2021-DSE
MATH EP
M1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2021

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Candidate Number

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)
Question-Answer Book

8:30 am – 11:00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of TWO sections, A and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

(7) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

* A 0 3 1 E 0 0 1 *

SECTION A (50 marks)

1. The table below shows the probability distribution of a discrete random variable X, where a and b are constants.

x	-1	0	1	2	3	4
$P(X=x)$	a	0.15	0.15	b	0.05	0.25

It is given that $E(5X+1)=10$.

(a) Find a and b.

(b) Let C be the event that $X>0$ and D be the event that $X\leq 2$. Find $P(C\mid D)$.

(6 marks)

2. The probability that a person has disease D is 0.12. Test T is used to show whether a person has disease D or not. For a person who has disease D, the probability that test T shows that the person has disease D is 0.97. For a person who does not have disease D, the probability that test T shows that the person does not have disease D is 0.89.

(a) Find the probability that test T shows a correct result.

(b) Find the probability that test T shows that a person has disease D.

(c) Given that a person is shown to have disease D by test T, is the probability that the person actually has disease D less than 0.6 ? Explain your answer.

(6 marks)

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3. In an examination, there are 10 questions. For each question, the probability that Peter knows how to do the question is 0.8. For each question that Peter knows how to do, the probability that he carelessly answers the question wrongly is 0.1; otherwise, Peter will answer the question correctly for the question that he knows how to do. For questions that Peter does not know how to do, he will answer them wrongly. Peter gets grade A if he answers 8 or more questions correctly.

(a) Find the probability that Peter gets grade A.

(b) Find the probability that Peter knows how to do all the questions and gets grade A.

(c) Given that Peter gets grade A, find the probability that he knows how to do all the questions.

(7 marks)

4. Mary conducts a survey to estimate the proportion p of children in a city who learn recorder. In a random sample of 40 children from the city, 28 of them learn recorder.

(a) (i) Find the sample proportion of children who learn recorder.

(ii) Find an approximate 90% confidence interval for p.

(b) Mary now wants to construct an approximate 99% confidence interval for p such that the width of the confidence interval does not exceed 0.1. Using the result of (a)(i), estimate the least number of children that Mary should survey.

(6 marks)

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5. Let $f(x) = e^{-x^{\frac{1}{3}}}$.

(a) Let $g(u) = e^{-u}(u^2 + 2u + 2)$, where $u = x^{\frac{1}{3}}$. Find the constant β such that $\frac{dg(u)}{dx} = \beta f(x)$.

(b) Express, in terms of e, the area of the region bounded by the curve $y = f(x)$, the x-axis, the y-axis and the straight line $x = 8$.

(6 marks)

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6. (a) Expand e^{-6x} in ascending powers of x as far as the term in x^4.

(b) Find the constant k such that the coefficient of x^4 in the expansion of $e^{-6x}(1 - kx^2)^5$ is -26.

(5 marks)

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7. Let $y = \dfrac{e^x}{x^3 - x + 2}$, where $0 \le x \le 5$. Find

(a) $\dfrac{dy}{dx}$,

(b) the greatest value and the least value of y. (7 marks)

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8. Let $f(x)$ be a function such that $f'(x)=\frac{2^{kx}}{1+2^{kx}}$, where k is a constant. The straight line $8x-9y+10=0$ touches the curve $y=f(x)$ at the point A. It is given that the x-coordinate of A is 1. Find

(a) k,

(b) $f(x)$.

(7 marks)

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SECTION B (50 marks)

9. The weight of each potato in a large farm follows a normal distribution with a mean of 200 grams and a standard deviation of σ grams. The classification of the potatoes is as follows:

Weight of a potato (W grams)	$W < 180$	$180 \leq W < 230$	$W \geq 230$
Classification	*small*	*medium*	*big*

It is given that 21.19% of the potatoes in the farm are *small*.

(a) Find the percentage of *medium* potatoes in the farm. (3 marks)

(b) The potatoes in the farm are now inspected one by one. Find the probability that the 4th potato inspected is the 2nd *big* potato inspected. (3 marks)

(c) From the farm, 5 potatoes are randomly selected.

(i) Find the probability that there are exactly 1 *big* potato and 2 *small* potatoes.

(ii) Given that there is exactly 1 *big* potato, find the probability that there are at least 2 *small* potatoes.

(5 marks)

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10. The number of commercial emails that John receives each hour follows a Poisson distribution with a mean of 1.3 per hour, while the number of non-commercial emails that he receives each hour follows a Poisson distribution with a mean of 0.9 per hour.

(a) Find the probability that the number of non-commercial emails that John receives in a certain hour is fewer than 3. (3 marks)

(b) Find the probability that the number of commercial emails that John receives in 6 hours is 5. (2 marks)

(c) Find the probability that the number of emails that John receives in a certain hour is 2. (3 marks)

(d) Given that the number of emails that John receives in a certain hour is 2, find the probability that both emails are non-commercial emails. (3 marks)

(e) Given that the number of emails that John receives in a certain hour is fewer than 3, find the probability that John does not receive commercial email in that hour. (3 marks)

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11. Let $f(x)=\left(\frac{x}{2-x}\right)^{\frac{1}{2}}$, where $0 \le x \le 1$.

(a) Find $f'(x)$ and $f''(x)$. (3 marks)

(b) Define $J=\int_0^{0.5} f(x)\,dx$ and $K=\int_{0.5}^{1} f(x)\,dx$.

(i) Using the trapezoidal rule with 5 sub-intervals, estimate J.

(ii) Using the fact that $\int_0^1 f(x)\,dx=\frac{\pi-2}{2}$ and the result of (b)(i), estimate K.

(iii) Someone claims that $\frac{J}{K}<0.44$. Do you agree? Explain your answer.

(8 marks)

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12. A tank is used for collecting rain water. During a certain shower, rain water flows into the tank for 7 minutes. Let V m^3 be the volume of rain water in the tank. It is given that

$$\frac{dV}{dt} = \sqrt{t+1}\sqrt{3-\sqrt{t+1}} \quad (0 \le t \le 7),$$

where t is the number of minutes elapsed since rain water starts flowing into the tank. The tank is empty at $t = 0$ and the rate of change of the volume of rain water in the tank attains its maximum value when $t = T$.

(a) Find T. (4 marks)

(b) Find the exact value of V when $t = T$. (5 marks)

(c) The tank is in the shape of an inverted right circular cone of height 1 m and base radius 6 m. The tank is held vertically. Let h m be the depth of rain water in the tank. Find

(i) the constant Q such that $\frac{dV}{dt} = Qh^2\frac{dh}{dt}$,

(ii) $\left.\frac{dh}{dt}\right|_{t=T}$.

(5 marks)

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