SECTION A (50 marks)

1. The table below shows the probability distribution of a discrete random variable X, where k is a constant:

x	8	11	k	27	32
$P(X = x)$	0.2	0.1	0.3	0.3	0.1

It is given that $\mathrm{Var}(X) = 66$. Find k, $E(3X+5)$ and $\mathrm{Var}(3X+5)$. (6 marks)

Answers written in the margins will not be marked.

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2. Let A and B be two events. Denote the complementary events of A and B by A' and B' respectively. Suppose that $P(A' \cap B) = 0.12$ and $P(B' \mid A') = 2P(A)$.

(a) By considering $P(A' \cap B')$, or otherwise, find $P(A)$.

(b) If A and B are independent, find $P(B)$.

(6 marks)

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3. A lucky draw is held in a shop. In each day, there is a big prize in this lucky draw. When the big prize is won, the lucky draw on that day stops. For each draw, the probability of winning the big prize is 0.2 .

(a) Write down the mean and the variance of the number of draws for winning the big prize in a day.

(b) Within the first 4 draws in a day, are winning the big prize and not winning the big prize of equal chance? Explain your answer.

(c) Find the probability of not winning the big prize within the first 4 draws in each day for 5 days.

(7 marks)

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4. In each month, the probability that a store offers a discount to its products is 0.35 . If a discount is offered in a certain month, then the probability that the store makes a profit in that month is 0.7 ; otherwise, the probability of making a profit in that month is 0.28 .

(a) Find the probability that the store makes a profit in a certain month.

(b) Given that the store makes a profit in a certain month, find the probability that the store offers a discount in that month.

(c) Find the probability that the store makes a profit in at least 2 months out of 12 months.

(6 marks)

Answers written in the margins will not be marked.

5. Define $f(x)=\frac{6-x}{x+3}$ for all $x>-3$.

(a) Prove that $f(x)$ is decreasing.

(b) Find $\lim_{x\to\infty} f(x)$.

(c) Find the exact value of the area of the region bounded by the graph of $y=f(x)$, the x-axis and the y-axis.

(6 marks)

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6. (a) Expand e^{-18x} in ascending powers of x as far as the term in x^2.

(b) Let n be a positive integer. If the coefficient of x^2 in the expansion of $e^{-18x}(1+4x)^n$ is -38, find n.

(6 marks)

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7. Consider the curve $C: y=(x-2)\sqrt{3x+6}-8x$, where $x>-2$.

(a) Find $\frac{dy}{dx}$.

(b) Someone claims that two of the tangents to C are horizontal lines. Do you agree? Explain your answer.

(6 marks)

Answers written in the margins will not be marked.

8. (a) Express $7^{\frac{-1}{\ln 7}}$ in terms of e.

(b) By considering $\frac{d}{dx}(x7^{-x})$, find $\int x7^{-x}\,dx$.

(c) Define $h(x)=x7^{-x}$ for all real numbers x. It is given that the equation $h'(x)=0$ has only one real root α. Find α. Also express $\int_0^{\alpha} h(x)\,dx$ in terms of e.

(7 marks)

SECTION B (50 marks)

9. The number of matches won by a basketball team in a season follows a Poisson distribution with a mean of 3 matches per season. The points scored by the team in a match follows a normal distribution with a mean of 66 points and a standard deviation of 10 points.

(a) Find the probability that the team wins fewer than 6 matches in a certain season. (3 marks)

(b) Find the probability that the team scores higher than 70 points in a certain match. (2 marks)

(c) The team receives a certificate if the team wins a match and scores more than 70 points in that match. The team is awarded a bonus in a certain season if the team receives more than 2 certificates in that season.

(i) Find the probability that the team wins exactly 3 matches in a certain season and is awarded a bonus in that season.

(ii) If the team wins exactly 4 matches in a certain season, find the probability that the team is awarded a bonus in that season.

(iii) Given that the team wins fewer than 6 matches in a certain season, find the probability that the team is awarded a bonus in that season.

(7 marks)

10. In city H, the water consumption (in m^3) of each family in a certain month follows a normal distribution with a mean of μ m^3 and a standard deviation of $4\ m^3$.

(a) A survey is conducted to estimate μ.

(i) A random sample of 16 families is selected and their water consumptions (in m^3) in that month are recorded as follows:

17 17 18 19 19 20 20 21 21 21 22 23 23 23 24 24

Find a 95% confidence interval for μ.

(ii) Find the least sample size to be taken such that the width of a 99.5% confidence interval for μ is less than 3.

(7 marks)

(b) Suppose that $\mu = 20$. If the water consumption of a family in that month lies between $18\ m^3$ and $23\ m^3$, the family is regarded as *ordinary*.

(i) Find the percentage of *ordinary* families in city H.

(ii) The families in city H are randomly selected one by one and their water consumptions in that month are recorded. The recording stops when 3 *ordinary* families are found. Given that more than 6 families are selected in this recording process, find the probability that the water consumptions of exactly 9 families are recorded.

(6 marks)

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11. A steel factory has two machines, P and Q, for producing steel. The two machines start production at the same time. The manager of the factory models the rates of change of the amount of steel produced (in thousand tonnes per month) by P and Q respectively by

$$\mathrm{p}(t) = 2t\ln(t^2+4) \quad \text{and} \quad \mathrm{q}(t) = \frac{4\ln(2e^t+1)}{e^{-t}+2} \quad (0 \le t \le 4),$$

where t is the number of months elapsed since the steel production begins. Denote the total amount of steel produced by P in the first 4 months by α thousand tonnes. Let α_1 be the estimate of α by using the trapezoidal rule with 4 sub-intervals.

(a) (i) Find α_1.

(ii) Is α_1 an over-estimate or an under-estimate? Explain your answer.

(6 marks)

(b) Let β thousand tonnes be the total amount of steel produced by Q in the first 4 months.

(i) Using the substitution $u = \ln(2e^t+1)$, find β.

(ii) The manager claims that the total amount of steel produced by Q in the first 4 months exceeds 30% of the sum of the total amount of steel produced by P and Q in the first 4 months. Do you agree? Explain your answer.

(6 marks)

12. A tank contains some water. Water is now leaking from the tank. Let V m^3 be the volume of water in the tank. It is given that

$$V = \frac{64}{he^{kt}+4},$$

where $t\ (\geq 0)$ is the number of hours elapsed since the leaking begins and h and k are constants.

(a) Express $\ln\left(\frac{64}{V}-4\right)$ as a linear function of t.

(1 mark)

(b) It is given that the graph of the linear function obtained in (a) passes through the origin and the point $(2,1)$. Find

(i) h and k,

(ii) $\frac{dV}{dt}$,

(iii) the value of V when $\frac{dV}{dt}$ attains its least value.

(8 marks)

(c) The owner of the tank finds that $S = V^{\frac{2}{3}}$, where S m^2 is the wet total surface area of the tank.

(i) Find the value of $\frac{dS}{dt}$ when $\frac{dV}{dt}$ attains its least value.

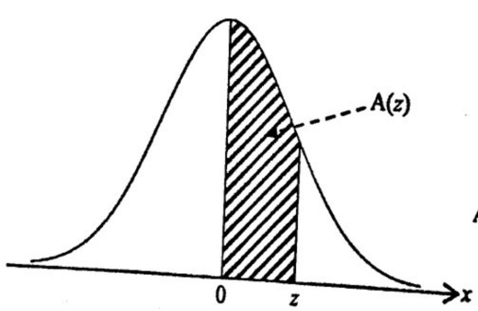
(ii) The owner claims that $\frac{dS}{dt}$ attains its least value when $\frac{dV}{dt}$ attains its least value. Is the claim correct? Explain your answer.

(4 marks)

Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note: An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$