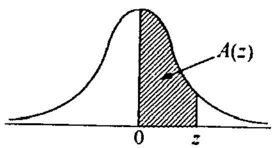
標準正態曲線下的面積

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

註： 本表所列數字，爲曲線下由 $z=0$ 至正值 z 之間的面積所佔的比例。負值 z 所含的面積可利用對稱性求得。



$$A(z)=\int_0^z \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}\,dx$$

96-ASL
M&S

HONG KONG EXAMINATIONS AUTHORITY

HONG KONG ADVANCED LEVEL EXAMINATION 1996

MATHEMATICS AND STATISTICS AS-LEVEL

9.00 am–12.00 noon (3 hours)

This paper must be answered in English

1. This paper consists of Section A and Section B.
2. Answer ALL questions in Section A, using the AL(C1) answer book.
3. Answer any FOUR questions in Section B, using the AL(C2) answer book.
4. Unless otherwise specified, numerical answers should either be exact or given to 4 decimal places.

SECTION A (40 marks)
Answer **ALL** questions in this section.
Write your answers in the AL(C1) answer book.

1. A stem-and-leaf diagram for the test scores of 30 students is shown below:

Stem (tens)	Leaf (units)
1	0
2	
3	0 2
4	4 5 8 9
5	0 1 2 6 8 8 9 9
6	2 3 3 5 5 8
7	1 2 2 4 4 4
8	2 5
9	1

(a) Find the mean, mode and interquartile range of these scores.

(b) If the score 71 is an incorrect record and the correct score is 11, which of the statistics in (a) will have different values?

Find the correct values of these statistics.

(6 marks)

2. Let $y = \frac{\ln x}{x}$ $(x > 0)$, find $\frac{dy}{dx}$.

Hence or otherwise, find $\int \frac{\ln x}{x^2} dx$

(5 marks)

3. Sketch the curve $y = \frac{x-1}{x-3}$ $(x \neq 3)$ and indicate the asymptotes and intercepts.

(6 marks)

4. Figure 1 shows a unit square target for shooting on the rectangular coordinate plane. The target is divided into three regions I, II and III by the curves $y = \sqrt{x}$ and $y = x^3$. The scores for hitting the regions I, II and III are 10, 20 and 30 points respectively.

(a) Find the areas of the three regions.

(b) Suppose a child shoots randomly at the target twice and both shots hit the target. Find the probability that he will score 40 points.

(7 marks)

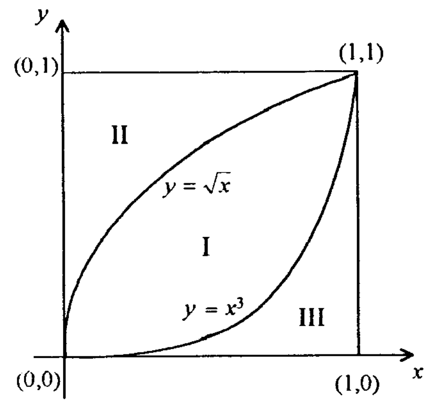


Figure 1

5. At any time t (in hours), the relationship between the number N of tourists at a ski-resort and the air temperature θ °C can be modelled by

$$N = 2930 - (\theta + 440)[\ln(\theta + 49)]^2 ,$$

where $-45 \leq \theta \leq -40$.

(a) Express $\frac{dN}{dt}$ in terms of θ and $\frac{d\theta}{dt}$.

(b) At a certain moment, the air temperature is -40 °C and it is falling at a rate of 0.5 °C per hour. Find, to the nearest integer, the rate of increase of the number of tourists at that moment.

(6 marks)

6. A company buys equal quantities of fuses, in 100-unit lots, from two suppliers A and B. The company tests two fuses randomly drawn from each lot, and accepts the lot if both fuses are non-defective.

It is known that 4% of the fuses from supplier A and 1% of the fuses from supplier B are defective. Assume that the quality of the fuses are independent of each other.

(a) What is the probability that a lot will be accepted?

(b) What is the probability that an accepted lot came from supplier A?

(6 marks)

7. A reporter wished to find the mean number of children per family in a village. She visited the only primary school in the village and asked all the pupils there what the number of children, including themselves, was in each of their families. The data obtained are presented in the following frequency table:

Number of children in the family	1	2	3	4	5
Number of pupils	10	11	6	9	4

The reporter concludes that the mean number of children per family is

$$\frac{1\times 10+2\times 11+3\times 6+4\times 9+5\times 4}{10+11+6+9+4}$$

Is her conclusion justified? Explain your answer.

(4 marks)

SECTION B (60 marks)

Answer any **FOUR** questions in this section. Each question carries 15 marks.

Write your answers in the AL(C2) answer book.

8. There are several bags on a table each containing six cards numbered 0, 1, 2, 3, 4 and 5 respectively.

(a) (i) Find the coefficient of x^5 in the expansion of $(1+x+x^2+x^3+x^4+x^5)^2$.

(ii) John takes two bags away from the table and randomly draws a card from each of them. Using (a)(i), or otherwise, find the probability that the sum of the numbers on the two cards drawn is 5.

(4 marks)

(b) (i) Expand $(1-x^6)^4$.

(ii) Find the coefficient of x^r, where r is a non-negative integer, in the expansion of $(1-x)^{-4}$ for $|x|<1$.

(iii) Using (b)(i) and (b)(ii), or otherwise, find the coefficient of x^8 in the expansion of $\left(\frac{1-x^6}{1-x}\right)^4$ for $|x|<1$.

(7 marks)

(c) Joan takes four bags away from the table and randomly draws a card from each of them. Using (b)(iii), or otherwise, find the probability that the sum of the numbers on the four cards drawn is 8.

(4 marks)

9. The population size P of a species of reptiles living in a jungle increases at a rate of

$$\frac{dP}{dt} = 5e^{\frac{t^2}{10}} - 2t \qquad (t \geq 0),$$

where t is the time in month. It is known that $P = 10$ when $t = 0$.

(a) Use the trapezoidal rule with 6 sub-intervals to estimate $\int_0^6 e^{\frac{t^2}{10}} dt$.

Hence estimate P, to the nearest integer, at $t = 6$.

(7 marks)

(b) A chemical plant was recently built near the jungle. Pollution from the plant affects the growth of the population of the reptiles from $t = 6$ onwards. An ecologist suggests that the population size of the species of reptiles can then be approximated by

$$P = kte^{-0.04t} - 50 \qquad (t \geq 6).$$

(i) Using (a), find the value of k correct to 1 decimal place.

(ii) Determine the time at which the population size will attain its maximum. Hence find the maximum population size correct to the nearest integer.

(iii) Use the graph in Figure 2 to find the value of t, correct to the nearest integer, when the species of reptiles becomes extinct due to pollution.

(8 marks)

The graph of $y = e^{0.04t}$

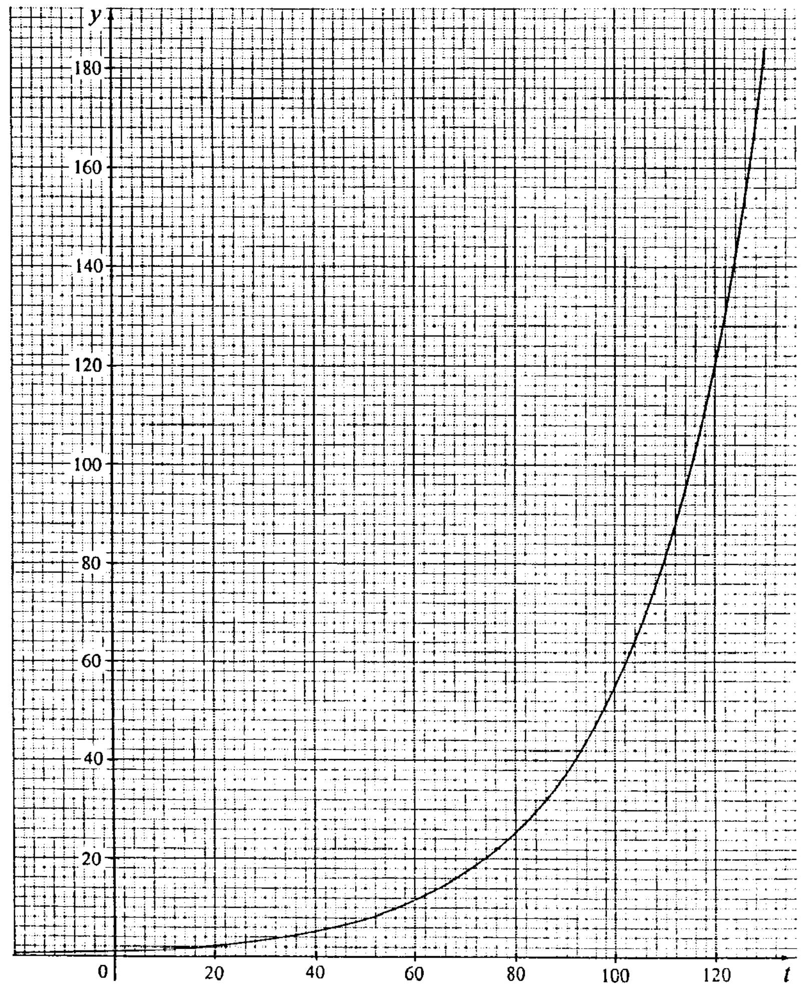


Figure 2

10. The monthly cost $C(t)$ at time t of operating a certain machine in a factory can be modelled by

$$C(t) = ae^{bt} - 1 \qquad (0 < t \leq 36),$$

where t is in month and $C(t)$ is in thousand dollars.

Table 1 shows the values of $C(t)$ when $t = 1, 2, 3, 4$.

Table 1

t	1	2	3	4
$C(t)$	1.21	1.44	1.70	1.98

(a) (i) Express $\ln[C(t)+1]$ as a linear function of t.

(ii) Use Table 1 and the graph paper on Page 8 to estimate graphically the values of a and b correct to 1 decimal place.

(iii) Using the values of a and b found in (a)(ii), estimate the monthly cost of operating this machine when $t = 36$.

(8 marks)

(b) The monthly income $P(t)$ generated by this machine at time t can be modelled by

$$P(t) = 439 - e^{0.2t} \qquad (0 < t \leq 36),$$

where t is in month and $P(t)$ is in thousand dollars.

The factory will stop using this machine when the monthly cost of operation exceeds the monthly income.

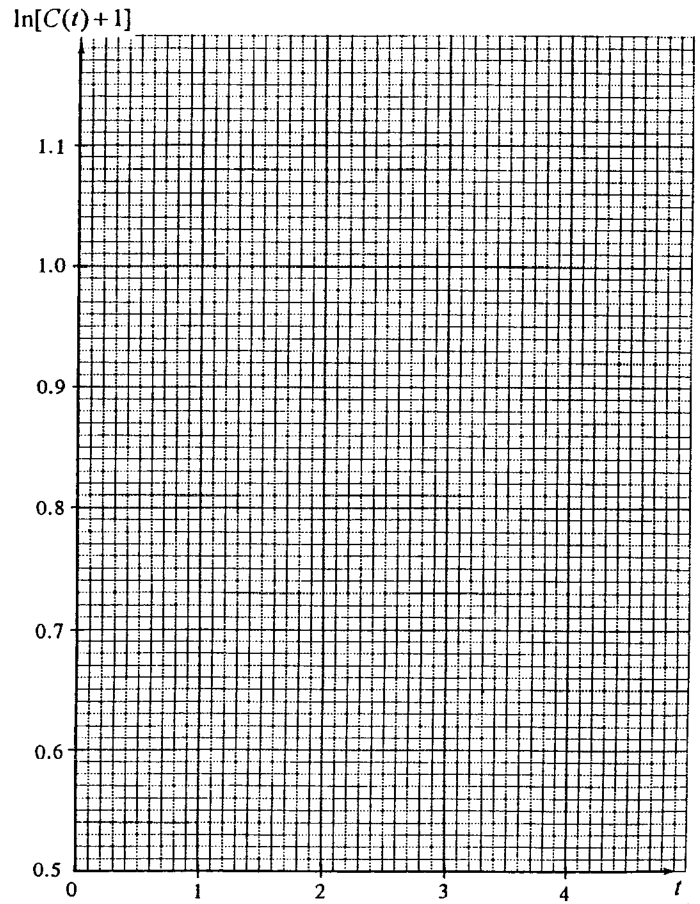
(i) Find the value of t when the factory stops using this machine. Give the answer correct to the nearest integer.

(ii) What is the total profit generated by this machine? Give the answer correct to the nearest thousand dollars.

(7 marks)

Candidate Number	Centre Number	Seat Number	Page Total

10(Cont'd) **If you attempt Question 10, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**



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11. A machine discharges soda water once for each cup of soda water purchased. The amount of soda water in each discharge is independently normally distributed with mean 210 ml and standard deviation 15 ml .

(a) Find the probability that the amount of a cup of soda water is between 200 ml and 220 ml .

(2 marks)

(b) Suppose cups of capacity 240 ml each are used.

(i) Find the probability that a discharge will overflow.

(ii) What is the probability that there will be exactly 1 overflow out of 30 discharges?

(iii) If Sam buys a cup of soda water from the machine every day starting on 1st July, find the probability that he will get the second overflow on 31st July.

(5 marks)

(c) The vendor has decided to use cups of capacity 220 ml each and to repair the machine so that, on the average, 80 in 100 cups contain more than 205 ml of soda water in each and only 1 in 100 discharges overflows. The amount of soda water in each discharge is still independently normally distributed.

(i) What will the new mean and standard deviation of the amount of soda water in each discharge be? Give the answers correct to 1 decimal place.

(ii) If a discharge from the repaired machine overflows, find the probability that the amount of soda water in this discharge exceeds 225 ml . Give the answer correct to 2 decimal places.

(8 marks)

12. A factory supplies batches of low-quality electronic chips. Each batch contains 6 chips, some of which may be defective. Table 2 (Page 12) shows the frequency distribution of defective chips in each of 80 randomly selected batches.

(a) It is suggested that the number of defective chips in a batch could be modelled by a binomial distribution with the probability that a chip is defective being 0.1. Fill in the missing expected frequencies, under this distribution, in Table 2.

(2 marks)

(b) A buyer claims that the number of defective chips in a batch could be approximated by a Poisson distribution with mean λ. He has calculated some expected frequencies as shown in Table 2.

(i) Determine λ correct to 1 decimal place.

(ii) Fill in the missing expected frequencies, under this distribution, in Table 2.

(3 marks)

(c) A buyer compares the two distributions in (a) and (b) and adopts the one which fits the observed data better. He buys 4 batches of chips from the factory and classifies a batch as *good* if all the chips in the batch are non-defective.

(i) Find the probability that at least 3 of these 4 batches are *good*.

(ii) Suppose that at least 3 of these 4 batches are *good* and the buyer buys 6 more batches. Find the probability that exactly 8 of these 10 batches are *good*.

(10 marks)

Candidate Number	Centre Number	Seat Number	Page Total

12(Cont'd) **If you attempt Question 12, fill in the details in the first three boxes above and tie this sheet INSIDE your answer book.**

Table 2 Observed and expected frequencies of defective chips in each of 80 randomly selected batches

Number of Defective Chips	Observed Frequency	Expected Frequency *	
		Binomial	Poisson
0	33	42.5	32.5
1	29	28.3	
2	13		
3	4		
4	1	0.1	
5	0	0.0	
6	0	0.0	

* Correct to 1 decimal place.

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13. In city A, the occurrences of rainstorms are assumed to be independent. The number of occurrences may be modelled by a Poisson distribution with mean occurrence rate of 2 rainstorms per year.

(a) Find the probability of having more than two rainstorms in a particular year.

(3 marks)

(b) Last year, more than two rainstorms occurred. Estimate the number of years which will elapse before the next occurrence of more than two rainstorms in a year. Give the answer correct to the nearest integer.

(3 marks)

(c) Past experience suggests that the probability of having at least one serious landslide in a year depends on the number of rainstorms in that year as follows:

Number of rainstorms	0	1 or 2	3 or more
Probability of having at least one serious landslide	0.2	0.3	0.5

Find the probability that, in city A,

(i) there is no serious landslide in a particular year ;

(ii) no rainstorm has occurred if there is no serious landslide in a particular year ;

(iii) there is no serious landslide for at most 2 out of 5 years.

(9 marks)

END OF PAPER