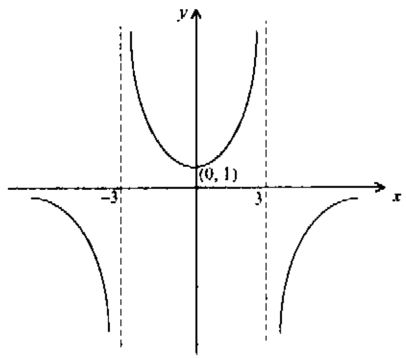
Solution	Marks	Remarks
1. (a) $(1+ax)^{\frac{1}{3}} = 1 + \frac{1}{3}ax + \frac{\frac{1}{3}(\frac{1}{3}-1)}{2\cdot 1}(ax)^2 + \ldots$	1A+1A	1A for the 1st & 2nd term 1A for the 3rd term
$= 1 + \frac{1}{3}ax - \frac{a^2}{9}x^2 + \ldots$	1A	
(b) $-\frac{a^2}{9} = -1$		
$a = 3$ or -3	1A	
	(4)	
2. (a) Range = (26 − 18) °C = 8 °C	1A	
(b) (i) Median = 21.5 °C		
$= [\frac{9}{5}(21.5) + 32]$ °F		
= 70.7 °F	1A	
Interquartile range = $\left\{\left[\frac{9}{5}(22.5)+32\right] - \left[\frac{9}{5}(20)+32\right]\right\}$ °F	1A	for 22.5−20
= 4.5 °F	1A	
(ii) Mean = $[\frac{9}{5}(22) + 32]$ °F = 71.6 °F	1A	
Standard deviation = $\frac{9}{5} \times 2$ °F = 3.6 °F	1A	
	(6)	
3. 	1A	for the 2 vertical asymtotes
	1A+1A+1A	for each part of the curves
	1A	for the local minimum
	(5)	

Solution	Marks	Remarks
4. (a) $V(t) = \int 200(t-15)dt$	1A	
$= 100t^2 - 3000t + c$	1A	
$\because V(0) = 20\,000$, $\therefore c = 20\,000$	1A	
Hence $V(t) = 100t^2 - 3000t + 20000$ for $0 \le t \le k$.		
(b) $\because V(k) = 0$		
$\therefore 100k^2 - 3000k + 20000 = 0$	1M	
$k^2 - 30k + 200 = 0$		
$(k-20)(k-10) = 0$		
$k = 10$ or 20 (rejected)	1A	
$k = 10$		
(c) $V(5) - V(0)$	1M	
$= 100(5)^2 - 3000(5) + 20000 - 20000$		
$= -12500$		
Alternatively, $\int_0^5 200(t-15)dt$	1M	
$= \left[100t^2 - 3000t\right]_0^5$		
$= -12500$		
$\therefore$ The total depreciation in the first 5 years is \$12 500.	1A	
	(7)	

Solution	Marks	Remarks
5. (a) Number of ways in which the 10 students can take the seats		
$= \frac{10!}{2!\,4!\,4!}$	1A	
$= 3150$	1A	
(b) Number of ways in which the 10 students can take the seats with the 2 students from school A are next to each other		
$= \frac{9!}{4!\,4!}$	1A	
$= 630$		
The probability that the 2 students from school A are next to each other		
$= \frac{630}{3150}$	1M	
$= \frac{1}{5}$	1A	
Alternatively, The probability that the 2 students from school A are next to each other		
$= 2 \cdot \frac{1}{10} \cdot \frac{1}{9} + 8 \cdot \frac{1}{10} \cdot \frac{2}{9}$ (or $\frac{9!2!}{10!}$)	2A	Marks can be awarded independent of part (a).
$= \frac{1}{5}$	1A	
	(5)	

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Solution	Marks	Remarks
6. (a) Let X be the number of cars passing through the auto-toll in a minute, then $X \sim Po(5)$.	1M	
$P(X > 5)$		
$= 1 - \sum_{x=0}^{5} \frac{5^x e^{-5}}{x!}$	1A	
≈ 0.3840	1A	a-1 for r.t. 0.384
(b) Out of the next 4 minutes, let Y be the number of minutes in which more than 5 cars will pass through the auto-toll, then $Y \sim B(4, 0.3840)$.	1M	
$P(Y = 3)$		
$\approx C_3^4(0.3840)^3(1-0.3840)$	1M	For binomial formula
$= 0.1395$ (or 0.1396)	1A	a-1 for r.t. 0.140
	(6)	

Solution	Marks	Remarks
7. Let A_1 be the event that the original motor breaks down, A_2 be the event that the backup motor breaks down and W be the even that the machine is working.		
(a) $P(A_1 A_2)$		
$= 0.15 \times 0.24$	1A	
$= 0.036$	1A	
(b) $P(W) = 1 - P(A_1 A_2)$		
$= 1 - 0.036$	1M	
$= 0.964$		
Alternatively, $P(W) = P(\overline{A_1}) + P(A_1 \overline{A_2})$		
$= 0.85 + 0.15 \times 0.76$	1M	
$= 0.964$		
The probability that the machine is operated by the original motor		
$= \frac{P(\overline{A_1})}{P(W)}$		
$= \frac{0.85}{0.964}$	1M	
≈ 0.8817	1A	a-1 for r.t. 0.882
(c) The prob. that the 1st break down of the machine occurs on the 10th day		
$= (0.036)(1-0.036)^{10-1}$	1M	
$= 0.0259$	1A	a-1 for r.t. 0.026
	(7)	

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Solution	Marks	Remarks
8. (a) $\because$ $N(0)=16$		
$\therefore$ $\dfrac{40}{1+b}=16$	1M	
$b=1.5$	1A	
$\because$ $N(7)=17.4$		
$\therefore$ $\dfrac{40}{1+1.5e^{-7r}}=17.4$	1M	
$e^{-7r}=\dfrac{1}{1.5}\left(\dfrac{40}{17.4}-1\right)$		
$r=\dfrac{1}{-7}\ln\left[\dfrac{1}{1.5}\left(\dfrac{40}{17.4}-1\right)\right]$	1M	
$=0.02$	1A	
(b) $N(t)=\dfrac{40}{1+be^{-rt}}$ (or $\dfrac{40}{1+1.5e^{-0.02t}}$)		
$N'(t)=\dfrac{-40(-bre^{-rt})}{(1+be^{-rt})^2}$ (or $\dfrac{-40(-1.5)(0.02)e^{-0.02t}}{(1+1.5e^{-0.02t})^2}$)	1M+1A	
$=\dfrac{40bre^{-0.02t}}{(1+be^{-rt})^2}$ (or $\dfrac{1.2e^{-0.02t}}{(1+1.5e^{-0.02t})^2}$)		
>0		
$\therefore$ $N(t)$ is increasing.	1	
(c) $\because$ $\lim\limits_{t\to\infty}e^{-rt}=0$		
$\therefore$ $N_o=\lim\limits_{t\to\infty}\dfrac{40}{1+be^{-rt}}$ (or $\lim\limits_{t\to\infty}\dfrac{40}{1+1.5e^{-0.02t}}$)	1M	
$=40$	1A	
(d) (i) $N''(t)$		
$=\dfrac{[(1+1.5e^{-0.02t})(1.2)-1.2e^{-0.02t}(2)(1.5)](1+1.5e^{-0.02t})(-0.02)e^{-0.02t}}{(1+1.5e^{-0.02t})^4}$	1M	
$=\dfrac{0.012e^{-0.02t}(3e^{-0.02t}-2)}{(1+1.5e^{-0.02t})^3}$	1A	
(ii) From (i), $N''(t)\begin{cases}>0 & \text{when } t<t_0\\ =0 & \text{when } t=t_0\\ <0 & \text{when } t>t_0\end{cases}$	1M	For Solving $N''(t)=0$
where $t_0=-\dfrac{1}{0.02}\ln\dfrac{2}{3}\approx 20.2733$		
$\therefore$ The rate of increase is the greatest when $t=t_0\approx 20.2733$	1M	For checking maximum
$\because$ $N'(20)\approx 0.199999$		
$N'(21)\approx 0.199989$		
$\therefore$ The company should start to advertise on the 20th day after the first week.	1A	

Solution	Marks	Remarks
9. (a) $b\approx\dfrac{7.49-7.95}{8-3.4}$		
$=-0.1$	1A	
Sub. (8, 7.49) into $\ln N(x)=-0.1x+\ln a$		
$7.49\approx\ln a-0.8$	1M	
$a\approx 4000$	1A	
(b) (i) $N(x)=ae^{bx}=4000e^{-0.1x}$	1M	
Daily profit (in dollars) of selling $N(x)$ clams:		
$P(x)=N(x)\cdot x-(2N(x)+5000)$	1A	for $2N(x)+5000$
$=(x-2)N(x)-5000$		
$=4000(x-2)e^{-0.1x}-5000$	1A	
(ii) $P'(x)=4000[(x-2)(-0.1e^{-0.1x})+e^{-0.1x}]$	1A	
$=400e^{-0.1x}(12-x)$		
$P'(x)\begin{cases}>0 & \text{if } 0<x<12\\ =0 & \text{if } x=12\\ <0 & \text{if } x>12\end{cases}$	1	
$\therefore$ $P(x)$ attains its maximum when $x=12$.		
Hence the selling price of each clam = \$12	1A	
the number of clams sold per day $=N(12)$		
$=4000e^{-0.1(12)}$		
≈ 1205	1A	
(c) The difference between the numbers of clams sold on the n-th and $(n-1)$-th days after the launch of the promotion programme		
$=M(n)-M(n-1)$		
$=[1500+1000(1-e^{-0.1n})]-[1500+1000(1-e^{-0.1(n-1)})]$	1M	
$=1000(-e^{-0.1n}+e^{-0.1n}\cdot e^{0.1})$		
$=1000e^{-0.1n}(e^{0.1}-1)$	1A	
If $M(n)-M(n-1)<15$	1M	
then $e^{-0.1n}<\dfrac{15}{1000(e^{0.1}-1)}$		
$n>19.475$	1M	
$\therefore$ The promotion programme should run for 20 days.	1A	

Solution	Marks	Remarks
10. (a) $y = x^x$		
$\ln y = x \ln x$	1A	
$\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$		
$\frac{dy}{dx} = x^x(1+\ln x)$	1A	
(b) $\frac{d^2y}{dx^2} = x^x \frac{d}{dx}(1+\ln x) + (1+\ln x)\frac{d}{dx}x^x$		
$= x^x \cdot \frac{1}{x} + (1+\ln x)x^x(1+\ln x)$	1A	
$= x^{x-1} + x^x(1+\ln x)^2$		
> 0 for $1 \le x \le 2$	1A	
y is concave upward (or convex) for $1 \le x \le 2$		
$\therefore$ I would be overestimated if the trapezoidal rule is used to estimate I.	1	
(c) $I + J = \int_1^2 x^x(1+\ln x)dx$	1A	
$= \left[x^x\right]_1^2$ by (a)		
$= 3$	1	

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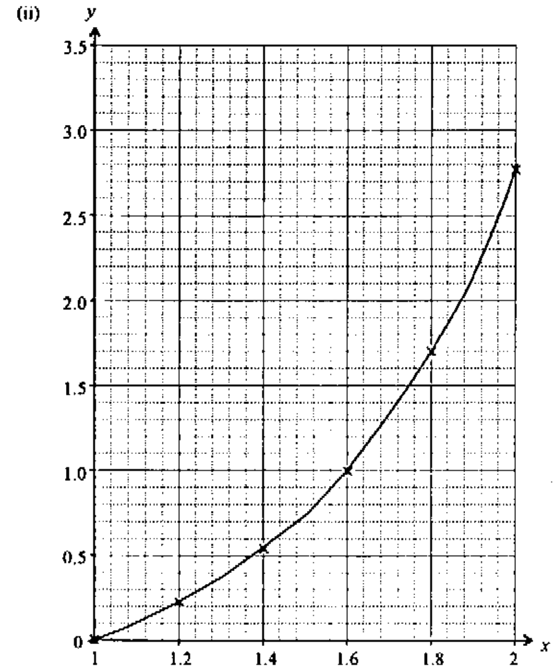
(d) (i)

x	1	1.2	1.4	1.6	1.8	2
$x^x \ln x$	0	0.22691	0.53893	0.99700	1.69321	2.77259

Solution	Marks	Remarks
(table above)	1A	
$J_0 \approx \frac{0.2}{2}[2.77259 + 2(0.22691 + 0.53893 + 0.99700 + 1.69321)]$	1M	
$= 0.9685$	1A	

(ii)



Solution	Marks	Remarks
(graph)	1A+1M	
From the plotted graph, $y = x^x \ln x$ is concave upward (or convex) for $1 \le x \le 2$.		
$\therefore$ J_0 is an overestimate of J.	1M	
(iii) The estimation can be improved by increasing the number of sub-intervals.	1	
(iv) I_0 is an underestimate of I because the value 3 for $I + J$ is exact and J_0 is an overestimate of J.	1	

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Solution	Marks	Remarks

11. (a) Let X be the number of FICs per day, then $X \sim Po(4)$.

$P(X=0) = \frac{4^0 e^{-4}}{0!}$ — 1M

≈ 0.0183 — 1A

(b) Let Y be the number of FICs which are related to house fires in 5 FICs, then $Y \sim B(5, 0.6)$.

$P(Y \geq 2) = 1 - P(Y=0) - P(Y=1)$

$= 1 - C_0^5(0.4)^5 - C_1^5(0.6)(0.4)^4$ — 1M+1A

≈ 0.9130 — 1A

(c) Let H and L be the events of "a FIC is related to a house fire" and "a FIC is *large*". Let A be the amount of a FIC.

(i) $P(L|H) = P(A > 20\,000)$

$= P(Z > \frac{200\,000 - 100\,000}{50\,000})$ — 1M

$= P(Z > 2)$

≈ 0.0228 — 1A

$P(L|\bar{H}) = P(A > 20\,000)$

$= P(Z > \frac{200\,000 - 150\,000}{20\,000})$

$= P(Z > 2.5)$

≈ 0.0062 — 1A

$P(L) = P(L|H)P(H) + P(L|\bar{H})P(\bar{H})$

$\approx 0.0228(0.6) + 0.0062(0.4)$ — 1M

≈ 0.0162 — 1A

(ii) $P(H|L) = \frac{P(L|H)\,P(H)}{P(L)}$

$\approx \frac{0.0228 \times 0.6}{0.0162}$ — 1M

≈ 0.8444 — 1A

(iii) P(5 FICs and at least 2 of them are *large*)

= P(2 or more out of 5 FICs are *large*)$P(X=5)$

$= [1-(1-0.0162)^5 - 5(0.0162)(1-0.0162)^4]\frac{e^{-4}4^5}{5!}$ — 1M+1A

≈ 0.0004 — 1A

Solution	Marks	Remarks

12. (a) & (b) Note: Under B(5, 0.4), expected freq $= 60 \times C_x^5 (0.4)^x (0.6)^{5-x}$.

Merit Points	Observed Frequency	Expected Frequency *	
		Binomial	Normal
0	4	4.67	4.00
1	14	15.55	14.50
2	23	20.74	22.98
3	15	13.82	14.50
4	4	4.61	3.64
5	0	0.61	0.37

* Correct to 2 decimal places.

1A+1A — For the 3rd column: 1A for any one being correct; 1A for the remaining two

1A+1A — For the 4th column: 1A for any one being correct; 1A for the remaining two

(b) $X \sim N(\mu, \sigma^2)$

$P(X < 9000) = \frac{4.00}{60}$

$P(Z < \frac{9000-\mu}{\sigma}) = \frac{4.00}{60}$ (≈ 0.06667)

$\frac{9000-\mu}{\sigma} \approx -1.50$ ....(1) — 1A

$P(X < 12000) = \frac{14.50 + 4.00}{60}$

$P(Z < \frac{12000-\mu}{\sigma}) = \frac{18.50}{60}$ (≈ 0.30833)

$\frac{12000-\mu}{\sigma} \approx -0.50$ ....(2) — 1A

Solving (1) and (2), we have $\mu \approx 13500$, — 1A

$\sigma \approx 3000$. — 1A

(c) $N(13500, 3000^2)$ is used to model the sales volumes.

(i) The probability that a salesman will get a warning

$= P(X=0)^3 + 3P(X=0)^2 P(X=1) + 3P(X=0)P(X=1)^2 + 3P(X=0)^2 P(X=2)$ — 1M

$= \left(\frac{4.00}{60}\right)^3 + 3\left(\frac{4.00}{60}\right)^2\left(\frac{14.50}{60}\right) + 3\left(\frac{4.00}{60}\right)\left(\frac{14.50}{60}\right)^2 + 3\left(\frac{4.00}{60}\right)^2\left(\frac{22.99}{60}\right)$ — 1A

≈ 0.0203 — 1A

(ii) The probability that a salesman got no merit points in at least 2 of the previous 3 months

$\approx 1 - \frac{3P(X=0)P(X=1)^2}{0.0203}$ — 1M

$\approx 1 - \frac{3\left(\frac{4.00}{60}\right)\left(\frac{14.50}{60}\right)^2}{0.0203}$ — 1A

≈ 0.4246

The number of salesmen who are expected to get no merit points in at least 2 of the previous 3 months

$\approx 10 \times 0.4246$ — 1M

≈ 4 — 1A

Solution	Marks	Remarks
13. Let L cm be the length of the front portion of Mr. Wong's necktie.		
(a) $P(44 < L < 45)$		
$= P(\frac{44-44.6}{1.2} < Z < \frac{45-44.6}{1.2})$	1M	
$\approx P(-0.5 < Z < 0.3333)$		
$\approx 0.1915 + 0.1293$ (or $0.1915 + 0.1306$)	1A	For either
≈ 0.3208 (or 0.3221)	1A	
(b) Let Y be the number of trials that Mr. Wong gets the first *perfect tying*, then $Y \sim \text{Geometric}(p)$, where		
$p \approx 0.3208$ (or 0.3221)		
$E(Y) = \frac{1}{p}$	1M	
≈ 3.1172 (or 3.1046)	1A	
(c) P(not more than 3 trials)		
= P(1 trial) + P(2 trials) + P(3 trials)		
$= p + p(1-p) + p(1-p)^2$	1M	or $1-(1-p)^3$
≈ 0.6867 (or 0.6885)	1A	
(d) Let T be the event that Mr. Wong has to go to work by taxi.		
(i) $P(T) \approx 1 - 0.6867$ (or $1 - 0.6885$)		
$= 0.3133$ (or 0.3115)		
P(less than 2 T out of 6 days)		
$\approx C_0^6(0.6867)^6 + C_1^6(0.6867)^5(0.3133)$	1M+1A	
(or $C_0^6(0.6885)^6 + C_1^6(0.6885)^5(0.3115)$)		
≈ 0.3919 (or 0.3957)	1A	
(ii) $P(Y=5 \mid T) \approx \frac{(1-p)^4 p}{P(T)}$	1M	
≈ 0.2179 (or 0.2184)	1A	
(iii) Probability required		
$\approx 5(0.3133)^2(0.6867)^4$ (or $5(0.3115)^2(0.6885)^4$)	1M+1A	
≈ 0.1091 (or 0.1090)	1A	