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**香港考試及評核局**  
**HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY**

**2022年香港中學文憑考試**  
**HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2022**

**數學**                      **延伸部分**                      **單元一 (微積分與統計)**  
**MATHEMATICS**    **EXTENDED PART**    **MODULE 1 (CALCULUS AND STATISTICS)**

**評卷參考**  
**MARKING SCHEME**

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**Hong Kong Diploma of Secondary Education Examination**  
**Mathematics Extended Part Module 1 (Calculus and Statistics)**

**General Marking Instructions**

1. It is very important that all markers should adhere as closely as possible to the marking scheme. In many cases, however, candidates will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question. Markers should be patient in marking alternative solutions not specified in the marking scheme.
2. In the marking scheme, marks are classified into the following three categories:

‘M’ marks	awarded for correct methods being used;
‘A’ marks	awarded for the accuracy of the answers;
Marks without ‘M’ or ‘A’	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, ‘M’ marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, ‘A’ marks for the corresponding answers should NOT be awarded (unless otherwise specified).
3. For the convenience of markers, the marking scheme was written as detailed as possible. However, it is still likely that candidates would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, markers should exercise their discretion in marking candidates’ work. In general, marks for a certain step should be awarded if candidates’ solution indicated that the relevant concept/technique had been used.
4. In marking candidates’ work, the benefit of doubt should be given in the candidates’ favour.
5. In the marking scheme, ‘r.t.’ stands for ‘accepting answers which can be rounded off to’ and ‘f.t.’ stands for ‘follow through’. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles.
6. Unless otherwise specified in the question, numerical answers should either be exact or given to 4 decimal places. Answers not accurate up to the required degree of accuracy should not be accepted.

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Solution	Marks	Remarks
1. (a) $0.1 + a + b = 1$ $a + b = 0.9$	1M	
$E(X) = 4.6$ $4a + 6b = 4.6$ $2a + 3b = 2.3$	1M	
Solving, we have $a = 0.4$ and $b = 0.5$ .	1A	for both correct
$\text{Var}(X)$ $= (4^2)(0.4) + (6^2)(0.5) - 4.6^2$ $= 3.24$	1A	
(b) By central limit theorem, $\bar{X} \sim N\left(4.6, \left(\frac{\sqrt{3.24}}{\sqrt{225}}\right)^2\right)$ , i.e. $N(4.6, 0.12^2)$ .	1M	can be absorbed
The required probability $\approx P\left(Z > \frac{4.75 - 4.6}{0.12}\right)$ $= 0.1056$	1M 1A	
	----- (7)	
2. (a) $\frac{\text{Var}(X)}{\text{Var}(Y)}$ $= \frac{\text{Var}(Y)}{4^2}$ $= 9$	1M 1A	<div style="border: 1px dashed black; padding: 5px; display: inline-block;">                     either one                 </div>
$E(Y)$ $= 200 - 4E(X)$ $= 164.8$	1A	
(b) $E(Y)$ $= 164.8$ $\neq 144$ $= \text{Var}(Y)$		
Since $E(Y) \neq \text{Var}(Y)$ , it is not possible that $Y$ follows a Poisson distribution.	1A	ft.
(c) Assume that $X \sim B(n, p)$ . $E(X) = np$ and $\text{Var}(X) = np(1-p)$ .		
$\text{Var}(X) = (1-p)E(X)$ $9 = (1-p)8.8$ $p = \frac{-1}{44} < 0$		
Thus, it is not possible that $X$ follows a binomial distribution.	1A	ft.
	----- (5)	

Solution	Marks	Remarks
<p>3. (a) <math>P(A'   B) = 5P(A   B)</math>  <math>\frac{P(A' \cap B)}{P(B)} = 5 \left( \frac{P(A \cap B)}{P(B)} \right)</math>  <math>P(A' \cap B) = 5P(A \cap B)</math></p> <p><math>P(A \cap B) + P(A' \cap B) = p</math>  <math>P(A \cap B) + 5P(A \cap B) = p</math>  <math>P(A \cap B) = \frac{p}{6}</math></p> <p><math>P(A)</math>  <math>= P(A \cap B) + P(A \cap B')</math>  <math>= P(A \cap B) + P(A \cap B) + 0.45</math>  <math>= \frac{p}{3} + 0.45</math></p>	<p>1M</p> <p>1M</p> <p>1</p>	<p>either one</p>
<p>(b) Assume that <math>A</math> and <math>B</math> are independent.  <math>P(A \cap B) = P(A)P(B)</math>  <math>\frac{p}{6} = \left( \frac{p}{3} + 0.45 \right) p</math>  <math>2p + 2.7 = 1</math> (since <math>p \neq 0</math>)  <math>p = -0.85 &lt; 0</math></p> <p>Thus, <math>A</math> and <math>B</math> are not independent.</p>	<p>1M</p> <p>1A</p>	<p>f.t.</p>
<p>(c) <math>P(A) + P(C)</math>  <math>= \frac{p}{3} + 0.45 + 0.6</math>  <math>= 1.05 + \frac{p}{3}</math>  <math>&gt; 1</math>  <math>\geq P(A \cup C)</math></p> <p>Thus, <math>A</math> and <math>C</math> are not mutually exclusive.</p>	<p>1M</p> <p>1A</p>	<p>f.t.</p>
<p><math>P(A \cap C)</math>  <math>= P(A) + P(C) - P(A \cup C)</math>  <math>= \frac{p}{3} + 0.45 + 0.6 - P(A \cup C)</math>  <math>\geq \frac{p}{3} + 0.05</math> (<math>P(A \cup C) \leq 1</math>)  <math>\neq 0</math></p> <p>Thus, <math>A</math> and <math>C</math> are not mutually exclusive.</p>	<p>1M</p> <p>1A</p>	<p>f.t.</p>
	<p>----- (7)</p>	

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Solution	Marks	Remarks
<p>4. (a) A 95% confidence interval for <math>\mu</math></p> $= \left( \frac{150}{100} - 1.96 \left( \frac{0.4}{\sqrt{100}} \right), \frac{150}{100} + 1.96 \left( \frac{0.4}{\sqrt{100}} \right) \right)$ $= (1.4216, 1.5784)$	<p>1M+1A</p> <p>1A</p>	
<p>(b) Let <math>n</math> be the number of students.</p> <p>The width of the new confidence interval <math>= (2)(1.96) \left( \frac{0.4}{\sqrt{n}} \right)</math></p> <p>When <math>n &lt; 100</math>, we have <math>\frac{0.4}{\sqrt{n}} &gt; \frac{0.4}{\sqrt{100}}</math>.</p> <p>Thus, the width of the new confidence interval is greater.</p>	<p>1M</p> <p>1A</p>	<p>f.t.</p> <p>----- (5)</p>
<p>5. (a) <math>e^{\frac{-kx}{2}}</math></p> $= 1 + \left( \frac{-kx}{2} \right) + \frac{1}{2!} \left( \frac{-kx}{2} \right)^2 + \dots$ $= 1 - \frac{k}{2}x + \frac{k^2}{8}x^2 + \dots$	<p>1M</p> <p>1A</p>	
<p>(b) (i) <math>y = 64e^{-kx}</math></p> $\ln y = -kx + \ln 64$	<p>1A</p>	
<p>(ii) <math>\sqrt{y}(1-2x)^5</math></p> $= 8e^{\frac{-kx}{2}}(1-2x)^5$ $= 8 \left( 1 - \frac{k}{2}x + \frac{k^2}{8}x^2 + \dots \right) \left( 1 + C_1^5(-2x) + C_2^5(-2x)^2 + \dots + (-2x)^5 \right)$ $= 8 \left( 1 - \frac{k}{2}x + \frac{k^2}{8}x^2 + \dots \right) \left( 1 - 10x + 40x^2 + \dots - 32x^5 \right)$	<p>1M</p>	<p>for binomial expansion</p>
$8 \left( (1)(40) + \left( -\frac{k}{2} \right)(-10) + \left( \frac{k^2}{8} \right)(1) \right) = 449$ $320 + 40k + k^2 = 449$ $k^2 + 40k - 129 = 0$ $k = 3 \text{ or } k = -43 \text{ (rejected)}$ <p>Thus, the slope of the graph of the linear function in (b)(i) is <math>-3</math>.</p>	<p>1M</p> <p>1A</p>	<p>----- (6)</p>

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Solution	Marks	Remarks
<p>6. (a) <math>\frac{dy}{dx}</math></p> $= \frac{(6+2x^2)(-8x) - (9-4x^2)(4x)}{(6+2x^2)^2}$ $= \frac{-21x}{(3+x^2)^2}$	<p>1M</p> <p>1A</p>	
<p>(b) Let <math>h</math> be the <math>x</math>-coordinate of the point of contact. So, we have</p> $\frac{-21h}{(3+h^2)^2} = \frac{\frac{9-4h^2}{6+2h^2} + 2}{h-3}$ $-21h(h-3) = \frac{9-4h^2+12+4h^2}{2(3+h^2)}(3+h^2)^2$ $-21h(h-3) = \frac{21}{2}(3+h^2)$ $h^2 - 2h + 1 = 0$ $h = 1$ <p>The equation of <math>L</math> is</p> $y + 2 = \frac{-21(1)}{(3+(1)^2)^2}(x-3)$ $y + 2 = \frac{-21}{16}(x-3)$ $21x + 16y - 31 = 0$	<p>1M+1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	
	----- (7)	

Solution	Marks	Remarks
$\begin{aligned} 7. (a) \quad &g'(9) = 2g'(4) \\ &9^\beta 3^{\sqrt{9}} = (2)(4^\beta)(3^{\sqrt{4}}) \\ &\frac{9^\beta}{4^\beta} = \frac{2}{3} \\ &\left(\frac{3}{2}\right)^{2\beta} = \left(\frac{3}{2}\right)^{-1} \\ &\beta = \frac{-1}{2} \end{aligned}$	1	
$(b) \quad \text{Let } u = \sqrt{x} .$	1M	
$\text{So, we have } \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} .$		
$\begin{aligned} &g(x) \\ &= \int x^{-\frac{1}{2}} 3^{\sqrt{x}} dx \\ &= 2 \int 3^{\sqrt{x}} \left(\frac{1}{2}x^{-\frac{1}{2}}\right) dx \\ &= 2 \int 3^u du \\ &= 2 \left(\frac{3^u}{\ln 3}\right) + C \\ &= \frac{2}{\ln 3} 3^{\sqrt{x}} + C , \text{ where } C \text{ is a constant} \end{aligned}$	1M 1M	
$\begin{aligned} &g(4) = 0 \\ &\frac{2}{\ln 3} 3^{\sqrt{4}} + C = 0 \\ &C = \frac{-18}{\ln 3} \end{aligned}$	1M	
$\begin{aligned} &g(x) = \frac{2}{\ln 3} 3^{\sqrt{x}} - \frac{18}{\ln 3} \\ &g(9) \\ &= \frac{2}{\ln 3} 3^{\sqrt{9}} - \frac{18}{\ln 3} \\ &= \frac{36}{\ln 3} \end{aligned}$	1A	

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Solution	Marks	Remarks
$g(9)$ $= \int_4^9 g'(x) dx + g(4)$ $= \int_4^9 g'(x) dx$ $= \int_4^9 x^{-\frac{1}{2}} 3^{\sqrt{x}} dx$	1M	
Let $u = \sqrt{x}$ . So, we have $\frac{du}{dx} = \frac{1}{2} x^{-\frac{1}{2}}$ .	1M	
$g(9)$ $= \int_4^9 x^{-\frac{1}{2}} 3^{\sqrt{x}} dx$ $= 2 \int_2^3 3^u \left( \frac{1}{2} x^{-\frac{1}{2}} \right) dx$	1M	
$= 2 \int_2^3 3^u du$	1M	
$= 2 \left[ \frac{3^u}{\ln 3} \right]_2^3$	1M	
$= \frac{36}{\ln 3}$	1A	
-----(6)		



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Solution	Marks	Remarks																								
<p>8. (a) <math>f'(x) = 8ax^7 - 760x^4 - 8640x</math>  <math>f'(0) = 0</math></p> <p><math>f''(x) = 56ax^6 - 3040x^3 - 8640</math>  <math>f''(0) = -8640 &lt; 0</math>                      Thus, <math>f(x)</math> attains its maximum value at <math>x = 0</math>.</p>	1M																									
<p>(b) (i) <math>f'(-2) = 0</math>  <math>8a(-2)^7 - 760(-2)^4 - 8640(-2) = 0</math>  <math>a = 5</math></p>	1																									
<p>(ii) <math>f'(x) = 0</math>  <math>40x^7 - 760x^4 - 8640x = 0</math>  <math>x(x^6 - 19x^3 - 216) = 0</math>  <math>x(x^3 - 27)(x^3 + 8) = 0</math>  <math>x = 0, x = 3</math> or <math>x = -2</math></p>	1M																									
<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th><math>x</math></th> <th><math>x &lt; -2</math></th> <th><math>x = -2</math></th> <th><math>-2 &lt; x &lt; 0</math></th> <th><math>x = 0</math></th> <th><math>0 &lt; x &lt; 3</math></th> <th><math>x = 3</math></th> <th><math>x &gt; 3</math></th> </tr> </thead> <tbody> <tr> <td><math>f'(x)</math></td> <td>-</td> <td>0</td> <td>+</td> <td>0</td> <td>-</td> <td>0</td> <td>+</td> </tr> <tr> <td><math>f(x)</math></td> <td>↘</td> <td>-11136</td> <td>↗</td> <td>0</td> <td>↘</td> <td>-43011</td> <td>↗</td> </tr> </tbody> </table>	$x$	$x < -2$	$x = -2$	$-2 < x < 0$	$x = 0$	$0 < x < 3$	$x = 3$	$x > 3$	$f'(x)$	-	0	+	0	-	0	+	$f(x)$	↘	-11136	↗	0	↘	-43011	↗	1M	for testing
$x$	$x < -2$	$x = -2$	$-2 < x < 0$	$x = 0$	$0 < x < 3$	$x = 3$	$x > 3$																			
$f'(x)$	-	0	+	0	-	0	+																			
$f(x)$	↘	-11136	↗	0	↘	-43011	↗																			
<p>Thus, the least value of <math>f(x)</math> is <math>-43011</math>.</p>	1A																									
	----- (7)																									

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9. (a) The required probability $= (0.5)(0.3085) + (0.5)(0.1587)$ $= 0.2336$	1A -----(1)	
(b) The required probability $= \frac{(0.5)(0.3085)}{0.2336}$ $= \frac{3085}{4672}$	1M  1A -----(2)	r.t. 0.6603
(c) (i) The required probability $= \frac{e^{-2.1}(2.1^2)}{2!}(0.2336^2)$ $\approx 0.014734515$ $\approx 0.0147$	1M  1A	r.t. 0.0147
(ii) The required probability $= \frac{\frac{e^{-2.1}(2.1^1)}{1!}(0.5)(0.3085) + \frac{e^{-2.1}(2.1^2)}{2!}(2)(0.5)(0.3085)(0.5)(0.1587)}{\frac{e^{-2.1}(2.1^0)}{0!}(0.2336^0) + \frac{e^{-2.1}(2.1^1)}{1!}(0.2336^1) + \frac{e^{-2.1}(2.1^2)}{2!}(0.2336^2)}$ $\approx 0.234593001$ $\approx 0.2346$	1M+1M  1A	r.t. 0.2346
The required probability $= \frac{\frac{e^{-2.1}(2.1^1)}{1!}(0.2336)\left(\frac{3085}{4672}\right) + \frac{e^{-2.1}(2.1^2)}{2!}(2)(0.2336)\left(\frac{3085}{4672}\right)(0.2336)\left(1 - \frac{3085}{4672}\right)}{\frac{e^{-2.1}(2.1^0)}{0!}(0.2336^0) + \frac{e^{-2.1}(2.1^1)}{1!}(0.2336^1) + \frac{e^{-2.1}(2.1^2)}{2!}(0.2336^2)}$ $\approx 0.234593001$ $\approx 0.2346$	1M+1M  1A	r.t. 0.2346
(iii) The required probability $= \frac{e^{-2.1}(2.1^0)}{0!}(0.2336^0) + \frac{e^{-2.1}(2.1^1)}{1!}(0.2336^1) + \frac{e^{-2.1}(2.1^2)}{2!}(0.2336^2) + \dots$ $= \sum_{k=0}^{\infty} \frac{e^{-2.1}(2.1^k)}{k!}(0.2336^k)$ $= \sum_{k=0}^{\infty} \frac{e^{-2.1}(0.49056^k)}{k!}$ $= (e^{-2.1}) \sum_{k=0}^{\infty} \frac{(0.49056^k)}{k!}$ $= e^{-2.1} e^{0.49056}$ $= e^{-1.60944}$ $\approx 0.199999582 < 0.2$ Thus, the claim is agreed.	1M+1M        1M  1A -----(9)	f.t.

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Solution	Marks	Remarks
<p>10 (a) P(a certain athlete finishes the race in more than 12.1 seconds)</p> $= P\left(Z > \frac{12.1-12.3}{0.5}\right)$ $= P(Z > -0.4)$ $= 0.6554$	<p>1M</p> <p>1A</p> <p>----- (2)</p>	
<p>(b) The required probability</p> $= (0.6554)^8 + C_1^8 (0.6554)^7 (1-0.6554) + C_2^8 (0.6554)^6 (1-0.6554)^2$ $\approx 0.440775526$ $\approx 0.4408$	<p>1M+1M</p> <p>1A</p> <p>----- (3)</p>	<p>r.t. 0.4408</p>
<p>(c) (i) The required probability</p> $= (0.6554)^7$ $\approx 0.051945156$ $\approx 0.0519$	<p>1A</p>	<p>r.t. 0.0519</p>
<p>(ii) The required probability</p> $= C_2^7 (0.6554)^5 (1-0.6554)^2$ $\approx 0.301565757$ $\approx 0.3016$	<p>1M</p> <p>1A</p>	<p>r.t. 0.3016</p>
<p>(iii) (1) P(Peter can proceed to the next stage   Peter is the 3rd place in the group)</p> $= P(\text{the 3rd place in at least 2 other groups finishes the race in more than 12.1 seconds})$ $= P(\text{at least 6 athletes finish the race in more than 12.1 seconds in at least 2 other groups})$ $\approx C_2^5 (0.440775526)^2 (1-0.440775526)^3 +$ $C_3^5 (0.440775526)^3 (1-0.440775526)^2 +$ $C_4^5 (0.440775526)^4 (1-0.440775526) + (0.440775526)^5$ $\approx 0.72976499$ $\approx 0.7298$	<p>1M+1M</p> <p>1A</p>	<p>r.t. 0.7298</p>
<p>(2) P(Peter is the 2nd place in the group)</p> $= C_1^7 (0.6554)^6 (1-0.6554)$ $\approx 0.191184172$ <p>The required probability</p> $\approx 0.051945156 + 0.191184172 + (0.301565757)(0.72976499)$ $\approx 0.463201461$ $\approx 0.4632$	<p>1M</p> <p>1M</p> <p>1A</p> <p>----- (9)</p>	<p>r.t. 0.4632</p>

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Solution	Marks	Remarks
<p>11. (a) Let <math>g(x) = e^x \ln x</math>.</p> $\int_1^2 e^x \ln x \, dx$ $= \int_1^2 g(x) \, dx$ $\approx \frac{1}{2} \left( \frac{2-1}{5} \right) (g(1) + g(2) + 2(g(1.2) + g(1.4) + g(1.6) + g(1.8)))$ $\approx 2.082897622$ $\approx 2.0829$	<p>1M</p> <p>1A</p> <p>----- (2)</p>	
<p>(b) <math>\frac{d}{dx}(xe^x \ln x)</math></p> $= e^x \ln x + x \left( e^x \ln x + (e^x) \left( \frac{1}{x} \right) \right)$ $= (x+1)e^x \ln x + e^x$ <p>So, we have</p> $(x+1)e^x \ln x = \frac{d}{dx}(xe^x \ln x) - e^x$ $\int (x+1)e^x \ln x \, dx = xe^x \ln x - \int e^x \, dx$ $\int (x+1)e^x \ln x \, dx = xe^x \ln x - e^x + \text{constant}$ $\int \left( (x+1)e^x \ln x + \frac{1}{x} \right) dx$ $= xe^x \ln x - e^x + \int \frac{1}{x} \, dx$ $= xe^x \ln x - e^x + \ln x  + \text{constant}$ $= xe^x \ln x - e^x + \ln x + \text{constant}$	<p>1M</p> <p>1M</p> <p>1A</p> <p>----- (3)</p>	
<p>(c) (i) <math>\alpha</math></p> $= \int_1^2 \left( xe^x \ln x + \frac{1}{x} \right) dx$ $= \int_1^2 \left( (x+1)e^x \ln x + \frac{1}{x} \right) dx - \int_1^2 e^x \ln x \, dx$ $\approx \left[ xe^x \ln x - e^x + \ln x \right]_1^2 - 2.082897622$ $\approx 4.182882092$ $\approx 4.1829$	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p>for using the results of (a) and (b)</p> <p>r.t. 4.1829</p>

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Solution	Marks	Remarks
<p>(ii) <math>\frac{d}{dx}(e^x \ln x) = e^x \ln x + (e^x)\left(\frac{1}{x}\right)</math></p>	1M	
$\frac{d^2}{dx^2}(e^x \ln x)$ $= e^x \ln x + (e^x)\left(\frac{1}{x}\right) + (e^x)\left(\frac{1}{x}\right) - (e^x)\left(\frac{1}{x^2}\right)$ $= e^x\left(\ln x + \frac{2x-1}{x^2}\right)$	1M	
<p>Since <math>\frac{d^2}{dx^2}(e^x \ln x) &gt; 0</math> for <math>1 \leq x \leq 2</math>, the estimate of <math>\int_1^2 e^x \ln x \, dx</math> is an over-estimate.</p> <p>Then, the estimate of <math>\alpha</math> is an under-estimate. So, we have</p> $\alpha > 4.18288$ $> 4$	1A	
<p>Thus, the claim is agreed.</p>	1A	f.t.
	----- (8)	

**機密 (只限閱卷員使用)**  
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Solution	Marks	Remarks								
<p>12. (a) <math>\frac{du}{dt}</math>  <math>= -2e^{6-2t}</math>  <math>= -2u</math></p> <p><math>\frac{dN}{dt}</math>  <math>= \frac{dN}{du} \frac{du}{dt}</math>  <math>= (-Ae^{-u})(-2u)</math>  <math>= 2Aue^{-u}</math></p>	1A									
	1M	for chain rule								
	1A									
	-----(3)									
<p>(b) <math>\frac{d^2N}{dt^2}</math>  <math>= \frac{d}{du}(2Aue^{-u}) \frac{du}{dt}</math>  <math>= (2Ae^{-u} - 2Aue^{-u})(-2u)</math>  <math>= N(4u^2 - 4u)</math></p> <p><math>\therefore p(u) = 4u^2 - 4u</math></p>	1A									
	-----(2)									
<p>(c) (i) When <math>\frac{d}{dt} \left( \frac{dN}{dt} \right) = 0</math>, we have <math>u = 1</math> or <math>u = 0</math> (rejected)</p> <p><math>u = 1</math>  <math>e^{6-2t} = 1</math>  <math>t = 3</math></p> <p>Thus, <math>t_0 = 3</math>.</p>	1M									
	1A									
<p>(ii)</p> <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <thead> <tr> <th style="padding: 5px;"><math>t</math></th> <th style="padding: 5px;"><math>0 &lt; t &lt; 3</math></th> <th style="padding: 5px;"><math>t = 3</math></th> <th style="padding: 5px;"><math>t &gt; 3</math></th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;"><math>\frac{d}{dt} \left( \frac{dN}{dt} \right)</math></td> <td style="padding: 5px; text-align: center;">+</td> <td style="padding: 5px; text-align: center;">0</td> <td style="padding: 5px; text-align: center;">-</td> </tr> </tbody> </table>	$t$	$0 < t < 3$	$t = 3$	$t > 3$	$\frac{d}{dt} \left( \frac{dN}{dt} \right)$	+	0	-	1M	for testing
$t$	$0 < t < 3$	$t = 3$	$t > 3$							
$\frac{d}{dt} \left( \frac{dN}{dt} \right)$	+	0	-							
	1A	f.t.								
	-----(4)									
<p>(d) Note that <math>\lim_{t \rightarrow \infty} u = \lim_{t \rightarrow \infty} e^{6-2t} = 0</math>.</p> <p>The estimated number of bugs found after a very long time  <math>= \lim_{t \rightarrow \infty} N</math>  <math>= \lim_{t \rightarrow \infty} Ae^{-u}</math>  <math>= A</math></p>	1M									
	1A									
	-----(2)									