SECTION A (40 marks)
Answer **ALL** questions in this section.
Write your answers in the AL(C)1 answer book.

1. Let $y = xe^{\frac{1}{x}}$ where $x > 0$. Show that $x^4 \dfrac{d^2y}{dx^2} - y = 0$. (5 marks)

2. The binomial expansion of $(1 + ax)^{-4}$ in ascending powers of x is

   $1 + bx + cx^2 + 160x^3 + \cdots$, where a, b and c are constants.

   (a) Find the values of a, b and c.

   (b) State the range of values of x for which the expansion is valid.
   (6 marks)

3. 40 students participate in a 5-day summer camp. The stem-and-leaf diagram below shows the distribution of heights in cm of these students.

Stem (tens)	Leaf (units)
13	8
14	1 5 6 9
15	0 1 3 4 4 4 5 5 6 7 8 8 9
16	1 1 2 3 3 4 5 6 7 7 8 8
17	0 2 2 3 4 5 6 7
18	1 4

   (a) Find the median of the distribution of heights.

   (b) A student is to be selected randomly to hoist the school flag every day during the camp. Find the probability that

   (i) the fourth day will be the first time that a student taller than 170 cm will be selected,

   (ii) out of the 5 selected students, exactly 3 are taller than 170 cm.
   (6 marks)

4. A mobile phone company plans to invite a famous singer to help to promote its products. The Executive Director of the company estimates that the rate of increase of the number of customers can be modelled by

   $$\frac{dx}{dt} = 650e^{-0.004t} \qquad (0 \le t \le 365),$$

   where x is the number of customers of the company and t is the number of days which has elapsed since the start of the promotion campaign.

   (a) Suppose that at the start of the campaign, the company already has 57 000 customers. Express x in terms of t.

   (b) How many days after the start of the campaign will the number of customers be doubled?
   (6 marks)

5. John and Mary invite 8 friends to their Christmas party.

   (a) When playing a game, all of the 10 participants are arranged in a row. Find the number of arrangements that can be made if

   (i) there is no restriction,

   (ii) John and Mary are next to each other.

   (b) By the end of the party, the participants are arranged in 2 rows of 5 in order to take a photograph. Find the number of arrangements that can be made if

   (i) there is no restriction,

   (ii) John and Mary are next to each other.
   (6 marks)

6. A factory produces 3 kinds of ice-cream bars A, B and C in the ratio 1 : 2 : 5. It was reported that some ice-cream bars produced on 1 May, 1998 were contaminated. All ice-cream bars produced on that day were withdrawn from sale and a test was carried out. The test results showed that 0.8% of kind A, 0.2% of kind B and 0% of kind C were contaminated.

   (a) An ice-cream bar produced on that day is selected randomly. Find the probability that

      (i) the bar is of kind A and is NOT contaminated,

      (ii) the bar is NOT contaminated.

   (b) If an ice-cream bar produced on that day is contaminated, find the probability that it is of kind A.

   (6 marks)

7. 5 000 children are divided into 100 groups, each consisting of 50 children. The number of "over-weight" children are counted in each group and the numbers of groups having 0, 1, 2, ... "over-weight" children are recorded. The distributions, Poisson (λ) and Binomial (n, p), are respectively used to approximate the number of "over-weight" children in each group and some of the observed and expected frequencies are shown in the table below.

Observed and expected frequencies of the number of groups by number of "over-weight" children

Number of "over-weight" children	Observed frequency	Expected frequency *	
		Poisson (λ)	Binomial (n, p)
3	20	19.5	19.9
4	22	19.5	20.4
5	17	15.6	16.3

* Correct to 1 decimal place.

It is known that λ is an integer.

   (a) Find λ.

   (b) If the means of the two distributions are equal, find p.

   (5 marks)

SECTION B (60 marks)

Answer any **FOUR** questions in this section. Each question carries 15 marks.

Write your answers in the AL(C)2 answer book.

8. Mr. Lee has a fish farm in Sai Kung. Last week, the fish in his farm were affected by a certain disease. An expert told Mr. Lee that the number N of fish in his farm could be modelled by the function

$$N = \frac{5000\, e^{\lambda t}}{t} \qquad (0 < t < 120),$$

where λ is a constant and t is the number of days elapsed since the disease began to spread.

   (a) Suppose that the numbers of fish will be the same when $t = 15$ and $t = 95$.

      (i) Find the value of λ.

      (ii) How many days after the start of the spread of the disease will the number of fish decrease to the minimum?

   (8 marks)

   (b) The day that the number of fish decreased to the minimum is called the *Recovery Day*. It is suggested that from the *Recovery Day*, the fish will begin to gain weight according to the model

   $$\frac{dW}{ds} = \frac{3}{50}(e^{-\frac{s}{20}} - e^{-\frac{s}{10}}) \qquad (0 < s < 60),$$

   where s is the number of days elapsed since the *Recovery Day* and W is the mean weight of the fish in kg.

   Find the increase in mean weight of the fish in the first 15 days from the *Recovery Day*.

   How long will it take for the mean weight of the fish to increase 0.5 kg from the *Recovery Day*?

   (7 marks)

9. The curve in Figure 1 represents the graph of $y = ae^{-x} + bxe^{-x}$ for $x \geq 0$, where a and b are constants. The y-intercept of the curve is -3 and y attains its maximum when $x = \dfrac{3}{2}$.

Define $I = \displaystyle\int_{\frac{1}{2}}^{\frac{5}{2}} e^{-x} dx$ and $J = \displaystyle\int_{\frac{1}{2}}^{\frac{5}{2}} xe^{-x} dx$.

(a) Evaluate I. (2 marks)

(b) Find the values of a and b. (4 marks)

(c) Find the x-intercept and the coordinates of the point(s) of inflection of the curve. (4 marks)

(d) Let A be the area of the shaded region in Figure 1 bounded by the curve, the x-axis and the line $x = \dfrac{5}{2}$. Let J_0 be an estimate of J obtained by using the trapezoidal rule with 4 sub-intervals.

A student uses $A_0 = aI + bJ_0$ to estimate A.

(i) Find A_0.

(ii) The student made the following argument:

Since $\dfrac{d^2 y}{dx^2} < 0$ for $\dfrac{1}{2} < x < \dfrac{5}{2}$,
the curve is concave downward in the interval,
therefore J_0 is an underestimate of J,
and hence A_0 is an underestimate of A.

Determine whether the student's argument is correct or not. Explain your answer briefly.
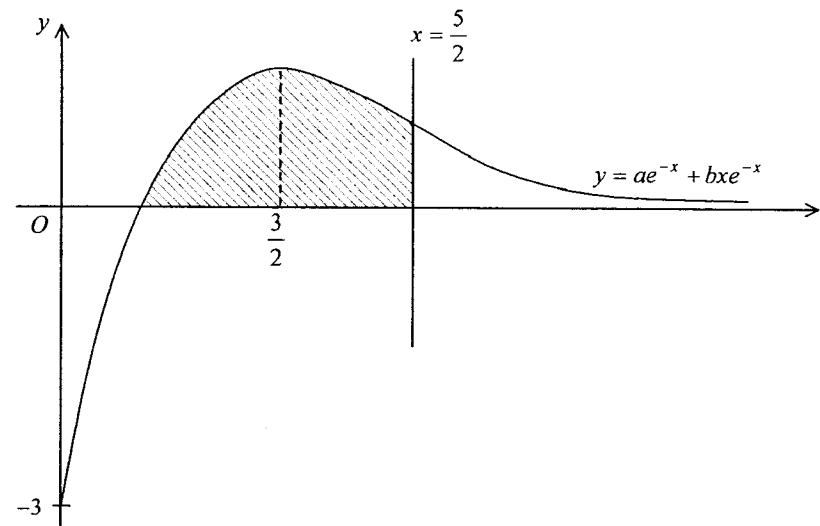
(5 marks)

Figure 1

10. A forest fire has started in a country. An official of the Department of Environmental Protection wants to estimate the number of trees destroyed in the fire when the fire is out of control. Let t be the number of days after the fire has started and $r(t)$, in hundred trees per day, be the rate of trees destroyed. The official obtained the following data:

t	2	3	4	5	6	7
$r(t)$	6.4	15.7	29.5	48.3	72.2	101.2

(a) It is suggested that $r(t)$ can be modelled by either one of the following functions

(I):　　$r(t) = \alpha\, t^{\beta}$　or

(II):　　$r(t) = \gamma\, e^{\lambda t}$,

where α, β, γ and λ are constants.

(i) Express $\ln r(t)$ in terms of $\ln t$ and t in (I) and (II) respectively.

(ii) Use the graph papers on Pages 8 and 9 to determine which function can better describe $r(t)$. Hence estimate graphically the two unknown constants in that function. Give your answers correct to 1 decimal place.

(10 marks)

(b) Assume the fire is out of control and the function in (a) which describes $r(t)$ better is used.

Estimate the total number, correct to the nearest hundred, of trees destroyed in the first 14 days of the fire. How many days more will it take for the total number of trees destroyed to be doubled?

(5 marks)

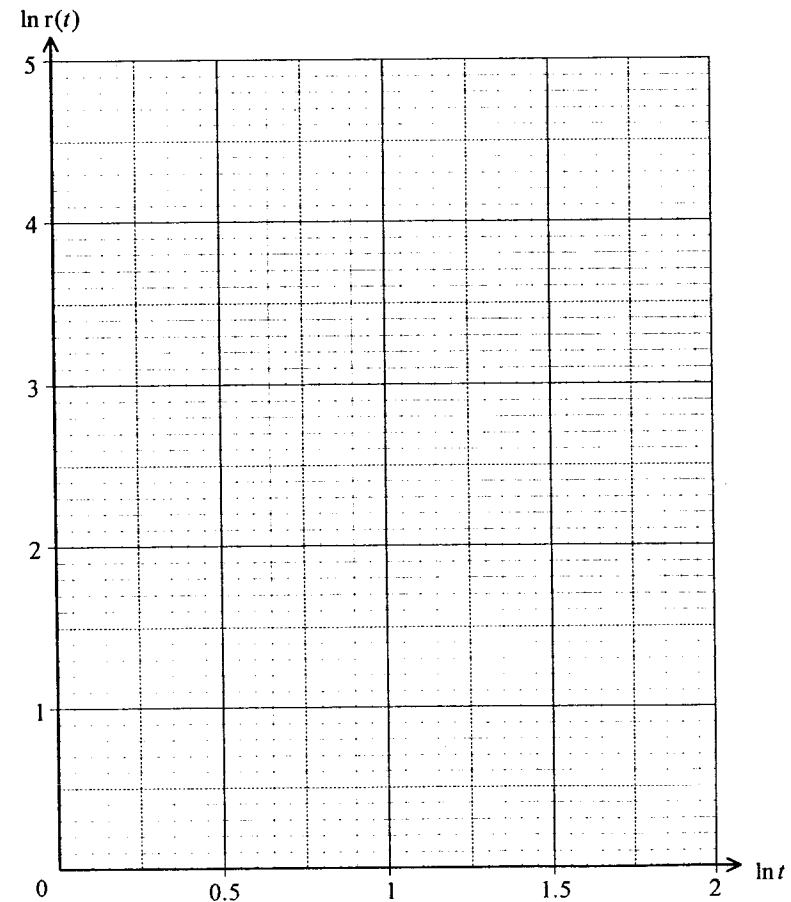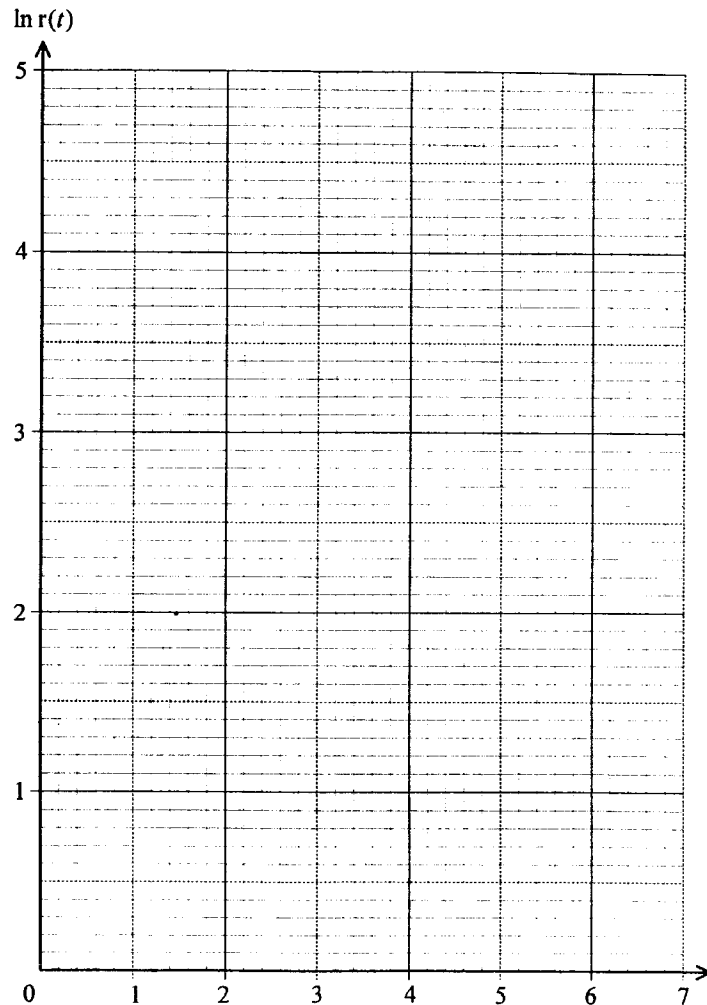10.(Cont'd)　If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.

Graph paper for function (I) in Question 10

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Graph paper for function (II) in Question 10

ln r(t)

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11.   Suppose that the number of printing mistakes on each page of a 200-page Mathematics book is independent of that on other pages, and it follows a Poisson distribution with mean 0.2 .

(a)   Find the probability that there is no printing mistake on page 23.
(2 marks)

(b)   Let page N be the first page which contains printing mistakes. Find

(i)   the probability that N is less than or equal to 3 ,

(ii)   the mean and variance of N .
(7 marks)

(c)   Let M be the number of pages which contain printing mistakes. Find the mean and variance of M .
(2 marks)

(d)   Suppose there is another 200-page Statistics book and there are 40 printing mistakes randomly and independently scattered through it. Let Y be the number of printing mistakes on page 23.

(i)   Which of the distributions – Bernoulli, binomial, geometric, Poisson or normal, does Y follow? Write down the parameter(s) of the distribution.

(ii)   Find the probability that there is no printing mistake on page 23.
(4 marks)

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12. A researcher wants to study the distribution of the number of cars that are speeding when they exit a tunnel in an hour. He has collected the data for 202 hours and the results are shown in Table 1. He suggests that the distribution can be modelled by a Poisson distribution.

(a) Use the data to calculate the sample mean and sample variance of the number of cars that are speeding in an hour. Do the results point to any objections to the use of a Poisson model? Explain. (3 marks)

(b) The researcher uses a Poisson distribution with mean being the sample mean calculated in (a) to fit the data. Some of his calculations are listed in Table 1. The model is acceptable if the maximum absolute discrepancy (see Table 1) is less than 1. Determine whether the model is acceptable by completing the table. (4 marks)

(c) Assume 40% of the cars which are speeding are private cars.

(i) Given that 2 cars are speeding, what is the probability that they are both private cars?

(ii) Given that 3 cars are speeding, what is the probability that 2 of them are private cars?

(3 marks)

(d) Assume the model in (b) is adopted and 40% of the cars which are speeding are private cars.

(i) Find the probability that in an hour, 2 cars are speeding and they are both private cars.

(ii) Find the probability that in an hour, 3 cars are speeding and 2 of them are private cars.

(iii) Given that in an hour, the total number of cars that are speeding is less than 4, find the probability that there are exactly 2 private cars speeding.

(5 marks)

12.(Cont'd) **If you attempt Question 12, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

Table 1   Observed and expected frequencies of number of hours by the number of cars speeding

Number of cars speeding	Observed frequency (f_O)	Expected frequency (f_E)*	Absolute discrepancy $\mid f_O - f_E \mid$ *
0	56	55.49	0.51
1	71	71.70	0.70
2	46		
3	20		
4	7	6.44	0.56
5	2	1.67	0.33
Total	202		

* Correct to 2 decimal places.

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13. The weight of each box of washing powder produced by a factory follows a normal distribution with mean 500 g and variance 25 g^2. The weights of boxes of washing powder are independent of each other. Every thirty minutes, a test consists of one or two parts will be performed as follows:

First part of the test
A randomly selected box of washing powder is weighed. If the weight of this box is greater than 510 g or less than 490 g, a black signal will be generated.

Second part of the test
(*Performed only when the weight of the box in the first part is greater than 508 g or less than 492 g and no black signal has been generated.*)
Another randomly selected box of washing powder is weighed.
  (I) A black signal will be generated if the weight of this box is greater than 510 g or less than 490 g.
  (II) A red signal will be generated if the weights of the two boxes in the first and second parts are **both between** 508 g and 510 g, or **both between** 490 g and 492 g.

(a) Find the probability that a black signal will be generated in the first part of a test. (2 marks)

(b) Find the probability that the second part has to be performed in a test. (3 marks)

(c) Find the probability that a black signal will be generated in a test. (3 marks)

(d) Given that the second part has to be performed in a test, find the probability that the weights of the two boxes selected are both between 508 g and 510 g. (3 marks)

(e) Given that the second part has to be performed in a test, find the probability that a red signal is generated. (2 marks)

(f) Find the probability that a red signal will be generated in a test. (2 marks)

END OF PAPER