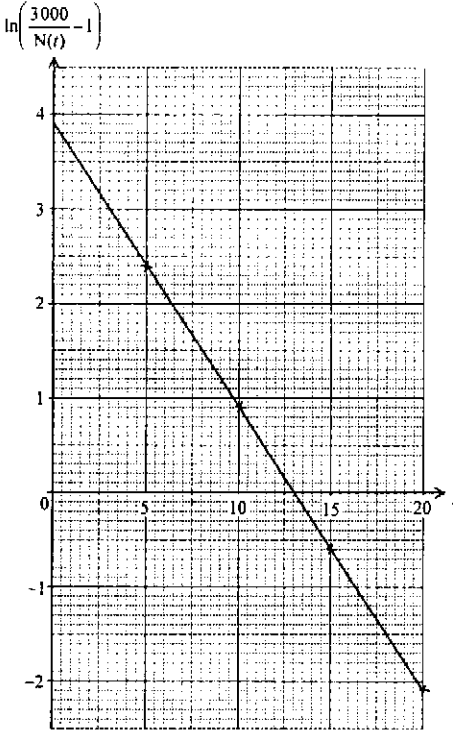


Solution	Marks	Remarks
1. (a) When $x = 1$, $e^y = \frac{2^1}{2} = 4$ $y = \ln 4$ (or $y = 2 \ln 2$) (or $y = 1.3863$)	1A	a-1 for r.t. 1.386
(b) $\therefore e^{xy} = \frac{x(x+1)^3}{x^2+1}$ $xy = \ln \left(\frac{x(x+1)^3}{x^2+1} \right)$ $xy = \ln x + 3 \ln(x+1) - \ln(x^2+1)$ $x \frac{dy}{dx} + y = \frac{1}{x} + \frac{3}{x+1} - \frac{2x}{x^2+1}$ When $x = 1$, $\frac{dy}{dx} + \ln 4 = 1 + \frac{3}{2} - 1$ $\frac{dy}{dx} = \frac{3}{2} - \ln 4$ (or 0.1137)	1A 1M+1M 1A	taking log on both sides (one side must correct) 1M for product rule 1M for differentiating log a-1 for r.t. 0.114
Alternatively, $e^{xy} \left(x \frac{dy}{dx} + y \right) = \frac{(x^2+1)[(x+1)^3 + 3(x+1)^2 x] - x(x+1)^3(2x)}{(x^2+1)^2}$ When $x = 1$, $e^{xy} \left(\frac{dy}{dx} + \ln 4 \right) = 6$ $\frac{dy}{dx} = \frac{3}{2} - \ln 4$	1M+1M+1A 1A	1M for differentiating e^{xy} 1M for product/quotient rule
(5)		
2. (a) $e^{-2x} = 1 - 2x + \frac{(-2x)^2}{2!} + \frac{(-2x)^3}{3!} + \dots$ $= 1 - 2x + 2x^2 - \frac{4x^3}{3} + \dots$	1A	
(b) $(1+x)^{\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)x + \left(\frac{1}{2!}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)x^2 + \left(\frac{1}{3!}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)x^3 + \dots$ $= 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \dots$	1M 1A	for any 3 terms
$\frac{(1+x)^{\frac{1}{2}}}{e^{2x}} = \left(1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} + \dots\right) \left(1 - 2x + 2x^2 - \frac{4x^3}{3} + \dots\right)$ $= 1 - \frac{3}{2}x + \frac{7}{8}x^2 - \frac{1}{48}x^3 + \dots$	1M 1A	applying the result in (a)
The expansion is valid for $ x < 1$. (or $-1 < x < 1$)	1A	pp-1 for missing '+...' in all cases
(6)		

Solution	Marks	Remarks
3. (a)	1A 1A 1A	for a box-and-whisker diagram for a correct box-and-whisker diagram with scale for all being correct
OR		
(b) Their chances of having a reaction time shorter than 1.1 seconds are equal. This is because (i) the median reaction times for both the boys and girls are 1.1 sec. or (ii) both the probabilities of having a reaction time being less than 1.1 sec for the boy and the girl are 0.5.	1A	
(5)		
4. (a) $N = \int (8t^{\frac{1}{3}} + 11t^{\frac{5}{6}}) dt$ $= 6t^{\frac{4}{3}} + 6t^{\frac{11}{6}} + c$ for some constant c . $\therefore N = 100$ when $t = 1$ $\therefore 100 = 6 + 6 + c \Rightarrow c = 88$ i.e. $N = 6t^{\frac{4}{3}} + 6t^{\frac{11}{6}} + 88$	1M+1A 1A 1A	1M for integration pp-1 for missing N or dt pp-1 for missing c
(b) When $t = 64$, $N = 6(64)^{\frac{4}{3}} + 6(64)^{\frac{11}{6}} + 88$ $= 13912$	1A	
(5)		

Solution	Marks	Remarks										
5. Let X be the no. of passengers using Octopus in a compartment.												
(a) $P(X=5) = C_5^{10}(0.6)^5(1-0.6)^5$ ≈ 0.200658 ≈ 0.2007 (p_1)	1A											
(b) $E(X) = np = 10 \times 0.6 = 6$ The mean number of passengers using Octopus in a compartment is 6.	1A+1A	$a-1$ for r.t. 0.201										
(c) The probability that the third compartment is the first one to have exactly 5 passengers using Octopus $= (1-0.200658)^2(0.200658)$ ≈ 0.1282	1M <u>1A</u> (6)	$(1-p_1)^2 p_1$ $a-1$ for r.t. 0.128										
6. (a) The number of different groups can be formed $= 5 \times 4 \times C_{10}^{10}$ (or $P_2^5 \times C_{10}^{10}$) $= 160160$	1M+1A	1M for P_2^5 or C_{10}^{10}										
(b) The possible numbers of boys are 10, 11, 12, 13, 14, 15, 16.	1A											
(c) Using (b) and the method of "try and error":												
<table border="1"> <thead> <tr> <th>Number of boys among the students</th> <th>Probability of having a time keeping group with all the time keepers being boys</th> </tr> </thead> <tbody> <tr> <td>10</td> <td>$\frac{C_{10}^{10}}{C_{10}^{16}} = \frac{1}{8008}$</td> </tr> <tr> <td>11</td> <td>$\frac{C_{10}^{11}}{C_{10}^{16}} = \frac{1}{728}$</td> </tr> <tr> <td>12</td> <td>$\frac{C_{10}^{12}}{C_{10}^{16}} = \frac{3}{364}$</td> </tr> <tr> <td>⋮</td> <td></td> </tr> </tbody> </table>	Number of boys among the students	Probability of having a time keeping group with all the time keepers being boys	10	$\frac{C_{10}^{10}}{C_{10}^{16}} = \frac{1}{8008}$	11	$\frac{C_{10}^{11}}{C_{10}^{16}} = \frac{1}{728}$	12	$\frac{C_{10}^{12}}{C_{10}^{16}} = \frac{3}{364}$	⋮		1M	for $\frac{C_{10}^n}{C_{10}^{16}}$
Number of boys among the students	Probability of having a time keeping group with all the time keepers being boys											
10	$\frac{C_{10}^{10}}{C_{10}^{16}} = \frac{1}{8008}$											
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12	$\frac{C_{10}^{12}}{C_{10}^{16}} = \frac{3}{364}$											
⋮												
\therefore There are 12 boys among the students.	<u>1A</u> (6)											

Solution	Marks	Remarks
7. Let E_X be the event that cable X is operative, E_Y be the event that cable Y is operative, E_Z be the event that cable Z is operative, and F be the event that A and B are not able to make contact.		
(a) (i) $P(E_X' \cap E_Z') = (0.015)(0.030)$ $= 0.00045$ (p_1)	1A	$a-1$ for r.t. 0.0005 (method must be shown)
(ii) $P(E_X' \cap E_Y' \cap E_Z') = (0.015)(0.025)(0.030)$ $= 0.00001125$ (p_2)	1A	$a-1$ for r.t. 0.0000 (method must be shown)
(iii) $P(F) = P(E_X' \cap E_Z') + P(E_Y' \cap E_Z') - P(E_X' \cap E_Y' \cap E_Z')$ $= 0.00045 + (0.025)(0.030) - 0.00001125$ $= 0.00118875$ ≈ 0.001189 (p_3)	1M 1A	$p_1 + (0.025)(0.030) - p_2$ r.t. 0.001189 $a-1$ for r.t. 0.0012 (method must be shown)
(b) $P(F E_X') = P(E_Z')$ $= 0.030$ (p_4)	1A	or 0.03
(c) $P(E_X' F) = \frac{P(E_X')P(F E_X')}{P(F)}$ $= \frac{(0.015)(0.030)}{0.00118875}$ ≈ 0.3785489 ≈ 0.3785	1M <u>1A</u> (7)	$\frac{(0.015)p_4}{p_3}$ r.t. 0.3785

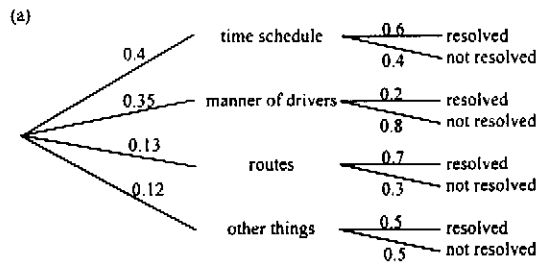
Solution	Marks	Remarks															
9. (a) (i) $N(t) = \frac{3000}{1 + ae^{-bt}} \Rightarrow \frac{3000}{N(t)} - 1 = ae^{-bt}$ $\Rightarrow \ln\left(\frac{3000}{N(t)} - 1\right) = -bt + \ln a$	1A	$pp-1$ for $-bt \ln e + \ln a$															
(ii) <table border="1" style="display: inline-table; vertical-align: middle;"> <thead> <tr> <th>t</th> <th>5</th> <th>10</th> <th>15</th> <th>20</th> </tr> </thead> <tbody> <tr> <td>$\ln\left(\frac{3000}{N(t)} - 1\right)$</td> <td>2.40</td> <td>0.90</td> <td>-0.60</td> <td>-2.09</td> </tr> <tr> <td></td> <td>(2.4)</td> <td>(0.9)</td> <td>(-0.6)</td> <td>(-2.1)</td> </tr> </tbody> </table>	t	5	10	15	20	$\ln\left(\frac{3000}{N(t)} - 1\right)$	2.40	0.90	-0.60	-2.09		(2.4)	(0.9)	(-0.6)	(-2.1)	1A	Correct to 1 d.p.
t	5	10	15	20													
$\ln\left(\frac{3000}{N(t)} - 1\right)$	2.40	0.90	-0.60	-2.09													
	(2.4)	(0.9)	(-0.6)	(-2.1)													
	1A	the line must pass through all the 4 points															
From the graph, $\ln a \approx 3.9$ $a \approx 49.4$ $b \approx \frac{-2.09 - 2.40}{20 - 5} \approx 0.3$	1A 1A	accept 3.85 – 3.95 accept 47.0 – 51.9															

Solution	Marks	Remarks
(b) (i) $N(t) = \frac{3000}{1 + ae^{-bt}}$ (or $\frac{3000}{1 + 49.4e^{-0.3t}}$) $N'(t) = \frac{3000abe^{-bt}}{(1 + ae^{-bt})^2}$ $= \frac{3000(49.4)(0.3)e^{-0.3t}}{(1 + 49.4e^{-0.3t})^2}$ (or $\frac{44460e^{-0.3t}}{(1 + 49.4e^{-0.3t})^2}$) $\therefore N'(t) > 0$ for all t $N(t)$ is increasing	1M 1A 1	accept $a \in [47.0, 51.9]$ and $3000ab \in [42300, 46710]$
(ii) If $N'(t) = \frac{1}{100}N(t)$ $\frac{3000abe^{-bt}}{(1 + ae^{-bt})^2} = \frac{1}{100} \frac{3000}{1 + ae^{-bt}}$ $e^{-bt} = \frac{1}{a(100b - 1)}$ $t = \frac{1}{0.3} \ln[a(100b - 1)]$	1M	$a \in [47.0, 51.9]$, $b = 0.3$ $a \in [47.0, 51.9]$, $b = 0.3$
OR $\frac{44460e^{-0.3t}}{(1 + 49.4e^{-0.3t})^2} = \frac{1}{100} \frac{3000}{1 + 49.4e^{-0.3t}}$ $1482e^{-0.3t} = 1 + 49.4e^{-0.3t}$ $t \approx 24.2242$	1M 1A	$t \in [24.0581, 24.3887]$
$\therefore N\left(\frac{1}{0.3} \ln[a(100b - 1)]\right) = \frac{3000}{1 + ae^{-b \frac{1}{0.3} \ln[a(100b - 1)]}} = 2900$	1M	
OR $N(24.2242) = \frac{3000}{1 + 49.4e^{-0.3(24.2242)}} \approx 2900$ \therefore The greatest number of migrants found at Mai Po is 2900.	1M 1A	
(iii) Suppose all the migrants leave Mai Po in x days. Then $\int_0^x 60\sqrt{s} ds = 2900$ $\left[40s^{\frac{3}{2}}\right]_0^x = 2900$ $x \approx 17.3870$ \therefore The number of days in which we can see the migrants is $24.2242 + 17.3870 \approx 42$	1M 1A 1A	for integration (including limits) r.t. 42

Solution	Marks	Remarks
10. Let X be the score on the questionnaire.		
(a) (i) $P(\text{classify as non-PD} \text{PD})$ $= P(X < 75 X \sim N(80, 5^2))$ $= P(Z < \frac{75-80}{5})$ $= P(Z < -1)$ $\approx 0.5 - 0.3413$ $= 0.1587$	1A	
(ii) $P(\text{classify as PD} \text{non-PD})$ $= P(X > 75 X \sim N(65, 5^2))$ $= P(Z > \frac{75-65}{5})$ $= P(Z > 2)$ $\approx 0.5 - 0.4772$ $= 0.0228$	1A	
(b) The probability that out of 10 PDs, not more than 2 will be misclassified $\approx (1 - 0.1587)^{10} + C_1^{10}(0.1587)(1 - 0.1587)^9 + C_2^{10}(0.1587)^2(1 - 0.1587)^8$ ≈ 0.7971	1M+1M 1A	1M for 2 nd or 3 rd term 1M for all
(c) Let x_0 be the required critical level of score. $P(X < x_0 X \sim N(80, 5^2)) = 0.01$ $P(Z < \frac{x_0 - 80}{5}) = 0.01$ $\frac{x_0 - 80}{5} \approx -2.3267$ $x_0 \approx 68.3665$	1A 1M 1A	accept -2.325 to -2.33, for the case ' $Z < \dots$ ' only accept 68.35 to 68.375
(d) If a teenager is classified by the sociologist, then $P(\text{classify as PD} \text{non-PD})$ $= P(X > 68.3665 X \sim N(65, 5^2))$ $= P(Z > 0.6733)$ $\approx 0.5 - 0.2496$ $= 0.2504$ $\therefore P(\text{misclassified}) \approx (0.01)(0.1) + (0.2504)(0.9)$ ≈ 0.2264	1M 1A	accept 68.35 to 68.375 accept 0.67 to 0.675
If a teenager is classified by the criminologist, then $P(\text{misclassified}) \approx (0.1587)(0.1) + (0.0228)(0.9)$ ≈ 0.0364	1A	accept 0.2498 to 0.2514 for either accept 0.2258 to 0.2273
$\therefore 0.2264 > 0.0364$ \therefore The probability of teenagers misclassified by the sociologist is greater than that by the criminologist.	1	

Solution	Marks	Remarks
11. (a) $f(x) = \frac{6(2-x) + (6x-4)}{(2-x)^2} = \frac{8}{(2-x)^2} > 0$ for $x \neq 2$	1	
(b) $\therefore \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \frac{6x-4}{2-x} = \infty$ and $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} \frac{6x-4}{2-x} = -\infty$ $\therefore x = 2$ is a vertical asymptote to C_1 .	1A	
$\therefore \lim_{x \rightarrow \pm\infty} f(x) = \lim_{x \rightarrow \pm\infty} \frac{6x-4}{2-x} = -6$ $\therefore y = -6$ is a horizontal asymptote to C_1 .	1A	
(c) $\therefore f(-2) = g(-2)$ and $f(1) = g(1)$ $\therefore \begin{cases} -4 = a \left(\frac{e^0 - 1}{e^{-2}} \right) + b \\ 2 = a \left(\frac{e^3 - 1}{e} \right) + b \end{cases}$ $\begin{cases} a = \frac{6e}{e^3 - 1} \\ b = -4 \end{cases}$	1A+1A	
(d)	1A 1A 1A	points of intersection intercepts shape and asymptotes

Solution	Marks	Remarks
(e) $g(x) = \frac{6e}{e^3-1} \left(\frac{e^{x+2}-1}{e^x} \right) - 4$		
$g'(x) = \frac{6e}{e^3-1} \left(\frac{e^x e^{x+2} - (e^{x+2}-1)e^x}{e^{2x}} \right) = \frac{6e}{e^3-1} e^{-x}$	1M	must be simplified
$\therefore g'(x) > 0$ and hence $g(x)$ is (strictly) increasing for all values of x .	1M	The 2 method marks can be awarded only when all calculations and arguments are correct except the constant a in $g(x)$.
For $x < -3$, $f(x) > -6$ but $g(x) < -6$.		
For $x > 8$, $f(x) < -6$ but $g(x) > -6$.		
Thus C_1 and C_2 has no point of intersection beyond the range $-3 \leq x \leq 8$.	1	
(f) Area of the region bounded by C_1 and C_2		
$= \int_{-2}^1 (g(x) - f(x)) dx$	1M	
$= \int_{-2}^1 \left[\frac{6e}{e^3-1} \left(\frac{e^{x+2}-1}{e^x} \right) - 4 - \frac{6x-4}{2-x} \right] dx$		
$= \frac{6e}{e^3-1} \int_{-2}^1 (e^2 - e^{-x}) dx - \int_{-2}^1 4 dx - \int_{-2}^1 \left(-6 + \frac{8}{2-x} \right) dx$	1A for $\int (e^2 - e^{-x}) dx$	
$= \frac{6e}{e^3-1} [e^2 x + e^{-x}]_{-2}^1 + [2x]_{-2}^1 + 8[\ln(2-x)]_{-2}^1$	1A+1A	1A for $\int \frac{6x-4}{2-x} dx$
$\approx 12.94312254 + 6 - 11.09035489$		
≈ 7.8528	1A	$a-1$ for r.t. 7.853

Solution	Marks	Remarks
12. Let N be the number of complaints received on a given day and X be the number of complaints involving the time schedule.		
(a) 		
$P(\text{manner of drivers} \text{not resolved})$		
$= \frac{0.35 \times 0.8}{0.4 \times 0.4 + 0.35 \times 0.8 + 0.13 \times 0.3 + 0.12 \times 0.5} \quad \left(\frac{p_1}{p_2} \right)$	1M+1A+1A	1A for p_1 , 1A for p_2
≈ 0.5195	1A	1M for $\frac{p_1}{p_2}$
(b) (i) $P(N=5) = \frac{10^5 e^{-10}}{5!}$	1A	$a-1$ for r.t. 0.519
$\approx 0.0378 \quad (p_3)$	1A	
(ii) $P(N=5 \text{ and } X=3) = \frac{10^5 e^{-10}}{5!} (C_3^5 (0.4)^3 (0.6)^2)$	1M	$p_3 (C_3^5 (0.4)^3 (0.6)^2)$
≈ 0.0087	1A	$a-1$ for r.t. 0.009
(c) $n \geq 9$. (or $P(N=n \text{ and } X=9) = 0$ for $n < 9$)	1M	
$P(N=n \text{ and } X=9) = \frac{10^n e^{-10}}{n!} C_9^n (0.4)^9 (0.6)^{n-9}$	1A	
(d) (i) $\sum_{k=9}^{\infty} \frac{x^k}{(k-9)!} = x^9 + \frac{x^{10}}{1!} + \frac{x^{11}}{2!} + \frac{x^{12}}{3!} + \dots$	1A	
$= x^9 \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \right)$	1	
$= x^9 e^x$		
(ii) $P(X=9) = \sum_{n=9}^{\infty} P(N=n \text{ and } X=9)$		
$= \sum_{n=9}^{\infty} \frac{10^n e^{-10}}{n!} C_9^n (0.4)^9 (0.6)^{n-9}$	1M	
$= \sum_{n=9}^{\infty} \frac{10^n e^{-10}}{n!} \frac{n!}{(n-9)!} (0.4)^9 (0.6)^{n-9}$		
$= \frac{e^{-10} (0.4)^9}{9! (0.6)^9} \sum_{n=9}^{\infty} \frac{6^n}{(n-9)!}$	1A	
$= \frac{e^{-10} (0.4)^9}{9! (0.6)^9} 6^9 e^6 \quad (\text{by (b)(i)})$		
$= \frac{4^9 e^{-4}}{9!} \quad (\text{or } 0.0132)$	1A	$a-1$ for r.t. 0.013

Solution		Marks	Remarks																															
13. (a)	Sample mean = 3	1A																																
(b)	<table border="1"> <thead> <tr> <th rowspan="2">Number of medicinal herbs</th> <th colspan="3">Expected frequency *</th> </tr> <tr> <th>Po(3)</th> <th>B(7, 3/7)</th> <th>Normal</th> </tr> </thead> <tbody> <tr> <td>3</td> <td></td> <td>29.38</td> <td></td> </tr> <tr> <td>4</td> <td></td> <td>22.03</td> <td></td> </tr> <tr> <td>5</td> <td>10.08</td> <td></td> <td>11.73 (or 11.87)</td> </tr> <tr> <td>6</td> <td>5.04</td> <td></td> <td>5.32 (or 5.23)</td> </tr> <tr> <td>7</td> <td></td> <td></td> <td></td> </tr> <tr> <td>8</td> <td></td> <td>0</td> <td></td> </tr> </tbody> </table>	Number of medicinal herbs	Expected frequency *			Po(3)	B(7, 3/7)	Normal	3		29.38		4		22.03		5	10.08		11.73 (or 11.87)	6	5.04		5.32 (or 5.23)	7				8		0		1A+1A 1A+1A 1A+1A	marks in (b) can be awarded independent of (a) for 29.38 and 22.03 for 10.08 and 5.04 for 11.73 and 5.32
Number of medicinal herbs	Expected frequency *																																	
	Po(3)	B(7, 3/7)	Normal																															
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7																																		
8		0																																
(c)	The maximum error for Po(3) is less than 1 ($ 14.94 - 14 = 0.94$) while the maximum errors for B(7, 3/7) and the normal distributions are all greater than 1. \therefore The Poisson distribution is the best.	1	provided that entries in the Poisson column are correct																															
(d) (i)	Let p be the probability that there is no medicinal herb in the tea, then $p = e^{-3}$ (≈ 0.0498) The required probability = $(p)^3(1-p)$ $= (e^{-3})^3(1-e^{-3})$ ≈ 0.0001	1M 1M 1A	for Po(3) only $\alpha-1$ for r.t. 0.0001																															
(ii)	Let q be the probability that a cup of tea contains exactly 3 kinds of medicinal herbs, then $q = \frac{3^3 e^{-3}}{3!}$ ≈ 0.22404 (or 0.2240) The required probability = $1 - [(1-q)^{10} + C_1^{10} q(1-q)^9]$ ≈ 0.6924 (or 0.6923)	1M 1A	$\alpha-1$ for r.t. 0.692																															