SECTION A (40 marks)
Answer ALL questions in this section.
Write your answers in the AL(E) answer book.

1. Let $x = -\frac{5}{t^2} + 2e^{-3t}$ and $y = \frac{10}{t^2} + e^{2t}$ $(t \neq 0)$. If $\frac{dy}{dx} = -2$, find the value of t.

(5 marks)

2. An adventurer estimates the volume of his hot air balloon by $V(r) = \frac{4}{3}\pi r^3 + 5\pi$, where r is measured in metres and V is measured in cubic metres. When the balloon is being inflated, r will increase with time $t (\geq 0)$ in such a way that

$$r(t) = \frac{18}{3 + 2e^{-t}},$$

where t is measured in hours.

(a) Find the rate of change of volume of the balloon at $t = 2$. Give your answer correct to 2 decimal places.

(b) If the balloon is being inflated over a long period of time, what will the volume of the balloon be? Give your answer correct to 2 decimal places.

(5 marks)

3. A researcher modelled the number of bacteria $N(t)$ in a sample t hours after the beginning of his observation by $N(t) = 900\,a^{kt}$, where $a (> 0)$ and k are constants. He observed and recorded the following data:

t (in hours)	0.5	1.0	2.0	3.0
$N(t)$	1100	1630	2010	2980

The researcher made one mistake when writing down the data for $N(t)$.

Express $\ln N(t)$ as a linear function of t and use the graph paper on Page 2 to determine which one of the data was incorrect, and estimate the value of $N(2.5)$ correct to 3 significant figures.

(4 marks)

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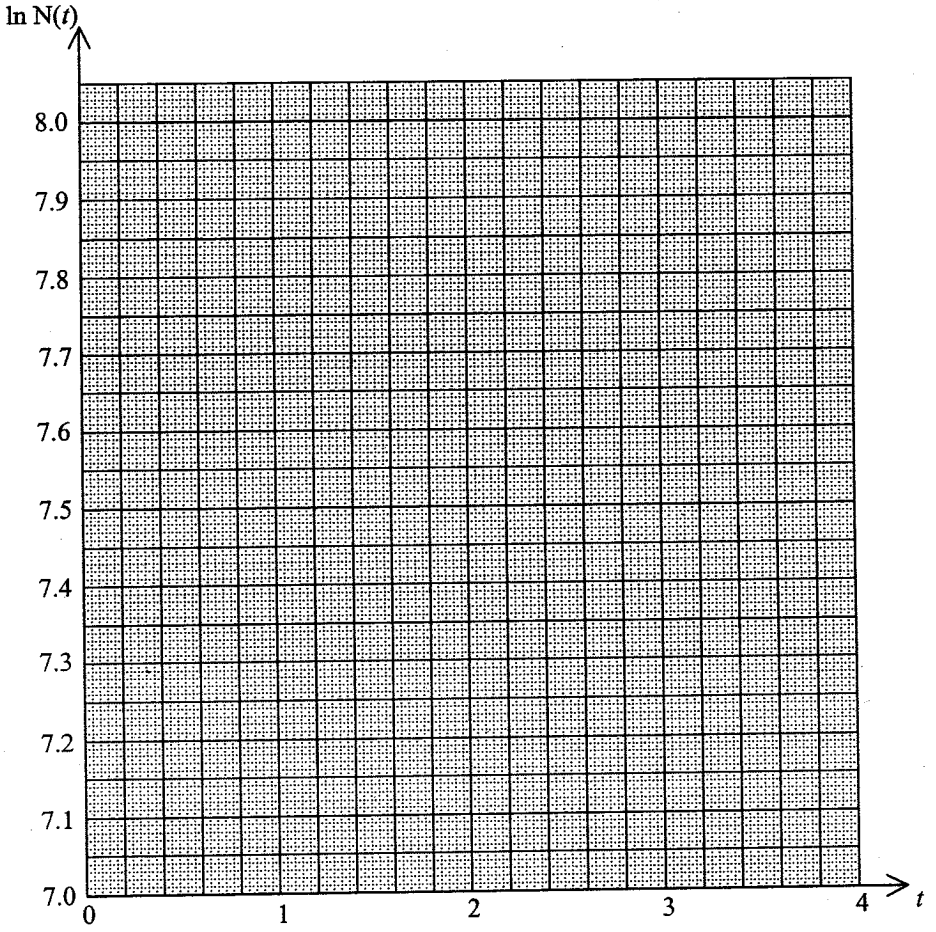
3. (Continued) **Fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**



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4. An engineer conducts a test for a certain brand of air-purifier in a smoke-filled room. The percentage of smoke in the room being removed by the air-purifier is given by $S\%$. The engineer models the rate of change of S by

$$\frac{dS}{dt} = \frac{8100t}{(3t+10)^3},$$

where $t (\geq 0)$ is measured in hours from the start of the test.

(a) Using the substitution $u = 3t + 10$ or otherwise, find the percentage of smoke removed from the room in the first 10 hours.

(b) If the air-purifier operates indefinitely, what will the percentage of smoke removed from the room be?

(5 marks)

5. Twelve boys and ten girls in a class are divided into 3 groups as shown in the table below:

	Group A	Group B	Group C
Number of boys	6	4	2
Number of girls	2	3	5

To choose a student as the class representative, a group is selected at random, then a student is chosen at random from the selected group.

(a) Find the probability that a boy is chosen as the class representative.

(b) Suppose that a boy is chosen as the class representative. Find the probability that the boy is from Group A.

(5 marks)

6. Assume that the number of passengers arriving at a bus stop per hour follows a Poisson distribution with mean 5. The probability that a passenger arriving at the bus stop is male is 0.65.

(a) Find the probability that 4 passengers arrive at the bus stop in an hour.

(b) Find the probability that 4 passengers arrive at the bus stop in an hour and exactly 2 of them are male.

(5 marks)

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7. Twenty two students in a class attended an examination. The stem-and-leaf diagram below shows the distribution of the examination marks of these students.

Stem (tens)	Leaf (units)
3	5 7
4	2 4 6
5	0 3 4 4 4 5
6	1 2 5 5 8
7	3 8 9
8	4 8
9	5

(a) Find the mean of the examination marks.

(b) Two students left the class after the examination and their marks are deleted from the stem-and-leaf diagram. The mean of the remaining marks is then increased by 1.2 and there are two modes. Find the two deleted marks.

(c) Two students are randomly selected from the remaining 20 students. Find the probability that their marks are both higher than 75.

(5 marks)

8. A flower shop has 13 roses of which 2 are red, 5 are white and 6 are yellow. Mary selects 3 roses randomly and the colours are recorded.

(a) Denote the red rose selected by R, the white rose by W and the yellow rose by Y.
List the sample space (i.e. the set of all possible combinations of the colours of roses selected, for example, $1R\,2W$ denotes that 1 red rose and 2 white roses are selected).

(b) Find the probability that Mary selects exactly one red rose.

(c) Given that Mary has selected exactly one red rose, find the probability that only one of the other two roses is white.

(6 marks)

SECTION B (60 marks)
Answer any FOUR questions in this section. Each question carries 15 marks.
Write your answers in the AL(C)2 answer book.

9. Lactic acid in large amounts is usually formed during vigorous physical exercise, which leads to fatigue. The amount of lactic acid, M, in muscles is measured in m mol/L. A student modelled the rate of change of the amount of lactic acid in his muscles during vigorous physical exercise by

$$\frac{dM}{dt} = \frac{12e^{\frac{2}{3}t}}{3+t} \qquad (0 \le t \le 4),$$

where t is the time measured in minutes from the start of the exercise.

(a) The student used the trapezoidal rule with 5 sub-intervals to estimate the amount of lactic acid formed after the first 2.5 minutes of exercise.

(i) Find his estimate.

(ii) Find $\frac{d^2}{dt^2}\left(\frac{12e^{\frac{2}{3}t}}{3+t}\right)$ and hence determine whether his estimate is an over-estimate or an under-estimate.

(5 marks)

(b) The student re-estimated the amount of lactic acid formed by expanding $\frac{12e^{\frac{2}{3}t}}{3+t}$ as a series in ascending powers of t.

(i) Expand $\frac{1}{3+t}$ and hence find the expansion of $\frac{12e^{\frac{2}{3}t}}{3+t}$ in ascending powers of t as far as the term in t^3.

(ii) By integrating the expansion of $\frac{12e^{\frac{2}{3}t}}{3+t}$ in (i), re-estimate the amount of lactic acid formed after the first 2.5 minutes of exercise.

(7 marks)

(c) The student wanted to predict the amount of lactic acid formed in his muscles after the first 4 minutes of exercise. He decided to use the method in (b) to estimate the amount of lactic acid formed. Briefly explain whether his method was valid.

(3 marks)

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10. Let $f(x)=\frac{ax+b}{cx+1}$ and $g(x)=-(x-3)(x+1)^3$, where a, b and c are constants. It is known that $f(0)=g(0)$, $f(3)=g(3)$ and $f(-2)=g(-2)$.

(a) (i) Find the values of a, b and c.

(ii) Find the horizontal and vertical asymptotes of the graph of $f(x)$.

(iii) Sketch the graph of $f(x)$ and its asymptotes. Indicate the point(s) where the curve cuts the y-axis.

(5 marks)

(b) (i) Find all relative extreme point(s) and point(s) of inflexion of $g(x)$.

(ii) On the diagram drawn in (a)(iii), sketch the graph of $g(x)$. Indicate all the relative extreme point(s) and the point(s) of inflexion, the point(s) where the graph cuts the coordinates axes and where it cuts the graph of $f(x)$.

(6 marks)

(c) Find the area enclosed by the graphs of $f(x)$ and $g(x)$.

(4 marks)

11. A food store manager notices that the **weekly sale** is declining, so he starts a promotion plan to boost the **weekly sale**. He models the rate of change of **weekly sale** G by

$$\frac{dG}{dt}=\frac{2t-8}{t^2-8t+20} \qquad (t\geq 0),$$

where G is the **weekly sale** recorded at the end of the week in thousands of dollars and t is the number of weeks elapsed since the start of the plan. Suppose that at the start of the plan (i.e. $t = 0$), the **weekly sale** is 50 thousand dollars.

(a) (i) Express G in terms of t.

(ii) At the end of which week after the start of the plan will the **weekly sale** be the same as at the start of the plan?

(5 marks)

(b) (i) At the end of which week after the start of the plan will the **weekly sale** drop to the least?

(ii) Find the increase between the **weekly sales** of the 5th and the 6th weeks after the start of the plan.

(iii) The store manager decides that once such increase of **weekly sales** between two consecutive weeks is less than 0.2 thousand dollars, he will terminate the promotion plan. At the end of which week after the start of the plan will the plan be terminated?

(6 marks)

(c) Let t_1 and t_2 be the roots of $\frac{d^2G}{dt^2}=0$, where $t_1<t_2$. Find t_2.

Briefly describe the behaviour of G and $\frac{dG}{dt}$ immediately before and after t_2.

(4 marks)

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12. Two researchers want to study the distribution of the number of car accidents at a certain road junction in a month. They have collected the data over 40 months as shown in the following table. They suggest that the distribution can be modelled by a Poisson distribution.

Number of car accidents	Observed number of months	Expected number of months *	
		Researcher A	Researcher B
0	12	12.99	12.05
1	15	14.61	14.46
2	9	8.22	b
3	4	a	3.47

* Correct to 2 decimal places.

(a) Researcher A uses the sample mean of the distribution as the mean of the Poisson distribution. Find the value of a in the table correct to 2 decimal places.

(3 marks)

(b) Researcher B tries to fit the data by using a Poisson distribution with another mean.

(i) Find the mean used by researcher B.

(ii) Find the value of b in the table correct to 2 decimal places.

(2 marks)

(c) The absolute values of the differences between observed and expected numbers are regarded as errors. The distribution with a smaller total sum of errors will fit the data better. Which Poisson distribution will fit the data better ?

(5 marks)

12. (Continued)

(d) Assume the Poisson distribution that fits the data better in (c) is adopted and 30% of car accidents involve a bus.

(i) Find the probability that the number of car accidents at the road junction in a month is 3 and only one of which involves a bus.

(ii) Find the probability that the number of car accidents at the road junction in a month is 3 and only the third car accident involves a bus.

(iii) Given that the number of car accidents at the road junction in a month is 3 and only one of which involves a bus, find the probability that the third car accident involves a bus.

(5 marks)

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13. The weight of each bag of self-raising flour in a batch produced by a factory follows a normal distribution with mean 400 g and standard deviation 10 g. A bag of flour with weight less than 376 g is **underweight**, and more than 424 g is **overweight**.

(a) Find the probability that a randomly selected bag of flour

(i) is **underweight**;

(ii) is **overweight**.

(3 marks)

(b) If a bag of flour is either **underweight** or **overweight**, it will be classified as a **substandard** bag by the director of the factory. The director randomly selects 50 bags as a sample from the batch.

(i) Find the probability that there is no **substandard** bag of flour in the sample.

(ii) Find the probability that there are no more than 2 **substandard** bags of flour in the sample.

(5 marks)

(c) A wholesaler is only concerned about the number of bags of flour which are **underweight**. The wholesaler re-analyses the sample of 50 bags of flour in (b).

(i) Find the probability that in the sample there is only 1 **substandard** bag and it is not **underweight**.

(ii) Find the probability that there are no more than 2 **substandard** bags in the sample and no **underweight** bag of flour in the sample.

(iii) Given that in the sample there are no more than 2 **substandard** bags, find the probability that there is no **underweight** bag in the sample.

(7 marks)

14. Suppose the number of customers visiting a supermarket per minute follows a Poisson distribution with mean 6.

(a) Find the probability that the number of customers visiting the supermarket in one minute is more than 2.

(3 marks)

(b) Suppose the amount $\$X$ spent by a customer in the supermarket follows a normal distribution $N(\mu, \sigma^2)$.

Probability distribution of the amount spent by a customer

Amount spent ($\$X$)	Probability *
$X < 100$	0.063
$100 \le X < 200$	0.364
$200 \le X < 300$	a_1
$300 \le X < 400$	a_2
$X \ge 400$	0.006

* Correct to 3 decimal places.

(i) Using the probabilities provided in the above table, find the values of μ and σ correct to 1 decimal place.
Hence find the values of a_1 and a_2 correct to 3 decimal places.

(ii) What is the median of the normal distribution?

(iii) Given that a customer spends less than \$200, find the probability that the customer spends more than \$50.

(iv) Find the probability that there are 5 customers visiting the supermarket in a minute and exactly 2 of them each spends less than \$200.

(12 marks)

END OF PAPER