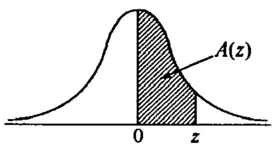
標準正態曲線下的面積

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

註： 本表所列數字，爲曲線下由 $z = 0$ 至正値 z 之間的面積所佔的比例。負値 z 所含的面積可利用對稱性求得。



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx$$

97-ASL
M&S

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG ADVANCED LEVEL EXAMINATION 1997

MATHEMATICS AND STATISTICS AS-LEVEL

9.00 am–12.00 noon (3 hours)

This paper must be answered in English

1. This paper consists of Section A and Section B.
2. Answer ALL questions in Section A, using the AL(C1) answer book.
3. Answer any FOUR questions in Section B, using the AL(C2) answer book.
4. Unless otherwise specified, numerical answers should either be exact or given to 4 decimal places.

SECTION A (40 marks)
Answer **ALL** questions in this section.
Write your answers in the AL(C1) answer book.

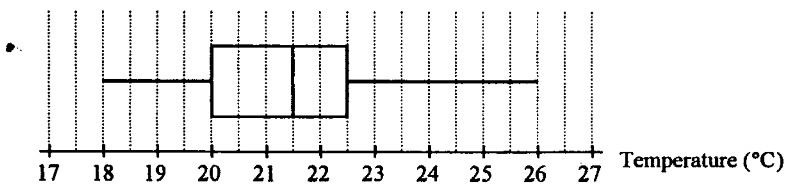
1. Let $|ax| < 1$.

(a) Expand $(1+ax)^{\frac{1}{3}}$ in ascending powers of x as far as the term in x^2 .

(b) If the coefficient of x^2 in the expansion of $(1+ax)^{\frac{1}{3}}$ is -1 , find all possible values of a .

(4 marks)

2. In an experiment, temperatures of a certain liquid under various experimental settings are measured. The box-and-whisker diagram for these temperatures (in °C) is constructed below:



(a) Find the range (in °C) of the temperatures.

(b) The temperature C (in °C) can be converted to the temperature F (in °F) according to the formula $F = \frac{9}{5}C + 32$.

(i) Find the median and interquartile range of the temperatures in °F .

(ii) If the mean and standard deviation of the temperatures are 22°C and 2°C respectively, find their values in °F .

(6 marks)

3. A rational function f(x) has the following properties:

(i) the horizontal asymptote of its graph is $y = 0$,

(ii) the vertical asymptotes of its graph are $x = -3$ and $x = 3$,

(iii)

	$x<-3$	$x=-3$	$-3<x<0$	$x=0$	$0<x<3$	$x=3$	$x>3$
f(x)				1			
f'(x)	<0		<0	0	>0		>0
f''(x)	<0		>0	>0	>0		<0

Using the information above, sketch the graph of f(x) .

(5 marks)

4. A machine depreciates with time t in years. Its value \$ V(t) is initially \$ 20 000 and will drop to \$ 0 when $t = k$ $(k \geq 0)$. The depreciation rate at time t is

$$V'(t) = 200(t-15) \qquad \text{for } 0 \leq t \leq k .$$

Find

(a) V(t) for $0 \leq t \leq k$,

(b) the value of k , and

(c) the total depreciation in the first 5 years.

(7 marks)

5. Ten seats are arranged in a row for 10 students from 3 different schools. There are 2 students from school A, 4 from school B and 4 from school C. Assume that students from the same school are indistinguishable.

(a) Find the number of ways in which these 10 students can take the seats.

(b) Find the probability that the 2 students from school A are sitting next to each other.

(5 marks)

6. On the average, 5 cars pass through an auto-toll every minute. Assuming that the cars pass through the auto-toll independently, find the probability that more than 5 cars will pass through the auto-toll

(a) in 1 minute,

(b) in any 3 of the next 4 minutes.

(6 marks)

7. A brewery has a backup motor for its bottling machine. The backup motor will be automatically turned on if the original motor breaks down during operating hours. The probability that the original motor breaks down during operating hours is 0.15 and when the backup motor is turned on, it has a probability of 0.24 of breaking down. Only when both the original and backup motors break down is the machine not able to work.

(a) What is the probability that the machine is not working during operating hours?

(b) If the machine is working, what is the probability that it is operated by the original motor?

(c) The machine is working today. Find the probability that the first break down of the machine occurs on the 10th day after today.

(7 marks)

SECTION B (60 marks)
Answer any **FOUR** questions in this section. Each question carries 15 marks.
Write your answers in the AL(C2) answer book.

8. A vehicle tunnel company wants to raise the tunnel fees. An expert predicts that after the increase in the tunnel fees, the number of vehicles passing through the tunnel each day will drop drastically in the first week and on the t-th day after the first week, the number $N(t)$ (in thousands) of vehicles passing through the tunnel can be modelled by

$$N(t) = \frac{40}{1+be^{-rt}} \qquad (t \geq 0),$$

where b and r are positive constants.

(a) Suppose that by the end of the first week after the increase in the tunnel fees, the number of vehicles passing through the tunnel each day drops to 16 thousand and by the end of the second week, the number increases to 17.4 thousand, find b and r correct to 2 decimal places.

(5 marks)

(b) Show that $N(t)$ is increasing.

(3 marks)

(c) As time passes, $N(t)$ will approach the average number N_a of vehicles passing through the tunnel each day before the increase in the tunnel fees. Find N_a .

(2 marks)

(d) The expert suggests that the company should start to advertise on the day when the rate of increase of the number of cars passing through the tunnel per day is the greatest. Using the values of b and r obtained in (a),

(i) find $N''(t)$, and

(ii) hence determine when the company should start to advertise.

(5 marks)

9. A stall sells clams only. The relationship between the selling price $\$x$ of each clam and the number $N(x)$ of clams sold per day can be modelled by

$$\ln N(x) = bx + \ln a \ ,$$

where a and b are constants. This relationship is represented by the straight line shown in Figure 1.

(a) Use the graph in Figure 1 to estimate the values of a and b correct to 1 significant figure.

(3 marks)

(b) Suppose the daily running cost of the stall is \$5 000 and the cost of each clam is \$2. Using the values of a and b estimated in (a),

(i) express the daily profit of selling $N(x)$ clams in terms of x, and

(ii) determine the selling price of each clam so that the daily profit of selling $N(x)$ clams will attain its maximum. What is then the number of clams sold per day? Give the answer correct to the nearest integer.

(7 marks)

(c) The stall has been running a promotion programme every day from April 15, 1997. The number $M(n)$ of clams sold on the n-th day of the programme is given by

$$M(n) = 1500 + 1000(1 - e^{-0.1n}) \ .$$

The stall will stop running the programme once the increase in the number of clams sold between two consecutive days falls below 15. Determine how many days the programme should be run. Give the answer correct to the nearest integer.

(5 marks)

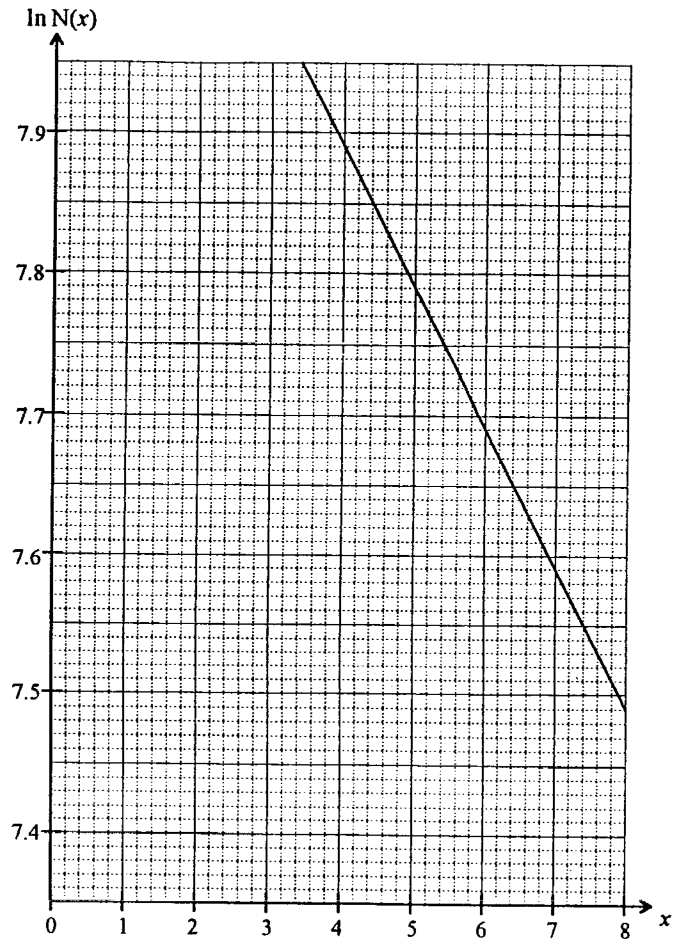


Figure 1

10. Let $y = x^x$, $I = \int_1^2 x^x \, dx$ and $J = \int_1^2 x^x \ln x \, dx$.

(a) Using logarithmic differentiation, find $\frac{dy}{dx}$.

(2 marks)

(b) By finding $\frac{d^2y}{dx^2}$, state whether I would be overestimated or underestimated if the trapezoidal rule is used to estimate I . Explain your answer briefly.

(3 marks)

(c) Using (a) or otherwise, show that $I + J = 3$.

(2 marks)

(d) Let J_0 be an estimate of J obtained by using the trapezoidal rule with 5 sub-intervals.

(i) Find J_0 .

(ii) Plot the graph of $y = x^x \ln x$ on the graph paper on Page 8. Hence state whether J_0 is an overestimate or underestimate of J . Explain your answer briefly.

(iii) How can the estimation be improved if the trapezoidal rule is applied again to estimate J ?

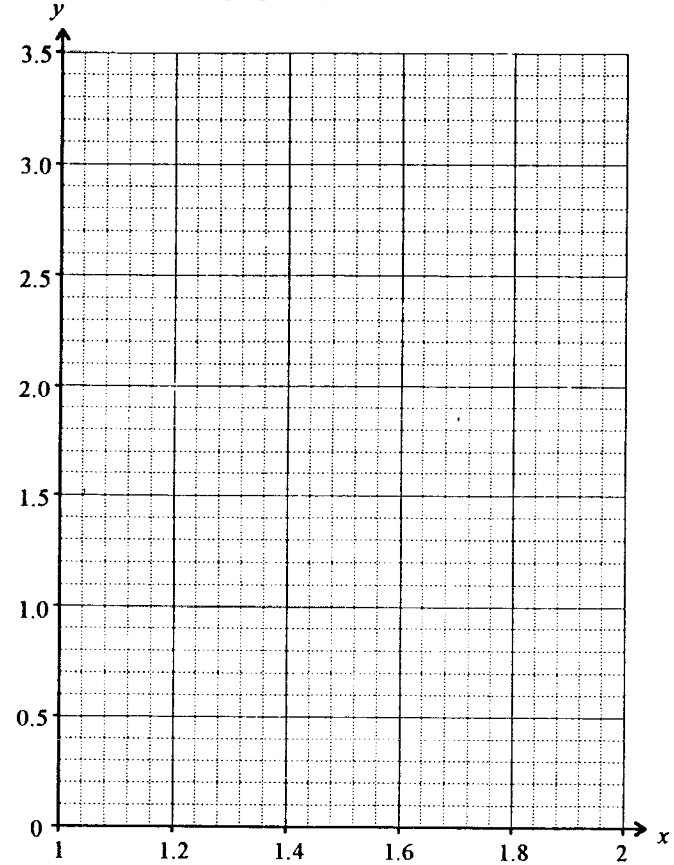
(iv) Let $I_0 = 3 - J_0$. State whether I_0 is an overestimate or underestimate of I . Explain your answer briefly.

(8 marks)

Candidate Number	Centre Number	Seat Number	Page Total

10(Cont'd) **If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

The graph of $y = x^x \ln x$ $(1 \le x \le 2)$



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11. The number of fire insurance claims (FICs) received by an insurance company is modelled by a Poisson distribution with mean 4 claims per day. The company found that 60% of the FICs are related to house fires.

(a) Find the probability that no FICs are received on a particular day.
(2 marks)

(b) If 5 FICs are received on a certain day, find the probability that at least 2 of them are related to house fires.
(3 marks)

(c) It is known that the amounts of FICs related and not related to house fires can be modelled respectively by normal distributions with the following means and standard deviations:

FICs	Mean	Standard deviation
Related to house fires	\$ 100 000	\$ 50 000
Not related to house fires	\$ 150 000	\$ 20 000

If the amount of a FIC is greater than \$ 200 000 , the FIC is said to be *large*.

(i) Find the probability that a certain FIC is *large*.

(ii) Given that a FIC is *large*. Find the probability that the FIC is related to a house fire.

(iii) Find the probability that on a particular day, the company receives 5 FICs and at least 2 of them are *large*.
(10 marks)

12. A company has a performance assessment system in which the salesmen will be awarded merit points according to their sales volumes every month. In Table 1 (Page 12), columns 1 and 2 show the assessment system and column 3 shows the frequency distribution of the merit points obtained by 60 salesmen in a certain month.

(a) Suppose the binomial distribution B(5, 0.4) is used to model the number of merit points obtained by the salesmen. Fill in the missing expected frequencies under this distribution in Table 1.

(2 marks)

(b) Suppose a normal distribution $N(\mu, \sigma^2)$ is used to model the sales volumes achieved by the salesmen. Using the class intervals as shown in Table 1, some expected frequencies are calculated and shown in column 5. Determine μ and σ, giving the answers correct to the nearest hundreds. Fill in the other expected frequencies.

(6 marks)

(c) A salesman will get a warning from the company if the total merit points obtained by him in the past 3 consecutive months is 2 or less. Suppose the merit points obtained by the salesmen every month are independent. From Table 1, choose one of the models in (a) and (b) which fits the observed data better and use it to answer the following:

(i) Find the probability that a salesman will get a warning in a particular month.

(ii) It is known that the company sent out warning letters to 10 salesmen in April 1997 according to the records of the previous 3 months. How many of these 10 salesmen are expected to get no merit points in at least 2 of these 3 months?

(7 marks)

Candidate Number	Centre Number	Seat Number	Page Total

12(Cont'd) **If you attempt Question 12, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

Table 1 Observed and expected frequencies of the merit points obtained by 60 salesmen in a certain month

Sales Volume ($$X$)	Merit Points	Observed Frequency	Expected Frequency *	
			B(5, 0.4)	$N(\mu, \sigma^2)$
$X < 9\,000$	0	4	4.67	4.00
$9\,000 \le X < 12\,000$	1	14		14.50
$12\,000 \le X < 15\,000$	2	23		
$15\,000 \le X < 18\,000$	3	15		14.50
$18\,000 \le X < 21\,000$	4	4	4.61	
$X \ge 21\,000$	5	0	0.61	

* Correct to 2 decimal places.

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13. Every morning, Mr. Wong wears a necktie to work. If the length of the front portion of his necktie is between 44 cm and 45 cm, he regards it to be a *perfect tying*. Otherwise, he has to tie it again until he gets the *perfect tying*. Suppose that the length of the front portion of his necktie can be modelled by a normal distribution with mean 44.6 cm and standard deviation 1.2 cm.

(a) Find the probability that Mr. Wong gets a *perfect tying* in one trial.
(3 marks)

(b) Find the mean number of trials to be taken by Mr. Wong to get the first *perfect tying*.
(2 marks)

(c) Find the probability that Mr. Wong gets the *perfect tying* in not more than 3 trials.
(2 marks)

(d) Mr. Wong will have to go to work by taxi only if he doesn't get the *perfect tying* in the first 3 trials in any morning.

(i) Find the probability that Mr. Wong will have to go to work by taxi in less than 2 out of 6 days.

(ii) Given that Mr. Wong has to go to work by taxi on a certain morning, find the probability that he could not get the *perfect tying* until the 5th trial.

(iii) Find the probability that in a certain week of 6 working days (Monday to Saturday), Mr. Wong will have to go to work by taxi on 2 consecutive mornings and he will not have to take a taxi on the other 4 mornings.
(8 marks)

END OF PAPER