2023-DSE
MATH EP
M1

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HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2023

Candidate Number

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)
Question-Answer Book

8:30 am – 11:00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of TWO sections, A and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

(7) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

* A 0 3 1 E 0 0 1 *

SECTION A (50 marks)

1. The table below shows the probability distribution of a discrete random variable X, where k is a constant and n is an integer.

x	-1	n	3	6
$P(X=x)$	0.1	0.3	k	0.2

(a) Find k. Hence express $\text{Var}(X)$ in terms of n.

(b) It is given that $\text{Var}(X)=3.69$. Find $\text{Var}(10X^2-9)$.

(6 marks)

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2. The number of customers in a coffee shop follows a Poisson distribution with a mean of 3 per minute. Let X be the number of customers in the coffee shop in an hour.

(a) Write down the values of $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

(b) Let $\overline{X}$ be the mean of n independent random observations of X. Find the least value of n such that the estimated value of $\mathrm{P}(178 \leq \overline{X} \leq 182)$ using central limit theorem is greater than 0.733.

(5 marks)

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3. A driver can choose either tunnel X or tunnel Y to go from a city to another city. A driver can choose either cash or electronic means to pay the tunnel fee. The probability that a driver chooses tunnel X is $\frac{9}{20}$. Given that a driver chooses tunnel X, the probability that the driver pays by cash is $\frac{4}{7}$. Given that a driver pays by electronic means, the probability that the driver chooses tunnel Y is $\frac{14}{41}$.

(a) Find the probability that a driver chooses tunnel X and pays by electronic means.

(b) Given that a driver chooses tunnel Y, find the probability that the driver pays by electronic means.

(c) Given that a driver pays by cash, find the probability that the driver chooses tunnel Y.

(7 marks)

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4. Let A and B be two events. Denote the complementary event of A by A'. It is given that $P(B \mid A') = \frac{2}{7}$ and $P(A \cup B) = \frac{37}{42}$.

(a) Find $P(A)$.

(b) It is given that A and B are independent.

(i) Find $P(B)$.

(ii) Determine whether A and B are mutually exclusive. Explain your answer.

(6 marks)

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5. (a) Expand e^{2x} in ascending powers of x as far as the term in x^3.

(b) Let $y=(1-ax)^4+3e^{2x}$, where a is a constant. It is given that the coefficient of x^3 in the expansion of y is -1368.

(i) Find a.

(ii) If $x=2^u$, find the value of $\dfrac{dy}{du}$ when $u=0$.

(7 marks)

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6. The principal $\$P$ is deposited into a bank at an interest rate of $r\%$ per annum, where r is a constant. The amount $\$A$ is modelled by $A = Pe^{\frac{rt}{100}}$, where t $(t \geq 0)$ is the number of years elapsed since the principal is deposited into the bank.

(a) Express $\ln A$ as a linear function of t.

(b) It is given that the slope and the intercept on the vertical axis of the graph of the linear function obtained in (a) are 0.05 and $4\ln 10$ respectively. When $t = k$, the amount is $\$2P$.

(i) Find the exact value of k.

(ii) Find the rate of change of the amount when $t = k$.

(6 marks)

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7. Consider the curve Γ: $y = \dfrac{(\ln x)^2 - 3\ln x + 2}{x}$, where $x > 0$.

(a) Express the x-intercept(s) of Γ in terms of e.

(b) Find the area of the region bounded by Γ, the x-axis and the straight lines $x = 1$ and $x = 9$.
(6 marks)

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8. Define $f(x) = x^{\frac{2}{3}}(x+2)^{\frac{1}{3}}$ for all $x > 0$.

(a) Find the constants A and B such that $f'(x) = \dfrac{Ax+B}{3x^{\frac{1}{3}}(x+2)^{\frac{2}{3}}}$.

(b) Find $f''(x)$.

(c) Using the trapezoidal rule with 4 sub-intervals, estimate $\int_1^2 f(x)dx$. Determine whether this estimate is an over-estimate or an under-estimate. Explain your answer.

(7 marks)

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SECTION B (50 marks)

9. In a city, the antibody level of a virus of a citizen follows a normal distribution with a mean of μ units and a standard deviation of 20 units.

(a) A medical inspection is conducted to estimate μ. A random sample of 30 citizens of the city is selected and their antibody levels are recorded below.

Antibody level (x units)	Number of citizens
$10 < x \leq 40$	4
$40 < x \leq 70$	9
$70 < x \leq 100$	10
$100 < x \leq 130$	7

Construct a 98.5% confidence interval for μ. (4 marks)

(b) Another random sample of 34 citizens of the city is selected. It is found that the mean of the antibody levels for this sample is 79 units. This sample is combined with the sample in (a). The upper limit of a constructed α% confidence interval for μ using the combined sample is 82.9. Find α correct to the nearest integer. (4 marks)

(c) Suppose $\mu = 76$. If the antibody level of a citizen is higher than 60 units, the citizen is classified as *well protected*; otherwise the citizen is *insufficiently protected*.

(i) Find the probability that a citizen of the city is *well protected*.

(ii) A random sample of 16 citizens of the city is selected and their antibody levels are inspected one by one. Given that at least 3 selected citizens are *insufficiently protected*, find the probability that exactly 3 among the final 5 citizens are *well protected*.

(5 marks)

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10. There are 20 telephonists on duty for a donation hotline in a charity show. For each telephonist, the number of calls received follows a Poisson distribution with a mean of 3.2 per minute. If the number of calls received by a telephonist exceeds 4 in a minute, the telephonist is regarded as *busy* in that minute.

(a) Find the probability that a certain telephonist is *busy* in a minute. (3 marks)

(b) Find the probability that no more than 3 telephonists are *busy* in a minute. (3 marks)

(c) Find the probability that exactly 3 telephonists are *busy* in a minute and the total number of calls received by these *busy* telephonists in that minute is 15 . (2 marks)

(d) Given that no more than 3 telephonists are *busy* in a minute, find the probability that the total number of calls received by these *busy* telephonists in that minute is 15 . (4 marks)

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11. Define $g(x) = \dfrac{x^3}{1+x^6}$ for all real numbers x.

(a) Solve the equation $g'(x) = 0$. (2 marks)

(b) Does $g(x)$ attain its extreme value at $x = 0$? Explain your answer. (1 mark)

(c) Find the greatest value and the least value of $g(x)$. (5 marks)

(d) Find the greatest slope of the tangent to the curve $y = g(x)$, where $0 < x < 1$. (4 marks)

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12. (a) Let $u=\dfrac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}$. Find a polynomial $\mathrm{p}(t)$ such that $\dfrac{\mathrm{d}u}{\mathrm{d}t}=\dfrac{\mathrm{p}(t)}{(t^2+\sqrt{2}t+1)^2}$. (2 marks)

(b) Using integration by substitution and the identity $t^4+1=(t^2+\sqrt{2}t+1)(t^2-\sqrt{2}t+1)$, find $\displaystyle\int\frac{t^2-1}{t^4+1}\,\mathrm{d}t$. (4 marks)

(c) A researcher studies the weight of a substance in an experiment. Let $\mathrm{W}(t)$ (in g) be the weight of the substance at time t, where t $(t\geq 0)$ is the number of hours elapsed since the study begins. The researcher models the rate of change of the weight of the substance by

$$\mathrm{W}'(t)=\frac{t^2-1}{t^4+1}+\left(\frac{1}{3}\right)^t.$$

(i) Find the change in the weight of the substance from $t=0$ to $t=2$.

(ii) It is given that $\mathrm{W}(0)=5$. Estimate the weight of the substance after a very long time.

(7 marks)

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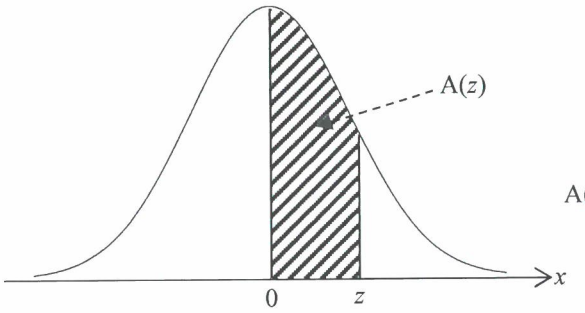
Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$