SECTION A (40 marks)
Answer **ALL** questions in this section.
Write your answers in the AL(C)1 answer book.

1. It is given that $e^{xy} = \dfrac{x(x+1)^3}{x^2+1}$, where $x > 0$.

   (a) Find the value of y when $x = 1$.

   (b) Find the value of $\dfrac{dy}{dx}$ when $x = 1$.

(5 marks)

2. (a) Expand e^{-2x} in ascending powers of x as far as the term in x^3 .

   (b) Using (a), expand $\dfrac{(1+x)^{\frac{1}{2}}}{e^{2x}}$ in ascending powers of x as far as the term in x^3 .

   State the range of values of x for which the expansion is valid.

(6 marks)

3. A test was carried out to see how quickly a class of students reacted to a visual instruction to press a particular key when they played a computer game. Their reaction times, measured in tenths of a second, are recorded and the statistics for the whole class are summarized below.

	Lower quartile	Upper quartile	Median	Minimum	Maximum
Boys	8	14	11	5	17
Girls	9	16	11	7	21

   (a) Draw two box-and-whisker diagrams in your answer book comparing the reaction times of boys and girls.

   (b) Suppose a boy and a girl are randomly selected from the class. Which one will have a bigger chance of having a reaction time shorter than 1.1 seconds? Explain.

(5 marks)

4. The total number of visits N to a web site increases at a rate of

$$\frac{dN}{dt} = t^{\frac{1}{3}}(8 + 11t^{\frac{1}{2}}) \qquad (0 \le t \le 100),$$

   where t is the time in weeks since January 1, 1999. It is known that $N = 100$ when $t = 1$.

   (a) Express N in terms of t .

   (b) Find the total number of visits to the web site when $t = 64$.

(5 marks)

5. 60% of passengers who travel by train use Octopus. A certain train has 12 compartments and there are 10 passengers in each compartment.

   (a) What is the probability that exactly 5 of the passengers in a compartment use Octopus?

   (b) What is the mean number of passengers using Octopus in a compartment?

   (c) What is the probability that the third compartment is the first one to have exactly 5 passengers using Octopus?

(6 marks)

6. At a school sports day, the timekeeping group for running events consists of 1 chief judge, 1 referee and 10 timekeepers. The chief judge and the referee are chosen from 5 teachers while the 10 timekeepers are selected from 16 students.

   (a) How many different timekeeping groups can be formed?

   (b) If it is possible to have a timekeeping group with all the timekeepers being boys, what are the possible numbers of boys among the 16 students?

   (c) If the probability of having a timekeeping group with all the timekeepers being boys is $\dfrac{3}{364}$, find the number of boys among the 16 students.

(6 marks)

7. Three control towers A, B and C are in telecommunication contact by means of three cables X, Y and Z as shown in Figure 1. A and B remain in contact only if Z is operative or if both cables X and Y are operative. Cables X, Y and Z are subject to failure in any one day with probabilities 0.015, 0.025 and 0.030 respectively. Such failures occur independently.

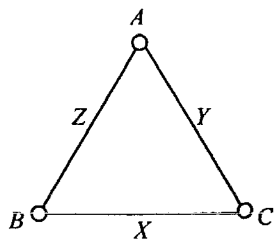

Figure 1

(a) Find, to 4 significant figures, the probability that, on a particular day,

(i) both cables X and Z fail to operate,

(ii) all cables X, Y and Z fail to operate,

(iii) A and B will not be able to make contact.

(b) Given that cable X fails to operate on a particular day, what is the probability that A and B are not able to make contact?

(c) Given that A and B are not able to make contact on a particular day, what is the probability that cable X has failed?

(7 marks)

SECTION B (60 marks)
Answer any **FOUR** questions in this section. Each question carries 15 marks.
Write your answers in the AL(C)2 answer book.

8. In a 100 m race, the speeds, S_A m/s and S_B m/s, of two athletes A and B respectively can be modelled by the functions

$$S_A = \frac{256}{9625}\left(\frac{1}{3}t^3 - \frac{47}{4}t^2 + 120t\right)$$

and $$S_B = \frac{183}{50}te^{-kt},$$

where k is a positive constant and t is the time measured from the start in seconds.

It is known that A finishes the race in 12.5 seconds and during the race, A and B attain their respective top speeds at the same time.

(a) Find the top speed of A during the race. (3 marks)

(b) Find the value of k. (3 marks)

(c) Suppose the model for B is valid for $0 \le t \le 12.5$. Use the trapezoidal rule with 5 sub-intervals to estimate the distance covered by B in 12.5 seconds. (3 marks)

(d) Find $\frac{\mathrm{d}^2 S_B}{\mathrm{d}t^2}$. Hence or otherwise, state with reasons whether B finishes the race ahead of A or not. (3 marks)

(e) In the same race, the speed, S_C m/s, of another athlete C is modelled by

$$S_C = \frac{50[\ln(t+2) - \ln 2]}{t+2}.$$

Determine whether or not C is the last one to finish the race among the three athletes. (3 marks)

9. An ecologist studies the birds at Mai Po Nature Reserve. Only 21% of the birds are "residents", i.e. found throughout the year. The remaining birds are migrants. The ecologist suggests that the number $N(t)$ of a certain species of migrants can be modelled by the function

$$N(t)=\frac{3000}{1+ae^{-bt}},$$

where a, b are positive constants and t is the number of days elapsed since the first one of that species of migrants was found at Mai Po in that year.

(a) This year, the ecologist obtained the following data:

t	5	10	15	20
$N(t)$	250	870	1 940	2 670

(i) Express $\ln\left(\frac{3000}{N(t)}-1\right)$ as a linear function of t.

(ii) Use the graph paper on Page 6 to estimate graphically the values of a and b correct to 1 decimal place.

(5 marks)

(b) Basing on previous observations, the migrants of that species start to leave Mai Po when the rate of change of $N(t)$ is equal to one hundredth of $N(t)$. Once they start to leave, the original model will not be valid and no more migrants will arrive. It is known that the migrants will leave at the rate $r(s)$ per day where $r(s)=60\sqrt{s}$ and s is the number of days elapsed since they started to leave Mai Po. Using the values of a and b obtained in (a)(ii),

(i) find $N'(t)$, and show that $N(t)$ is increasing;

(ii) find the greatest number of the migrants which can be found at Mai Po this year;

(iii) find the number of days in which the migrants can be found at Mai Po this year.

(10 marks)

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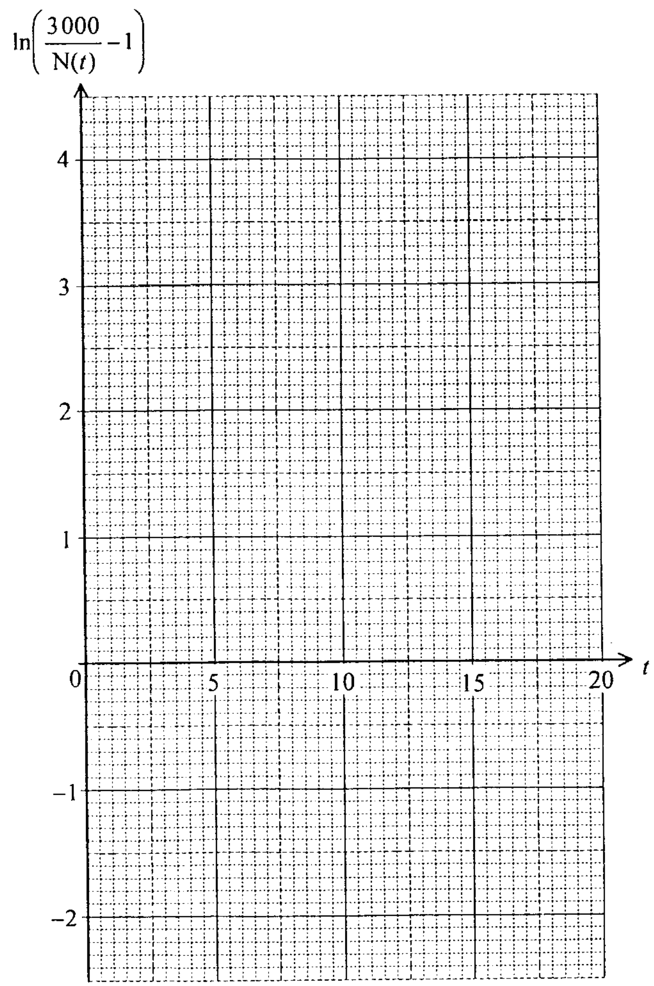
9.(Cont'd) **If you attempt Question 9, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**



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10. A criminologist has developed a questionnaire for predicting whether a teenager will become a delinquent. Scores on the questionnaire can range from 0 to 100, with higher values indicating a greater criminal tendency. The criminologist sets a **critical level** at 75, i.e., a teenager scores more than 75 will be classified as a potential delinquent (PD). Extensive studies have shown that the scores of those considered non-PDs follow a normal distribution with a mean of 65 and standard deviation of 5. The scores of those considered PDs follow a normal distribution with a mean of 80 and standard deviation of 5.

(a) Find the probability that

(i) a PD will be misclassified,

(ii) a non-PD will be misclassified.

(4 marks)

(b) What is the probability that out of 10 PDs, not more than 2 will be misclassified? (3 marks)

(c) If a sociologist wants to ensure that only 1 in 100 PDs should be misclassified, what **critical level** of score should be used?

(3 marks)

(d) It is known that 10% of all teenagers are PDs. Will the probability of teenagers misclassified by the sociologist in (c) be greater than that misclassified by the criminologist? Explain.

(5 marks)

11. Let $f(x)=\dfrac{6x-4}{2-x}$ $(x\neq 2)$ and

$g(x)=a\left(\dfrac{e^{x+2}-1}{e^{x}}\right)+b$, where a and b are constants.

Let C_1 and C_2 be the curves $y=f(x)$ and $y=g(x)$ respectively.
Figure 2 shows C_2 for $-3\leq x<8$.

(a) Show that $f'(x)>0$ for $x\neq 2$. (1 mark)

(b) Find the equations of the horizontal and vertical asymptotes to C_1 . (2 marks)

(c) It is given that $f(-2)=g(-2)$ and $f(1)=g(1)$. Find the exact values of a and b . (2 marks)

(d) Sketch C_1 on Figure 2 and indicate the asymptotes, intercepts and the points of intersection of the two curves. (3 marks)

(e) Find $g'(x)$. Hence explain briefly why there is no point of intersection of the two curves beyond the range $-3\leq x\leq 8$. (3 marks)

(f) Find the area of the region bounded by the two curves. (4 marks)

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11. (Cont'd) **If you attempt Question 11, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

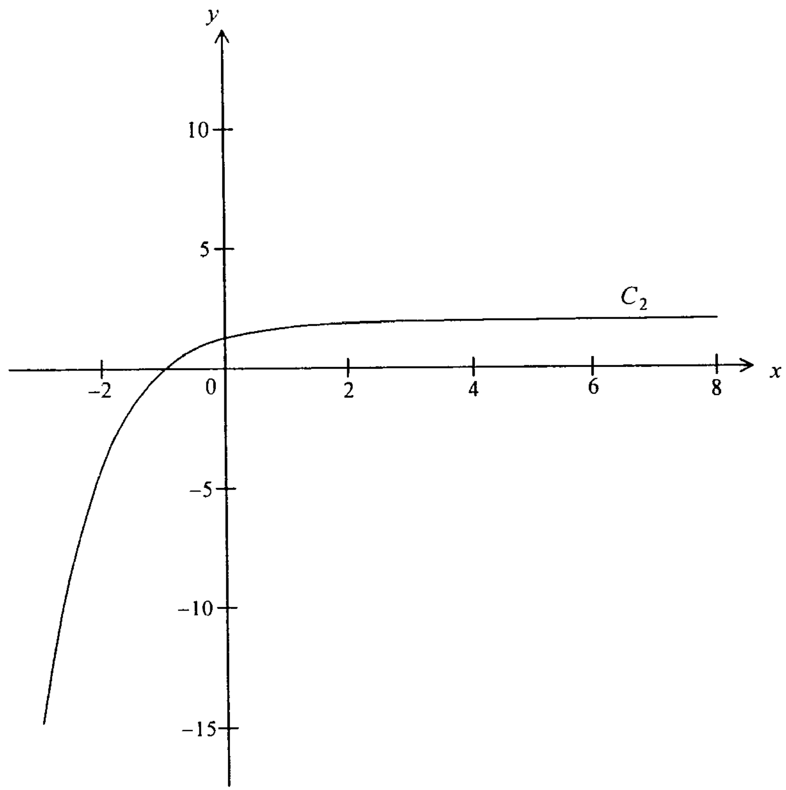

Figure 2

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12. A bus company finds that the number of complaints received per day follows a Poisson distribution with mean 10. 40% of the complaints involve the time schedule, 35% involve the manner of drivers, 13% involve the routes and 12% involve other things. These four kinds of complaints are mutually exclusive and can be resolved to the passenger's satisfaction with probabilities 0.6, 0.2, 0.7 and 0.5 respectively.

(a) If a complaint cannot be resolved to the passenger's satisfaction, find the probability that this complaint involves the manner of drivers.
(4 marks)

(b) Find the probability that on a given day,

(i) there are 5 complaints,

(ii) there are 5 complaints and 3 of them involve the time schedule.
(4 marks)

(c) Find the probability that on a given day, there are n complaints and 9 of them involve the time schedule. (2 marks)

(d) (i) Show that $\sum_{k=9}^{\infty} \frac{x^k}{(k-9)!} = x^9 e^x$.

(ii) Find the probability that, on a given day, there are 9 complaints involving the time schedule.
(5 marks)

13. A herbal tea for curing a certain disease is prepared and sold by many different shops. A researcher has collected the herbal tea from 100 randomly selected shops and counted the number of different kinds of medicinal herbs in the tea. The results are shown in the first two columns of Table 1.

(a) Find the sample mean number of different kinds of medicinal herbs in the herbal tea. (1 mark)

(b) The researcher tried to fit the data by a binomial ($n = 7$), a Poisson and a normal distribution. The sample mean was used as the mean of the distributions. Class intervals $(-0.5, 0.5]$, $(0.5, 1.5]$, … , $(7.5, 8.5]$ were used to calculate the expected frequencies under the normal distribution. Fill in the missing values in Table 1. (7 marks)

(c) Suppose the absolute values of the differences between observed and expected frequencies are regarded as errors. The distribution with the smallest maximum error is considered as the best fit. Which distribution is the best? (1 mark)

(d) A man drank a cup of this herbal tea bought from a randomly selected shop each day starting from the first day he felt sick. Find the probability, under the best distribution found in (c), that

(i) the fourth day is the first day that he actually got medicinal herbs in the tea;

(ii) in ten consecutive days he drank, at least two cups of this tea which contained exactly three kinds of medicinal herbs.

(6 marks)

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13. (Cont'd) **If you attempt Question 13, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

Table 1 Observed and expected frequencies of the number of medicinal herbs in a particular herbal tea

Number of medicinal herbs	Observed frequency	Expected frequency *		
		Poisson	Binomial	Normal
0	5	4.98	1.99	5.32
1	14	14.94	10.44	11.73
2	23	22.40	23.50	19.37
3	22	22.40		22.82
4	17	16.80		19.37
5	11		9.91	
6	5		2.48	
7	3	2.16	0.27	1.70
8	0	0.81		0.40
Total	100			

* Correct to 2 decimal places.

END OF PAPER