.7^2\times0.3\times\frac{1.200^3}{3!}e^{-1.200}$	
$\approx$ 0.0128	1A $a-1$ for r.t. 0.013
(iii) P($X = 3$ and the third car accident involves a bus $\mid X = 3$ and only one of which involves a bus)	
$= \frac{1}{3}$.	1M
P($X = 3$ and the third car accident involves a bus $\mid X = 3$ and one of which involves a bus)	
$= \frac{0.0128}{0.0383}\approx 0.3342$	1M
	------------(5)

Solution	Marks
13. Let Xg be the weight of a bag of self raising flour in the batch.	
(a) (i) P(a bag of flour is underweight) = $P(X < 376)$	
$= P(\frac{X-400}{10} < \frac{376-400}{10})$	1M
$= P(Z < -2.4)$	
≈ 0.0082	1A either one
(ii) P(a bag of flour is overweight) = $P(X > 424)$	
$= P(\frac{X-400}{10} > \frac{424-400}{10})$	
$= P(Z > 2.4)$	
≈ 0.0082	1A ----------(3)
(b) (i) P(a bag of flour is substandard)	
$= P(X < 376) + P(X > 424)$	
$\approx 0.0082 + 0.0082 = 0.0164$	1A
Let Y be the number of substandard bags in the sample.	
P(there is no substandard bags in the sample) = $P(Y = 0)$	
$= C_0^{50}\ 0.0164^0 \times (1-0.0164)^{50}$	1M
$= 0.9836^{50} \approx 0.4374$	1A (0.43745) $a-1$ for r.t. 0.437
(ii) $P(Y \leq 2)$	
$= P(Y=0) + P(Y=1) + P(Y=2)$	
$= C_0^{50} 0.0164^0 \times 0.9836^{50} + C_1^{50}\ 0.0164 \times 0.9836^{49} + C_2^{50}\ 0.0164^2 \times 0.9836^{48}$	1M
$\approx 0.43745 + 0.36469 + 0.14897$	
≈ 0.9511	1A (0.95111) $a-1$ for r.t. 0.951 ----------(5)
(c) Let W be the number of underweight bags in the sample.	
(i) $P(W=0,\ Y=1)$	
$= P(W=0 \mid Y=1) \cdot P(Y=1)$	
$= \frac{1}{2} \times C_1^{50} (0.0164)(0.9836)^{49}$	1M + 1M for $\frac{1}{2}$ and cond. prob.
≈ 0.1823	1A $a-1$ for r.t. 0.182
(ii) The required probability is $P(W=0,\ Y \leq 2)$	
$= P(W=0, Y=0) + P(W=0, Y=1) + P(W=0, Y=2)$	1M
$= P(Y=0) + P(W=0, Y=1) + P(W=0 \mid Y=2) \cdot P(Y=2)$	
$\approx 0.43745 + 0.18235 + \left(\frac{1}{2}\right)^2 \cdot C_2^{50} (0.0164)^2 (0.9836)^{48}$	1M for the last term
≈ 0.6570	1A (0.65704) (Accept 0.6569) $a-1$ for r.t. 0.657
(iii) The required probability is $P(W=0 \mid Y \leq 2)$	
$= \frac{P(W=0, Y \leq 2)}{P(Y \leq 2)}$	
$\approx \frac{0.65704}{0.95111}$	1M
≈ 0.6908	(Accept 0.6907) ----------(7)

Solution	Marks
14. (a) Let N be the number of customers visiting the supermarket in one minute. $P(N \le 2) = \sum_{k=0}^{2} \frac{6^k}{k!} e^{-6}$	1A
$= e^{-6} + \frac{6}{1!} e^{-6} + \frac{6^2}{2!} e^{-6}$ $\approx 0.002479 + 0.01487 + 0.04462$ ≈ 0.0620 $\therefore \quad P(N > 2) = 1 - P(N \le 2) \approx 0.9380$	1M+1A $a-1$ if 0.938 ----------(3)
(b) (i) $X \sim N(\mu, \sigma^2)$ $P(X < 100) = 0.063$ $P(Z < \frac{100-\mu}{\sigma}) = 0.063$ $\frac{100-\mu}{\sigma} \approx -1.53$ .... (1)	1A
$P(X \ge 400) = 0.006$ $p(Z \ge \frac{400-\mu}{\sigma}) = 0.006$ $\frac{400-\mu}{\sigma} \approx 2.51$ .... (2)	1A (Accept $\frac{200-\mu}{\sigma} \in [-0.185, -0.18]$)
Solving (1) and (2), we get $\mu \approx 213.6$	1A $a-1$ for more than 1 d.p. (Accept $\mu \in [213.3, 213.8]$)
$\sigma \approx 74.26 \approx 74.3$	1A $a-1$ for more than 1 d.p. (Accept $\sigma \in [74.1, 74.3]$)
$a_1 = P(200 \le X < 300)$ $= p(Z < \frac{300-213.6}{74.3}) - P(Z < \frac{200-213.6}{74.3})$ ≈ 0.4484 ≈ 0.448	1A $a-1$ for more than 3 d.p. (Accept $a_1 \in [0.448, 0.453]$)
$a_2 = P(300 \le X < 400)$ ≈ 0.117	1A $a-1$ for more than 3 d.p. (Accept $a_2 \in [0.115, 0.119]$)
(ii) For normal distribution, median = mean = 213.6	1M
(iii) $P(X > 50 \mid X \le 200)$ $= \frac{P(50 \le X < 200)}{P(X < 100) + P(100 \le X < 200)}$	1M
$= \frac{P(-2.20 \le Z < -0.18)}{0.063 + 0.364}$ $\approx \frac{0.4861 - 0.0714}{0.427}$ ≈ 0.9712	1A $a-1$ for more than 4 d.p. (Accept probability $\in$ [0.9620, 0.9749])

Solution	Marks
$\mathrm{P}(X > 50 \mid X \le 200)$ $= \dfrac{\mathrm{P}(50 \le X < 200)}{\mathrm{P}(X < 100) + \mathrm{P}(100 \le X < 200)}$	1M
$= \dfrac{\mathrm{P}(X < 200) - \mathrm{P}(X \le 50)}{\mathrm{P}(X \le 200)}$ $= \dfrac{\mathrm{P}(X < 200) - \mathrm{P}(Z \le -2.20)}{\mathrm{P}(X \le 200)}$ $= \dfrac{0.427 - 0.0139}{0.427}$ ≈ 0.9674	
	1A $a-1$ for more than 4 d.p. (Accept probability $\in [0.9620, 0.9749]$)
(iv) The required probability $= C_2^5\, \mathrm{P}(X < 200)^2 (1 - \mathrm{P}(X < 200))^3 \cdot \mathrm{P}(N = 5)$	
$= 10(0.063 + 0.364)^2 (1 - (0.063 + 0.364))^3 \cdot \dfrac{6^5}{5!} e^{-6}$	1M for Binomial/Poisson probability 1M for the multiplication rule (Binomial × Poisson)
$\approx 10(0.1823)(0.1881) \cdot (0.1606)$ ≈ 0.0551	1A $a-1$ for r.t. 0.055 (Accept probability $\in [0.0550, 0.0552]$) ----------(12)