2022-DSE
MATH EP
M1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2022

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Candidate Number									

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)
Question-Answer Book

8:30 am – 11:00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of TWO sections, A and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

(7) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

* A 0 3 1 E 0 0 1 *

SECTION A (50 marks)

1. The table below shows the probability distribution of a discrete random variable X, where a and b are constants.

x	0	4	6
$\mathrm{P}(X=x)$	0.1	a	b

It is given that $\mathrm{E}(X)=4.6$.

(a) Find a, b and $\mathrm{Var}(X)$.

(b) Let $\overline{X}$ be the mean of 225 independent random observations of X. Using central limit theorem, estimate $\mathrm{P}(\overline{X}>4.75)$.

(7 marks)

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2. Let X and Y be discrete random variables such that $Y = 200 - 4X$. It is given that $\mathrm{E}(X) = 8.8$ and $\mathrm{Var}(Y) = 144$.

(a) Find $\mathrm{Var}(X)$ and $\mathrm{E}(Y)$.

(b) Is it possible that Y follows a Poisson distribution? Explain your answer.

(c) Is it possible that X follows a binomial distribution? Explain your answer.

(5 marks)

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3. Let A and B be two events. Denote the complementary events of A and B by A' and B' respectively. It is given that $\mathrm{P}(A' \mid B) = 5\mathrm{P}(A \mid B)$ and $\mathrm{P}(A \cap B') = \mathrm{P}(A \cap B) + 0.45$. Suppose that $\mathrm{P}(B) = p$, where $p \neq 0$.

(a) Prove that $\mathrm{P}(A) = \dfrac{p}{3} + 0.45$.

(b) Are A and B independent? Explain your answer.

(c) Let C be an event such that $\mathrm{P}(C) = 0.6$. Are A and C mutually exclusive? Explain your answer.

(7 marks)

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4. Let T hours be the time spent by a student to complete an online exercise. It is given that the standard deviation of T is 0.4. A teacher wants to estimate μ, the mean of T. The teacher randomly selects 100 students, and finds that the total time spent by those students to complete the exercise is 150 hours.

(a) Construct a 95% confidence interval for μ.

(b) Several selected students encounter network problem when doing the exercise. The teacher constructs a new 95% confidence interval for μ after deleting the data of these students. Will the width of this new confidence interval be greater than, equal to or less than the width of the confidence interval obtained in (a)? Explain your answer.

(5 marks)

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5. Let k be a positive constant.

(a) Expand $e^{\frac{-kx}{2}}$ in ascending powers of x as far as the term in x^2 .

(b) Let $y = 64e^{-kx}$.

(i) Express $\ln y$ as a linear function of x .

(ii) It is given that the coefficient of x^2 in the expansion of $\sqrt{y}(1-2x)^5$ is 449 . Find the slope of the graph of the linear function obtained in (b)(i) .

(6 marks)

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6. Consider the curve C : $y = \dfrac{9-4x^2}{6+2x^2}$.

(a) Find $\dfrac{dy}{dx}$.

(b) If a tangent L to C passes through the point $(3, -2)$, find the equation of L .

(7 marks)

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7. Let $g(x)$ be a function such that $g'(x) = x^{\beta} 3^{\sqrt{x}}$ for all $x > 0$, where β is a constant. It is given that $g'(9) = 2g'(4)$.

(a) Prove that $\beta = \frac{-1}{2}$.

(b) If $g(4) = 0$, find the exact value of $g(9)$.

(6 marks)

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8. Let a be a constant. Define $\mathrm{f}(x) = ax^8 - 152x^5 - 4320x^2$ for all real numbers x .

(a) Prove that $\mathrm{f}(x)$ attains its maximum value at $x = 0$.

(b) It is given that $\mathrm{f}(x)$ attains its minimum value at $x = -2$.

(i) Prove that $a = 5$.

(ii) Find the least value of $\mathrm{f}(x)$.

(7 marks)

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SECTION B (50 marks)

9. In a hospital, the probability that a newborn baby boy is of weight above 3.7 kg is 0.3085, while the probability that a newborn baby girl is of weight above 3.7 kg is 0.1587. Assume that the probability for a newborn baby to be a boy is equal to the probability for the newborn baby to be a girl.

   (a) Find the probability that a newborn baby in the hospital is of weight above 3.7 kg. (1 mark)

   (b) If a newborn baby in the hospital is of weight above 3.7 kg, find the probability that the baby is a boy. (2 marks)

   (c) It is given that the number of newborn babies per day in the hospital follows a Poisson distribution with a mean of 2.1.

      (i) Find the probability that there are exactly 2 newborn babies and no newborn babies are of weight 3.7 kg or below on a certain day in the hospital.

      (ii) Given that there are at most 2 newborn babies and no newborn babies are of weight 3.7 kg or below on a certain day in the hospital, find the probability that there is exactly 1 newborn baby boy on that day in the hospital.

      (iii) Consider the following statement:

      'The probability that no newborn babies are of weight 3.7 kg or below on a certain day in the hospital is lower than 0.2.'

      Is the above statement correct? Explain your answer.

   (9 marks)

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10. There are 48 athletes in a running competition. In the first stage, the 48 athletes are randomly divided into 6 groups with 8 athletes in each group. It is given that the time taken by each athlete to finish the race follows a normal distribution with a mean of 12.3 seconds and a standard deviation of 0.5 seconds. Assume that the time taken by each athlete to finish the race is independent of each other.

(a) Find the probability that a certain athlete finishes the race in more than 12.1 seconds. (2 marks)

(b) Find the probability that at least 6 athletes finish the race in more than 12.1 seconds in a certain group. (3 marks)

(c) It is given that Peter finishes the race in 12.1 seconds in a certain group.

(i) Find the probability that Peter is the 1st athlete to finish the race in his group.

(ii) Find the probability that Peter is the 3rd athlete to finish the race in his group.

(iii) Among the 48 athletes, 16 of them proceed to the next stage according to the following rule:

The 1st and 2nd athletes to finish the race in each group proceed to the next stage. Among the 6 athletes who are the 3rd athlete to finish the race in each group, the 4 athletes who finish the race in the shortest times proceed to the next stage.

(1) Given that Peter is the 3rd athlete to finish the race in his group, find the probability that he proceeds to the next stage.

(2) Find the probability that Peter proceeds to the next stage.

(9 marks)

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11. (a) Using the trapezoidal rule with 5 sub-intervals, estimate $\int_1^2 e^x \ln x \, dx$. (2 marks)

(b) By considering $\frac{d}{dx}(xe^x \ln x)$, find $\int \left((x+1)e^x \ln x + \frac{1}{x} \right) dx$. (3 marks)

(c) Let α be the area of the region bounded by the curve $y = xe^x \ln x + \frac{1}{x}$, the x-axis and the straight lines $x = 1$ and $x = 2$.

(i) Using the results of (a) and (b), estimate α .

(ii) Someone claims that $\alpha > 4$. Do you agree? Explain your answer.

(8 marks)

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12. An engineer develops a software. Define $u = e^{6-2t}$, where t $(t \geq 0)$ is the number of months elapsed since the start of the development of the software. The engineer models the total number of bugs found N in the software by $N = Ae^{-u}$, where A is a positive constant.

(a) Find $\dfrac{\mathrm{d}u}{\mathrm{d}t}$. Hence, express $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ in terms of u . (3 marks)

(b) Find a polynomial $\mathrm{p}(u)$ in u such that $\dfrac{\mathrm{d}^2N}{\mathrm{d}t^2} = N\mathrm{p}(u)$. (2 marks)

(c) It is given that $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ attains its extreme value when $t = t_0$.

(i) Find t_0 .

(ii) Determine whether the extreme value of $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ is a maximum value or a minimum value. Explain your answer.

(4 marks)

(d) Estimate the total number of bugs found, in terms of A , after a very long time. (2 marks)

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END OF PAPER

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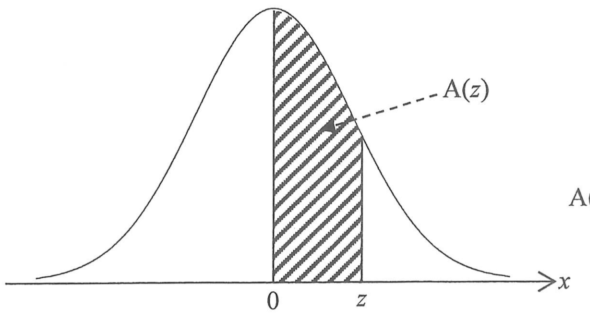
Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$