Solution	Marks	Remarks
1. $E(X)$		
$=0.2(8)+0.1(11)+0.3(k)+0.3(27)+0.1(32)$	1M	for either one
$=14+0.3k$		
$E(X^2)$		
$=0.2(8^2)+0.1(11^2)+0.3(k^2)+0.3(27^2)+0.1(32^2)$		
$=346+0.3k^2$		
$Var(X)=E(X^2)-(E(X))^2$		
$66=346+0.3k^2-(14+0.3k)^2$	1M	
$0.21k^2-8.4k+84=0$		
$k=20$	1A	
$E(3X+5)$		
$=3E(X)+5$	1M	for either one
$=3(14+0.3(20))+5$		
$=65$	1A	
$Var(3X+5)$		
$=9Var(X)$		
$=9(66)$		
$=594$	1A	
	----------(6)	
2. (a) Let $P(A)=a$.		
$P(A'\cap B')$		
$=P(B'\mid A')P(A')$	1M	for conditional probability
$=2a(1-a)$		
$=2a-2a^2$		
$P(A')=P(A'\cap B')+P(A'\cap B)$		
$1-a=2a-2a^2+0.12$	1M	
$2a^2-3a+0.88=0$		
$a=0.4$ or $a=1.1$ (rejected)		
Thus, we have $P(A)=0.4$.	1A	
(b) $P(A)P(B)=P(A\cap B)$	1M	
$P(A)P(B)=P(B)-P(A'\cap B)$		
$0.4P(B)=P(B)-0.12$	1M	for using the result of (a)
$0.6P(B)=0.12$		
$P(B)=0.2$	1A	
	----------(6)	

Solution	Marks	Remarks
3. (a) The mean = 5	1A	
The variance = 20	1A	
(b) The probability of winning the big prize within the first 4 draws		
$= 0.2 + (1-0.2)(0.2) + (1-0.2)^2(0.2) + (1-0.2)^3(0.2)$	1M	for $(1-p)^k p$
$= 1-(1-0.2)^4$		
$= 0.5904$	1A	
> 0.5		
Thus, winning the big prize and not winning the big prize within the first 4 draws are not of equal chance.	1A	f.t.
The probability of not winning the big prize within the first 4 draws		
$= (1-0.2)^4$	1M	for $(1-p)^k$
$= 0.4096$	1A	
The probability of winning the big prize within the first 4 draws		
$= 1-0.4096$		
$= 0.5904$		
$\neq 0.4096$		
Thus, winning the big prize and not winning the big prize within the first 4 draws are not of equal chance.	1A	f.t.
(c) The required probability		
$= (1-0.5904)^5$	1M	for q^5
[redacted]		
≈ 0.0115	1A	r.t. 0.0115
	----------(7)	
4. (a) The required probability		
$= (0.35)(0.7) + (1-0.35)(0.28)$	1M	for $pq + (1-p)r$
$= 0.427$	1A	
(b) The required probability		
$= \dfrac{(0.35)(0.7)}{0.427}$	1M	for denominator using (a)
$= \dfrac{35}{61}$	1A	r.t. 0.5738
[redacted]		
≈ 0.5738		
(c) The required probability		
$= 1-(1-0.427)^{12} - C_1^{12}(1-0.427)^{11}(0.427)$	1M	
[redacted]		
≈ 0.9875	1A	r.t. 0.9875
	----------(6)	

Solution	Marks	Remarks
5. (a) For all $x > -3$, $f'(x)$ $= \frac{(x+3)(-1)-(6-x)(1)}{(x+3)^2}$ $= \frac{-9}{(x+3)^2}$ < 0 Thus, $f(x)$ is decreasing.	1	
Note that $f(x) = \frac{9}{x+3} - 1$ for all $x > -3$. Thus, $f(x)$ is decreasing.	1	
(b) $\lim_{x\to\infty} f(x)$ $= \lim_{x\to\infty} \frac{\frac{6}{x}-1}{1+\frac{3}{x}}$ $= -1$	1A	
$\lim_{x\to\infty} f(x)$ $= \lim_{x\to\infty} \left(\frac{9}{x+3} - 1\right)$ $= -1$	1A	
(c) For $y = 0$, we have $x = 6$. The required area		
$= \int_0^6 f(x)\,dx$	1M	
$= \int_0^6 \frac{6-x}{x+3}\,dx$		
$= \int_0^6 \left(\frac{9}{x+3} - 1\right) dx$	1M	
$= [9\ln(x+3) - x]_0^6$	1M	
$= 9\ln 3 - 6$	1A	
For $y = 0$, we have $x = 6$. The required area		
$= \int_0^6 f(x)\,dx$	1M	
$= \int_0^6 \frac{6-x}{x+3}\,dx$		
$= \int_3^9 \frac{6-(u-3)}{u}\,du$ (by letting $u = x+3$)	1M	
$= \int_3^9 \left(\frac{9}{u} - 1\right) du$		
$= [9\ln u - u]_3^9$	1M	

Solution	Marks	Remarks
(a) e^{-18x}		
$=1+(-18x)+\frac{(-18x)^2}{2!}+\cdots$	1M	
$=1-18x+162x^2+\cdots$	1A	
(b) $(1+4x)^n$		
$=1+C_1^n(4x)+C_2^n(4x)^2+\cdots+C_n^n(4x)^n$	1M	
$=1+4C_1^nx+16C_2^nx^2+\cdots+4^nx^n$		
$16C_2^n-72C_1^n+162=-38$	1M	
$16\left(\frac{n(n-1)}{2}\right)-72n+162=-38$		
$n^2-10n+25=0$	1M	
$n=5$	1A	
	----------(6)	
(a) $\frac{dy}{dx}$		
$=(3x+6)^{\frac{1}{2}}+\frac{1}{2}(3)(3x+6)^{\frac{-1}{2}}(x-2)-8$	1M	
$=\sqrt{3x+6}+\frac{3(x-2)}{2\sqrt{3x+6}}-8$		
$=\frac{9x+6}{2\sqrt{3x+6}}-8$	1A	
(b) Note that the slope of a horizontal tangent is 0.		
$\frac{9x+6}{2\sqrt{3x+6}}-8=0$	1M	
$9x+6=16\sqrt{3x+6}$		
$(9x+6)^2=256(3x+6)$	1M	
$27x^2-220x-500=0$		
$x=10$ or $x=\frac{-50}{27}$		
$\left.\frac{dy}{dx}\right\|_{x=10}=\frac{9(10)+6}{2\sqrt{3(10)+6}}-8=0$	1M	for testing
$\left.\frac{dy}{dx}\right\|_{x=\frac{-50}{27}}=\frac{9\left(\frac{-50}{27}\right)+6}{2\sqrt{3\left(\frac{-50}{27}\right)+6}}-8=-16\neq 0$		
So, we have $x=10$ only.		
Hence, only one tangent to C is a horizontal line.		
Thus, the claim is disagreed.	1A	f.t.
	----------(6)	

Solution	Marks	Remarks
8. (a) $\ln 7^{\frac{-1}{\ln 7}}$		
$= \frac{-1}{\ln 7}(\ln 7)$		
$= -1$		
$7^{\frac{-1}{\ln 7}}$	1A	
$= e^{-1}$		
$= \frac{1}{e}$		
(b) $\frac{d}{dx}(x7^{-x})$	1M	for $\frac{d}{dx}(7^{-x}) = -7^{-x}\ln 7$
$= 7^{-x} - x(7^{-x}\ln 7)$		
So, we have $x7^{-x} = \frac{1}{\ln 7}\left(7^{-x} - \frac{d}{dx}(x7^{-x})\right)$.		
$\int x7^{-x}dx$		
$= \frac{1}{\ln 7}\left(\int 7^{-x}dx - x7^{-x}\right)$	1M	
$= \frac{1}{\ln 7}\left(\frac{-7^{-x}}{\ln 7} - x7^{-x}\right) + \text{constant}$	1A	
$= \frac{-1}{\ln 7}\left(\frac{1}{\ln 7} + x\right)7^{-x} + \text{constant}$		
(c) For $h'(x) = 0$, we have $7^{-x}(1 - x\ln 7) = 0$.		
So, we have $\alpha = \frac{1}{\ln 7}$.	1A	r.t. 0.5139
$\int_0^{\alpha} h(x)dx$		
$= \left[\frac{-1}{\ln 7}\left(\frac{1}{\ln 7} + x\right)7^{-x}\right]_0^{\frac{1}{\ln 7}}$	1M	
$= \frac{-1}{\ln 7}\left(\frac{2(7^{\frac{-1}{\ln 7}})}{\ln 7} - \frac{1}{\ln 7}\right)$		
$= \frac{1}{(\ln 7)^2}\left(1 - \frac{2}{e}\right)$ (by (a))	1A	
$= \frac{e-2}{e(\ln 7)^2}$		
	---------(7)	

	Marks	Remarks
(a) The required probability		
$=\frac{3^0e^{-3}}{0!}+\frac{3^1e^{-3}}{1!}+\frac{3^2e^{-3}}{2!}+\frac{3^3e^{-3}}{3!}+\frac{3^4e^{-3}}{4!}+\frac{3^5e^{-3}}{5!}$	1M+1M	1M for the 6 cases + 1M for Poisson probability
$=18.4e^{-3}$		
≈ 0.916082058	1A	r.t. 0.9161
≈ 0.9161		
	----------(3)	
(b) The required probability		
$=P\left(Z>\frac{70-66}{10}\right)$	1M	
$=P(Z>0.4)$		
$=0.5-0.1554$		
$=0.3446$	1A	
	----------(2)	
(c) (i) The required probability		
$=(0.3446)^3\left(\frac{3^3e^{-3}}{3!}\right)$	1M	
≈ 0.009168006		
≈ 0.0092	1A	r.t. 0.0092
(ii) The required probability		
$=C_3^4(0.3446)^3(1-0.3446)+(0.3446)^4$	1M	
≈ 0.121379753		
≈ 0.1214	1A	r.t. 0.1214
(iii) The probability that the team is awarded a bonus in a certain season if the team wins exactly 5 matches in that season		
$=C_3^5(0.3446)^3(1-0.3446)^2+C_4^5(0.3446)^4(1-0.3446)+(0.3446)^5$		
≈ 0.226845138		
The required probability		
$\approx\frac{0.009168006+(0.121379753)\left(\frac{3^4e^{-3}}{4!}\right)+(0.226845138)\left(\frac{3^5e^{-3}}{5!}\right)}{18.4e^{-3}}$	1M+1M	1M for numerator + 1M for denominator
≈ 0.057237086		
≈ 0.0572	1A	r.t. 0.0572
	----------(7)	

Solution	Marks	Remarks
0. (a) (i) The sample mean		
$=\frac{17+17+18+19+19+20+20+21+21+21+22+23+23+23+24+24}{16}$		
$=20.75\text{ m}^3$	1A	
A 95% confidence interval for μ		
$=\left(20.75-1.96\left(\frac{4}{\sqrt{16}}\right), 20.75+1.96\left(\frac{4}{\sqrt{16}}\right)\right)$	1M+1A	1A for 1.96
$=(18.79, 22.71)$	1A	
(ii) Let n be the sample size.		
$2(2.81)\left(\frac{4}{\sqrt{n}}\right)<3$	1M+1A	1A for 2.81
$n>56.15004444$		
Thus, the least sample size is 57.	1A	
	-----(7)	
(b) (i) The required percentage		
$=P\left(\frac{18-20}{4}<Z<\frac{23-20}{4}\right)\times 100\%$	1M	
$=P(-0.5<Z<0.75)\times 100\%$		
$=(0.1915+0.2734)\times 100\%$		
$=0.4649\times 100\%$		
$=46.49\%$	1A	
(ii) Take $p=0.4649$.		
The required probability		
$=\frac{C_2^8(1-p)^6p^3}{1-p^3-C_1^3(1-p)p^3-C_2^4(1-p)^2p^3-C_3^5(1-p)^3p^3}$	1M+1M+1M	1M for using (b)(i) + 1M for numerator +1M for denominator
≈ 0.160443919		
≈ 0.1604	1A	r.t. 0.1604
The required probability		
$=\frac{C_2^8(1-0.4649)^6(0.4649)^3}{(1-0.4649)^6+C_1^6(1-0.4649)^5(0.4649)+C_2^6(1-0.4649)^4(0.4649)^2}$	1M+1M+1M	1M for using (b)(i) + 1M for numerator+1M for denominator
≈ 0.160443919		
≈ 0.1604	1A	r.t. 0.1604
	-----(6)	

	Marks	Remarks
11. (a) (i) α_1		
$=\frac{1}{2}\left(\frac{4-0}{4}\right)(\text{p}(0)+\text{p}(4)+2(\text{p}(1)+\text{p}(2)+\text{p}(3)))$	1M	
$=2\ln 281216000$		
≈ 38.90926723	1A	r.t. 38.9093
≈ 38.9093		
(ii) $\frac{\text{dp}(t)}{\text{d}t}$		
$=\frac{4t^2}{t^2+4}+2\ln(t^2+4)$	1A	
$\frac{\text{d}^2\text{p}(t)}{\text{d}t^2}$		
$=4\left(\frac{(t^2+4)(2t)-(t^2)(2t)}{(t^2+4)^2}\right)+\frac{4t}{t^2+4}$	1M	
$=\frac{32t}{(t^2+4)^2}+\frac{4t}{t^2+4}$	1A	
$=\frac{4t(t^2+12)}{(t^2+4)^2}$		
$\frac{\text{d}^2\text{p}(t)}{\text{d}t^2}=0$ when $t=0$ and $\frac{\text{d}^2\text{p}(t)}{\text{d}t^2}>0$ for $0<t\le 4$.		
Thus, α_1 is an over-estimate.	1A	f.t.
	----------(6)	
(b) (i) Let $u=\ln(2e^t+1)$.		
So, we have $\frac{\text{d}u}{\text{d}t}=\frac{2e^t}{2e^t+1}$.	1A	
β		
$=\int_0^4 \text{q}(t)\,\text{d}t$	1M	
$=\int_0^4 \frac{4e^t\ln(2e^t+1)}{(2+e^{-t})e^t}\text{d}t$		
$=\int_0^4 \frac{4e^t\ln(2e^t+1)}{2e^t+1}\text{d}t$		
$=2\int_{\ln 3}^{\ln(2e^4+1)} u\,\text{d}u$	1M	
$=\left[u^2\right]_{\ln 3}^{\ln(2e^4+1)}$		
$=(\ln(2e^4+1))^2-(\ln 3)^2$	1A	r.t. 20.9043
≈ 20.90433138		
≈ 20.9043		

Solution	Marks	Remarks
(ii) By (a)(ii), α_1 is an over-estimate of α.		
$\alpha_1 > \alpha$		
$\alpha_1 + \beta > \alpha + \beta$	1M	
$\frac{\beta}{\alpha+\beta} > \frac{\beta}{\alpha_1+\beta}$		
$\frac{\beta}{\alpha_1+\beta}$		
$=\frac{(\ln(2e^4+1))^2-(\ln 3)^2}{2\ln 281216000+(\ln(2e^4+1))^2-(\ln 3)^2}$		
≈ 0.349491284		
>0.3		
So, we have $\frac{\beta}{\alpha+\beta} > \frac{\beta}{\alpha_1+\beta} > 30\%$.		
Thus, the claim is agreed.	1A	f.t.
By (a)(ii), α_1 is an over-estimate of α.		
$\alpha_1 > \alpha$		
$0.3(\alpha_1+\beta) > 0.3(\alpha+\beta)$	1M	
$0.3(\alpha_1+\beta)$		
$=0.3(2\ln 281216000+(\ln(2e^4+1))^2-(\ln 3)^2)$		
≈ 17.94407958		
$<\beta$		
So, we have $\beta > 0.3(\alpha_1+\beta) > 0.3(\alpha+\beta)$.		
Thus, the claim is agreed.	1A	f.t.
	----------(6)	

	Marks	Remarks
12. (a) $V=\dfrac{64}{he^{kt}+4}$ $\dfrac{64}{V}-4=he^{kt}$ $\ln\left(\dfrac{64}{V}-4\right)=kt+\ln h$	1A	
	----(1)	
(b) (i) $\ln h=0$ $h=1$	1A	
$k=\dfrac{1-0}{2-0}$ $k=0.5$	1A	
(ii) $V=\dfrac{64}{e^{0.5t}+4}$ $\dfrac{dV}{dt}$ $=-64(e^{0.5t}+4)^{-2}(0.5)e^{0.5t}$	1M	
$=\dfrac{-32e^{0.5t}}{(e^{0.5t}+4)^2}$	1A	
(iii) $\dfrac{d}{dt}\left(\dfrac{dV}{dt}\right)$ $=\dfrac{-32((e^{0.5t}+4)^2(0.5e^{0.5t})-(e^{0.5t})2(e^{0.5t}+4)(0.5e^{0.5t}))}{(e^{0.5t}+4)^4}$ $=\dfrac{16e^{0.5t}(e^{0.5t}-4)}{(e^{0.5t}+4)^3}$	1A	
For $\dfrac{d}{dt}\left(\dfrac{dV}{dt}\right)=0$, we have $\dfrac{16e^{0.5t}(e^{0.5t}-4)}{(e^{0.5t}+4)^3}=0$. So, we have $t=4\ln 2$.	1M	

t	$0\le t<4\ln 2$	$t=4\ln 2$	$t>4\ln 2$
$\dfrac{d}{dt}\left(\dfrac{dV}{dt}\right)$	$-$	0	+

1M — for testing

Therefore, $\dfrac{dV}{dt}$ attains its least value when $t=4\ln 2$.

The required value of V

$=\dfrac{64}{4+4}$

$=8$ — 1A

----(8)

Solution

Solution	Marks	Remarks
(c) (i) $\frac{dS}{dt}=\frac{2}{3}V^{\frac{-1}{3}}\frac{dV}{dt}$	1M	
When $t=4\ln 2$, $\frac{dS}{dt}=\frac{2}{3}(8)^{\frac{-1}{3}}\left(\frac{-32(4)}{(4+4)^2}\right)=\frac{-2}{3}$	1A	
$S=16(e^{0.5t}+4)^{\frac{-2}{3}}$ $\frac{dS}{dt}=16\left(\frac{-2}{3}(e^{0.5t}+4)^{\frac{-5}{3}}(0.5e^{0.5t})\right)$	1M	
$=\frac{-16e^{0.5t}}{3(e^{0.5t}+4)^{\frac{5}{3}}}$ When $t=4\ln 2$, we have $\frac{dS}{dt}=\frac{-2}{3}$.	1A	
(ii) $\frac{dS}{dt}=\frac{2}{3}V^{\frac{-1}{3}}\frac{dV}{dt}$ $\frac{d}{dt}\left(\frac{dS}{dt}\right)=\frac{2}{3}V^{\frac{-1}{3}}\frac{d}{dt}\left(\frac{dV}{dt}\right)-\frac{2}{9}V^{\frac{-4}{3}}\left(\frac{dV}{dt}\right)^2$	1M	
When $t=4\ln 2$, $\frac{d}{dt}\left(\frac{dS}{dt}\right)=\frac{2}{3}(8)^{\frac{-1}{3}}(0)-\frac{2}{9}(8)^{\frac{-4}{3}}(-2)^2=\frac{-1}{18}\neq 0$ Thus, the claim is not correct.	1A	f.t.
$\frac{d}{dt}\left(\frac{dS}{dt}\right)=\frac{16e^{0.5t}(e^{0.5t}-6)}{9(e^{0.5t}+4)^{\frac{8}{3}}}$	1M	
For $\frac{d}{dt}\left(\frac{dS}{dt}\right)=0$, we have $t=2\ln 6\neq 4\ln 2$. Thus, the claim is not correct.	1A	f.t.
	----------(4)	