SECTION A (40 marks)
Answer ALL questions in this section.
Write your answers in the AL(E) answer book.

1. A and B are two independent events. If $P(A)=0.4$ and $P(A\cup B)=0.7$, find $P(B)$.

(4 marks)

2. Let $u=e^{2x}$ and $\frac{dy}{du}=\frac{1}{u}-2u$.

(a) Express $\frac{du}{dx}$ and $\frac{dy}{dx}$ in terms of x.

(b) It is known that $y=1$ when $x=0$. Express y in terms of x.

(5 marks)

3. The ages of 35 members of a golf club are shown below:

Stem (tens)	Leaf (units)
1	a 8 8 9 9
2	0 1 2 3 3 4 7 8
3	1 2 2 5 b 9 9
4	0 2 5 5 6
5	2 2 5 5 8 8
6	0 1 c 6

It is known that the median and the range of the ages are 36 and 48 respectively, and the ages of the two eldest members differ by 1.

(a) Find the unknown digits a, b and c.

(b) The three members whose ages correspond to the three unknown digits a, b and c are replaced with three new members with ages 12, 38 and 68 respectively. Draw two box-and-whisker diagrams in your answer book comparing the age distributions of the members before and after replacement.

(6 marks)

4. The binomial expansion of $(1+ax)^{-\frac{1}{n}}$ in ascending powers of x is $1-\frac{4}{3}x+\frac{32}{9}x^2+\cdots$, where a is a constant and n is a positive integer.

(a) Find the values of a and n.

(b) State the range of values of x for which the expansion is valid.

(6 marks)

5. Suppose the rate of change of the accumulated bonus, R thousand dollars per month, for a group of salesmen can be modelled by

$$R=\frac{1200}{t^2+150} \qquad (0\le t\le 6),$$

where t is the time in months since January 1, 2001.

(a) Use the trapezoidal rule with 4 sub-intervals to estimate the total bonus for the first 6 months in 2001.

(b) Find $\frac{d^2R}{dt^2}$.

Hence or otherwise, state with reasons whether the approximation in (a) is an overestimate or an underestimate.

(6 marks)

6. 3 students are randomly selected from 10 students of different weights. Find the probability that

(a) the heaviest student is in the selection,

(b) the heaviest one out of the 3 selected students is the 4th heaviest among the 10 students,

(c) the 2 heaviest students are not both selected.

(7 marks)

7. In the election of the Legislative Council, 48% of the voters support Party A, 39% Party B and 13% Party C. Suppose on the polling day, 65%, 58% and 50% of the supporting voters of Parties A, B and C respectively cast their votes.

(a) A voter votes on the polling day. Find the probability that the voter supports Party B.

(b) Find the probability that exactly 2 out of 5 voting voters support Party B.

(6 marks)

SECTION B (60 marks)

Answer any FOUR questions in this section. Each question carries 15 marks.
Write your answers in the AL(C)2 answer book.

8. A chemical factory continually discharges a constant amount of biochemical waste into a river. The microorganisms in the waste material flow down the river and remove dissolved oxygen from the water during biodegradation. The concentration of dissolved oxygen (CDO) of the river is given by

$$G(x) = 2a - 12e^{-kx} + (a+12)e^{-2kx},$$

where $G(x)$ mg/L is the CDO of the river at position x km downstream from the location of discharge of the waste, and a, k are positive constants.

At the location of the discharge of waste (i.e. $x = 0$), the CDO of the river is 9 mg/L.

(a) (i) Show that $a = 3$.

(ii) Find the minimum CDO of the river.

(5 marks)

(b) Figure 1 shows a sketch of the graph of $G(x)$ against x. It is found that downstream from the location of the discharge of waste, a stretch of 2.85 km of the river has a CDO of 4.5 mg/L or below.

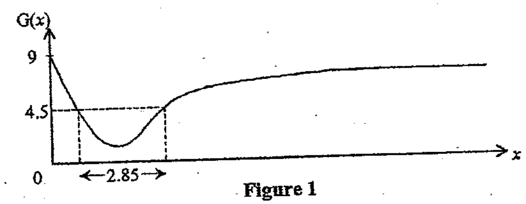

Figure 1

(i) Find the value of k correct to 1 decimal place.

(ii) Find $G''(x)$.
Hence determine the position of the river, to the nearest 0.1 km, where the rate of change of the CDO is greatest.

(iii) A river is said to be *healthy* if the CDO of the river is 5.5 mg/L or above. Will the river in this case become *healthy*? If yes, find the position of the river, to the nearest 0.1 km, where it becomes *healthy* again.

(10 marks)

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9. The spread of an epidemic in a town can be measured by the value of PPI (the proportion of population infected). The value of PPI will increase when the epidemic breaks out and will stabilize when it dies out.

The spread of the epidemic in town A last year could be modelled by the equation $P'(t)=\dfrac{0.04ake^{-kt}}{1-a}$, where $a, k>0$ and $P(t)$ was the PPI t days after the outbreak of the epidemic. Figure 2 shows the graph of $\ln P'(t)$ against t, which was plotted based on some observed data obtained last year. The initial value of PPI is 0.09 (i.e. $P(0)=0.09$).

(a) (i) Express $\ln P'(t)$ as a linear function of t and use Figure 2 to estimate the values of a and k correct to 2 decimal places.
Hence find $P(t)$.

(ii) Let μ be the PPI 3 days after the outbreak of the epidemic. Find μ.

(iii) Find the stabilized PPI.

(8 marks)

(b) In another town B, the health department took precautions so as to reduce the PPI of the epidemic. It is predicted that the rate of spread of the epidemic will follow the equation $Q'(t)=6(b-0.05)(3t+4)^{-\frac{3}{2}}$, where $Q(t)$ is the PPI t days after the outbreak of the epidemic in town B and b is the initial value of PPI.

(i) Suppose $b=0.09$.

(I) Determine whether the PPI in town B will reach the value μ in (a)(ii).

(II) How much is the stabilized PPI reduced in town B as compared with that in town A?

(ii) Find the range of possible values of b if the epidemic breaks out in town B. Explain your answer briefly.

(7 marks)

The graph of $\ln P'(t)$ against t

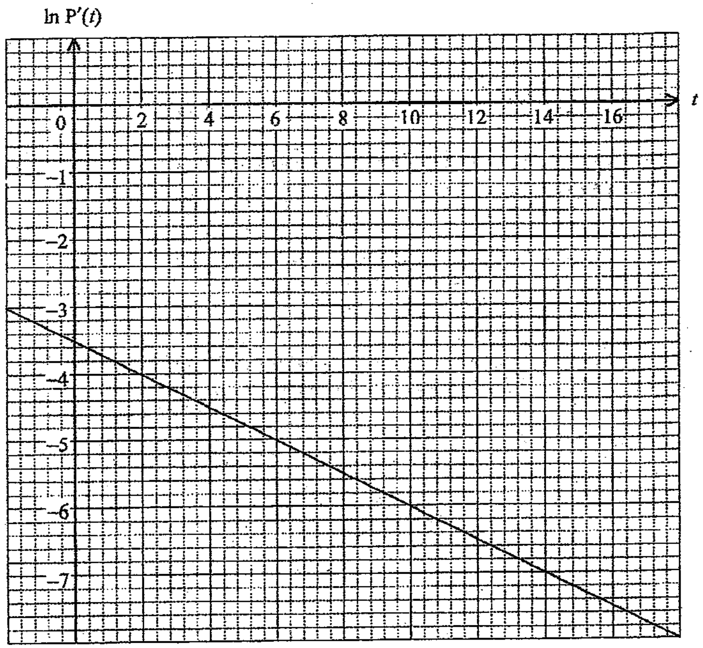

Figure 2

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The graph of $\ln P'(t)$ against t

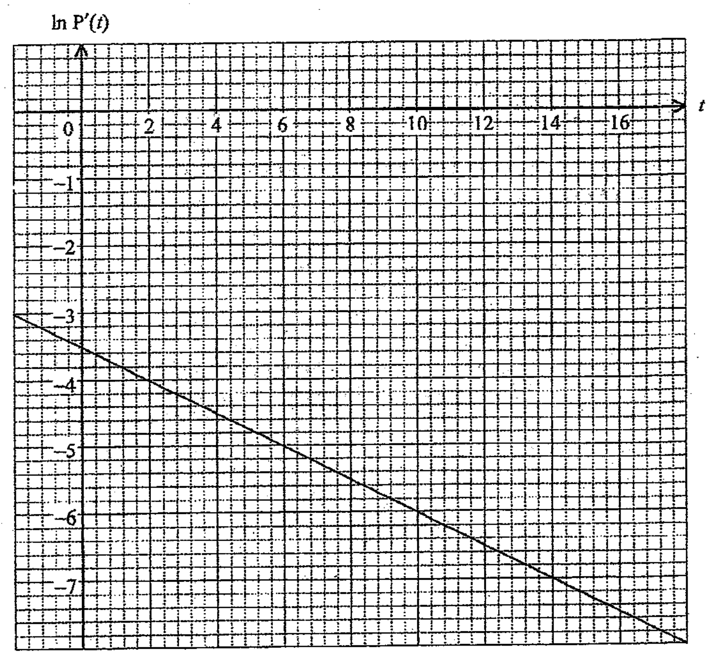

Figure 2

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11. A building has only two entrances A and B. Within a 15-minute period, the numbers of persons who entered the building by using entrances A and B follow the Poisson distributions with means 3.2 and 2.7 respectively.

(a) Find the probability that, on a given 15-minute period,

(i) no one entered the building by using entrance A;

(ii) no one entered the building by using entrance B;

(iii) at least one person entered the building;

(iv) exactly two persons entered the building.

(7 marks)

(b) Let X be the number of persons who entered the building within a 15-minute period. Suppose X follows a Poisson distribution with mean λ and k is the most probable number of persons who entered the building within a 15-minute period.

(i) By considering $P(X=k-1)$, $P(X=k)$ and $P(X=k+1)$, show that $\lambda-1 \le k \le \lambda$.

(ii) Suppose $\lambda=5.9$. For any 5 successive 15-minute periods, find the probability that the third time that exactly k persons entered the building within a 15-minute period will occur during the fifth 15-minute period.

(8 marks)

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12. Table 1 gives the probability distributions of the lifetimes of two brands of compact fluorescent lamps (CFLs). The lifetime of a Brand X CFL follows a normal distribution with mean μ hours and standard deviation 400 hours. The lifetime of a Brand Y CFL follows another normal distribution with mean 8 800 hours and standard deviation σ hours.

Table 1 Probability distributions of the lifetimes of brands X and Y CFLs

Lifetime of a CFL (in hours)	Probability*	
	Brand $X \sim N(\mu, 400^2)$	Brand $Y \sim N(8800, \sigma^2)$
Under 8 200	0.0808	0.1587
8 200 to 8 600	0.2638	b_1
8 600 to 9 000	a_1	b_2
9 000 to 9 400	0.2195	b_3
Over 9 400	a_2	0.1587

* Correct to 4 decimal places.

(a) Using the probabilities provided in Table 1, find μ and σ.

Hence find the values of a_1, a_2, b_1, b_2 and b_3 in Table 1.

(5 marks)

(b) Based on the results of (a), which brand of CFL would you choose to buy? Explain.

(1 mark)

(c) Figure 4 shows a lighting system formed by three lamps. The system will work only if lamp a works and either lamp b or lamp c works.

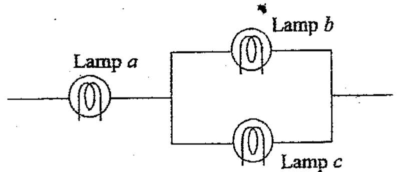

Figure 4

12. **(continued)**

(i) Suppose all the lamps in the system are brand X CFLs.

(I) Find the probability that the lifetime of the lighting system is more than 8 200 hours.

(II) It is known that the lifetime of the lighting system is less than 8 200 hours. Find the probability that only the lifetime of lamp a is less than 8 200 hours.

(ii) Suppose the lighting system is formed by 2 brand X and 1 brand Y CFLs. In order for the system to have a better chance of having a lifetime of more than 8 200 hours, where would you put the brand Y CFL in the system? Explain.

(9 marks)

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13. ***You may use the probabilities listed in Table 2 to answer this question.***

A salesman is promoting a new fertilizer which will improve the growth of potatoes. He claims that using the fertilizer, farmers will produce 65% of Grade A and 35% of Grade B potatoes (referred as *the claim* below). A farmer uses the fertilizer on his potatoes. In order to test the effectiveness of the fertilizer, he randomly selects 8 potatoes as a sample for testing.

(a) If *the claim* is valid, find the probability that there is at most 1 Grade A potato in the sample.

(2 marks)

(b) The farmer will reject *the claim* if there are not more than 3 Grade A potatoes in the sample.

(i) If *the claim* is valid, find the probability that the farmer will reject *the claim*.

(ii) If the fertilizer can only produce 20% Grade A and 80% Grade B potatoes, find the probability that the farmer will ***not*** reject *the claim*.

(5 marks)

(c) The farmer's wife takes 3 independent samples of 8 potatoes each to check *the claim*. She will reject *the claim* if not more than 3 Grade A potatoes are found in 2 or more of the 3 samples. If *the claim* is valid, find the probability that the farmer's wife will reject *the claim*.

(4 marks)

(d) Suppose *the claim* is valid. By comparing the methods described in (b) and (c), determine who, the farmer or his wife, will have a bigger chance of rejecting *the claim* wrongly.

(1 mark)

(e) The farmer's son will reject *the claim* if there are not more than k Grade A potatoes in a sample of 8 potatoes. Find the greatest value of k such that the probability of rejecting *the claim* is less than 0.05 given that *the claim* is valid.

(3 marks)

Table 2 Probabilities of two binomial distributions

Number of success	Probability*	
	B(8, 0.65)	B(8, 0.2)
0	0.0002	0.1678
1	0.0033	0.3355
2	0.0217	0.2936
3	0.0808	0.1468
4	0.1875	0.0459
5	0.2786	0.0092
6	0.2587	0.0011
7	0.1373	0.0001
8	0.0319	0.0000

* Correct to 4 decimal places.

END OF PAPER