HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION

MATHEMATICS
Extended Part
Module 1 (Calculus and Statistics)

(Sample Paper)

Time allowed: 2 hours 30 minutes

This paper must be answered in English

INSTRUCTIONS

1. This paper consists of Section A and Section B. Each section carries 50 marks.

2. Answer **ALL** questions in this paper.

3. All working must be clearly shown.

4. Unless otherwise specified, numerical answers must be either exact or correct to 4 decimal places.

Not to be taken away before the end of the examination session

Section A (50 marks)

1. Expand the following in ascending powers of x as far as the term in x^2 :

   (a) e^{-2x} ,

   (b) $\dfrac{(1+2x)^6}{e^{2x}}$.

(4 marks)

2. When a hot air balloon is being blown up, its radius r (in m) will increase with time t (in hr). They are related by $r=3-\dfrac{2}{2+t}$, where $t \ge 0$. It is known that the volume V (in m^3) of the balloon is given by $V=\dfrac{4}{3}\pi r^3$.

   Find the rate of change, in terms of π, of the volume of the balloon when the radius is 2.5 m.

(4 marks)

3. A political party conducted an opinion poll on a certain government policy. A random sample of 150 people was taken and 57 of them supported this policy.

   (a) Estimate the population proportion supporting this policy.

   (b) Find an approximate 90% confidence interval for the population proportion.

(4 marks)

4. Of all Year One students in a certain university, 90% are local students, among whom 5% are enrolled with a scholarship. For non-local Year One students, 35% of them are enrolled **without** a scholarship.

   (a) If a Year One student is selected at random, find the probability that the student is enrolled with a scholarship.

   (b) Given that a selected Year One student is enrolled with a scholarship, find the probability that this student is a non-local student.

(4 marks)

5. A manufacturer produces a large batch of light bulbs, with a mean lifetime of 640 hours and a standard deviation of 40 hours. A random sample of 25 bulbs is taken. Find the probability that the sample mean lifetime of the 25 bulbs is greater than 630 hours.

(5 marks)

6. Let $u=\sqrt{\dfrac{2x+3}{(x+1)(x+2)}}$, where $x>-1$.

(a) Use logarithmic differentiation to express $\dfrac{\mathrm{d}u}{\mathrm{d}x}$ in terms of u and x .

(b) Suppose $u=3^{y}$, express $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x .

(5 marks)

7. The random variable X has probability distribution $\mathrm{P}(X=x)$ for $x=1$, 2 and 3 as shown in the following table.

x	1	2	3
$\mathrm{P}(X=x)$	0.1	0.6	0.3

Calculate

(a) $\mathrm{E}(X)$,

(b) $\mathrm{Var}(3-2X)$.

(5 marks)

8. The monthly number of traffic accidents occurred in a certain highway follows a Poisson distribution with mean 1.7 . Assume that the monthly numbers of traffic accidents occurred in this highway are independent.

(a) Find the probability that at least four traffic accidents will occur in this highway in the first quarter of a certain year.

(b) Find the probability that there is exactly one quarter with at least four traffic accidents in a certain year.

(6 marks)

9. L is the tangent to the curve $C: y=x^3+7$ at $x=2$.

(a) Find the equation of the tangent L .

(b) Using the result of (a), find the area bounded by the y-axis, the tangent L and the curve C .

(7 marks)

10. The number N of fish, which are infected by a certain disease in a pool, can be modelled by

$$N = \frac{500}{1+ae^{-kt}} ,$$

where a , k are positive constants and t is the number of days elapsed since the outbreak of the disease.

The values of N when $t = 5, 10, 15, 20$ are as follows:

t	5	10	15	20
N	13	34	83	175

(a) Express $\ln\left(\frac{500}{N}-1\right)$ as a linear function of t .

(b) Using the graph paper on page 10, estimate graphically the values of a and k (correct your answers to 1 decimal place).

(c) How many days after the outbreak of the disease will the number of fish infected by the disease reach 270 ?

(6 marks)

Section B (50 marks)

11. The manager, Mary, of a theme park starts a promotion plan to increase **the daily number of visits** to the park. The rate of change of **the daily number of visits** to the park can be modelled by

$$\frac{dN}{dt} = \frac{k(25-t)}{e^{0.04t}+4t} \qquad (t \geq 0) ,$$

where N is **the daily number of visits** (in hundreds) recorded at the end of a day, t is the number of days elapsed since the start of the plan and k is a positive constant.

Mary finds that at the start of the plan, $N = 10$ and $\frac{dN}{dt} = 50$.

(a) (i) Let $v = 1 + 4te^{-0.04t}$, find $\frac{dv}{dt}$.

(ii) Find the value of k , and hence express N in terms of t .

(7 marks)

(b) (i) When will **the daily number of visits** attain the greatest value?

(ii) Mary claims that there will be more than 50 hundred visits on a certain day after the start of the plan. Do you agree? Explain your answer.

(3 marks)

(c) Mary's supervisor believes that **the daily number of visits** to the park will return to the original one at the start of the plan after a long period of time. Do you agree? Explain your answer.
(Hint: $\lim_{t\to\infty} te^{-0.04t} = 0$.)

(2 marks)

12. (a) Let $f(t)$ be a function defined for all $t \geq 0$. It is given that

$$f'(t) = e^{2bt} + ae^{bt} + 8 ,$$

where a and b are negative constants and $f(0) = 0$, $f'(0) = 3$ and $f'(1) = 4.73$.

(i) Find the values of a and b .

(ii) By taking $b = -0.5$, find $f(12)$.

(5 marks)

(b) Let $g(t)$ be another function defined for all $t \geq 0$. It is given that

$$g'(t) = \frac{33}{10} te^{-kt} ,$$

where k is a positive constant. Figure 1 shows a sketch of the graph of $g'(t)$ against t . It is given that $g'(t)$ attains the greatest value at $t = 7.5$ and $g(0) = 0$.

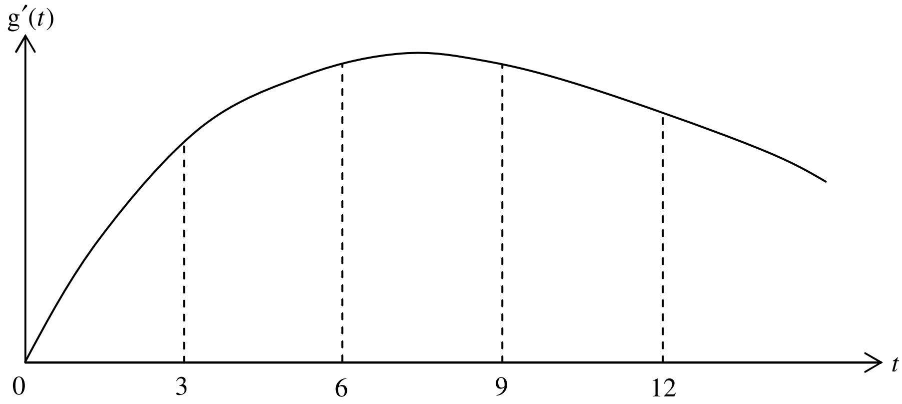


Figure 1

(i) Find the value of k .

(ii) Use the trapezoidal rule with four sub-intervals to estimate $g(12)$.

(6 marks)

(c) From the estimated value obtained in (b)(ii) and Figure 1, Jenny claims that $g(12) > f(12)$. Do you agree? Explain your answer.

(2 marks)

13. There are 80 operators in an emergency hotline centre. Assume that the number of incoming calls for the operators are independent and the number of incoming calls for each operator is distributed as Poisson with mean 6.2 in a ten-minute time interval (TMTI). An operator is said to be *idle* if the number of incoming calls received is less than three in a certain TMTI.

(a) Find the probability that a certain operator is *idle* in a TMTI.

(3 marks)

(b) Find the probability that there are at most two *idle* operators in a TMTI.

(3 marks)

(c) A manager, Calvin, checks the numbers of incoming calls of the operators one by one in a TMTI. What is the least number of operators to be checked so that the probability of finding an *idle* operator is greater than 0.9 ?

(4 marks)

14. The Body Mass Index (BMI) value (in $\mathrm{kg/m^2}$) of children aged 12 in a city are assumed to follow a normal distribution with mean μ $\mathrm{kg/m^2}$ and standard deviation 4.5 $\mathrm{kg/m^2}$.

(a) A random sample of nine children aged 12 is drawn and their BMI values (in $\mathrm{kg/m^2}$) are recorded as follows:

$$16.0\,,\ 18.3\,,\ 15.2\,,\ 17.8\,,\ 19.5\,,\ 15.9\,,\ 18.6\,,\ 22.5\,,\ 23.6$$

(i) Find an unbiased estimate for μ.

(ii) Construct a 95% confidence interval for μ.

(3 marks)

(b) Assume $\mu = 18.7$. If a random sample of 25 children aged 12 is drawn and their BMI values are recorded, find the probability that the sample mean is less than 17.8 $\mathrm{kg/m^2}$.

(4 marks)

(c) A child aged 12 having a BMI value greater than 25 $\mathrm{kg/m^2}$ is said to be *overweight*. Children aged 12 are randomly selected one after another and their BMI values are recorded until two *overweight* children are found. Assume that $\mu = 18.7$.

(i) Find the probability that a selected child is *overweight*.

(ii) Find the probability that more than eight children have to be selected in this sampling process.

(iii) Given that more than eight children will be selected in this sampling process, find the probability that exactly ten children are selected.

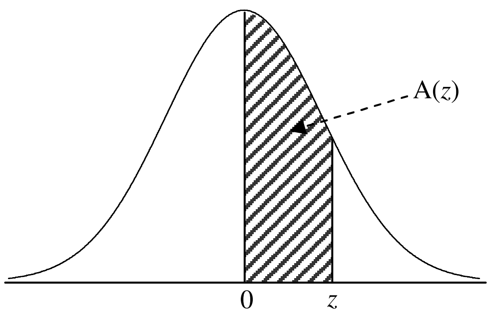
(8 marks)

END OF PAPER

Table: Area under the Standard Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the proportion of the area under the entire curve which is between $z = 0$ and a positive value of z. Areas for negative values of z are obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \, dx$$

10. (Continued)