Solution	Marks	Remarks
1. (a) $\mathrm{P}(B)$		
$=\mathrm{P}(B \mid A)\mathrm{P}(A)+\mathrm{P}(B \mid A')\mathrm{P}(A')$		
$=(0.45)(0.8)+(0.6)(1-0.8)$	1M	
$=0.48$	1A	
$\mathrm{P}(B\cap A)$ $=\mathrm{P}(B \mid A)\mathrm{P}(A)$ $=(0.45)(0.8)$ $=0.36$		
$\mathrm{P}(B\cap A')$ $=\mathrm{P}(B \mid A')\mathrm{P}(A')$ $=(0.6)(1-0.8)$ $=0.12$		
$\mathrm{P}(B)$ $=\mathrm{P}(B\cap A)+\mathrm{P}(B\cap A')$		
$=0.36+0.12$	1M	
$=0.48$	1A	
(b) $\mathrm{P}(A \mid B)$ $=\dfrac{\mathrm{P}(B \mid A)\mathrm{P}(A)}{\mathrm{P}(B)}$		
$=\dfrac{(0.45)(0.8)}{0.48}$	1M	
$=0.75$	1A	
(c) $\mathrm{P}(B\cap A)$ $=\mathrm{P}(B \mid A)\mathrm{P}(A)$ $=(0.45)(0.8)$ $=0.36$		
$\mathrm{P}(A\cup B)$ $=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$ $=0.8+0.48-0.36$		
$=0.92$	1A	
	----------(5)	

	Marks	Remarks
(a) (i) The sample proportion		
$= \frac{0.0915 + 0.3085}{2}$		
$= 0.2$	1A	
(ii) z		
$= \frac{0.3085 - 0.0915}{2\sqrt{\frac{0.2(1-0.2)}{64}}}$	1M	
$= 2.17$		
Thus, we have $\beta = 97$.	1A	
(b) Let n be the number of households.		
$1-(1-0.2)^n > 0.999$	1M	
$0.001 > 0.8^n$		
$\log 0.001 > \log(0.8^n)$		
$\log 0.001 > n \log 0.8$	1M	
$n > \frac{-3}{\log 0.8}$		
$n > 30.95655348$		
Thus, the least number of households is 31 .	1A	
	----------(6)	
(a) The required probability		
$= \mathrm{P}\left(Z > \frac{9.16-9}{0.125}\right)$	1M	
$= \mathrm{P}(Z > 1.28)$		
$= 0.1003$	1A	
(b) (i) $\mathrm{P}(X \leq 3)$		
$= 0.1003 + (1-0.1003)(0.1003) + (1-0.1003)^2(0.1003)$	1M	
≈ 0.271728757		
≈ 0.2717	1A	r.t. 0.2717
(ii) $\mathrm{E}(X)$		
$= \frac{1}{0.1003}$	1M	
≈ 9.970089731		
≈ 9.9701	1A	r.t. 9.9701
	----------(6)	

Solution	Marks	Remarks
4. (a) $p+0.25+0.5=1$	1M	
$p=0.25$		
$\mathrm{E}(Y)$		
$=-2(0.25)+2(0.25)+0.5m$	1M	
$=0.5m$		
$\mathrm{Var}(Y)$		
$=0.25(-2-0.5m)^2+0.25(2-0.5m)^2+0.5(m-0.5m)^2$	1M	
$=0.25(4+2m+0.25m^2+4-2m+0.25m^2)+0.125m^2$		
$=0.25m^2+2$	1	
$0.25+p+0.5=1$	1M	
$p=0.25$		
$\mathrm{E}(Y)$		
$=-2(0.25)+2(0.25)+0.5m$	1M	
$=0.5m$		
$\mathrm{Var}(Y)$		
$=(-2)^2(0.25)+(2)^2(0.25)+m^2(0.5)-(0.5m)^2$	1M	
$=0.25m^2+2$	1	
(b) Note that $\mathrm{E}(2Y-1)=2\mathrm{E}(Y)-1$ and $\mathrm{Var}(2Y-1)=4\mathrm{Var}(Y)$.	1M+1M	
$\mathrm{Var}(2Y-1)=8\mathrm{E}(2Y-1)$		
$4\mathrm{Var}(Y)=16\mathrm{E}(Y)-8$		
$4(0.25m^2+2)=16(0.5m)-8$		
$m^2-8m+16=0$		
$m=4$	1A	
	(7)	

Solution | Marks | Remarks

5. Note that $3x^2-24x+49=3(x-4)^2+1\neq 0$.

(a) $f'(x)=0$

$$\frac{12x-48}{(3x^2-24x+49)^2}=0$$

$x=4$

1M

x	$(-\infty,4)$	4	$(4,\infty)$
$f'(x)$	$-$	0	$+$

So, $f(x)$ attains its minimum value at $x=4$.

Thus, we have $\alpha=4$.

1A

$f'(x)=0$

$$\frac{12x-48}{(3x^2-24x+49)^2}=0$$

$x=4$

1M

$f''(x)$

$$=\frac{-108x^2+864x-1716}{(3x^2-24x+49)^3}$$

$f''(4)$

$=12$

>0

So, $f(x)$ attains its minimum value at $x=4$.

Thus, we have $\alpha=4$.

1A

(b) (i) Let $v=3x^2-24x+49$. Then, we have $\frac{dv}{dx}=6x-24$.

$f(x)$

$$=\int\frac{12x-48}{(3x^2-24x+49)^2}dx$$

$$=\int\frac{2}{v^2}dv$$

1M

$$=\frac{-2}{v}+C$$

$$=\frac{-2}{3x^2-24x+49}+C$$

Since $f(x)$ has only one extreme value, we have $f(4)=5$.

1M

$$\frac{-2}{3(4)^2-24(4)+49}+C=5$$

$C=7$

Thus, we have $f(x)=\frac{-2}{3x^2-24x+49}+7$.

1A

(ii) $\lim_{x\to\infty} f(x)$

$=7$

1A

----------(6)

Solution	Marks	Remarks
6. (a) $e^{kx}+e^{2x}$ $=\left(1+kx+\frac{(kx)^2}{2!}+\cdots\right)+\left(1+2x+\frac{(2x)^2}{2!}+\cdots\right)$	1M	for expanding e^{kx} or
$=2+(k+2)x+\frac{(k^2+4)}{2}x^2+\cdots$	1A	
(b) $(1-3x)^8$ $=1+C_1^8(-3x)+C_2^8(-3x)^2+\cdots$ $=1-24x+252x^2+\cdots$	1M	
$e^{kx}+e^{2x}-1$ $=1+(k+2)x+\frac{(k^2+4)}{2}x^2+\cdots$		
$(1)(k+2)+(-24)(1)=(1)\left(\frac{k^2+4}{2}\right)+(-24)(k+2)+(252)(1)$	1M+1M	
$k^2-50k+456=0$ $k=12$ or $k=38$	1A	
	----------(6)	

Solution | Marks | Remarks

(a) $\frac{dy}{dx}$

$= 2x\sqrt{h-x} + x^2\left(\frac{1}{2}\right)(h-x)^{\frac{-1}{2}}(-1)$

$= \frac{4hx-5x^2}{2\sqrt{h-x}}$

1M

$\frac{4h(4)-5(4)^2}{2\sqrt{h-4}} = 30$

$16h - 80 = 60\sqrt{h-4}$

$(16h-80)^2 = (60\sqrt{h-4})^2$

$16h^2 - 385h + 1300 = 0$

1M

$h = 20$ or $h = 4.0625$

Note that $16(4.0625) - 80 = -15 < 0$.

Thus, we have $h = 20$.

1

(b) For $\frac{dy}{dx} = 0$, we have $\frac{80x-5x^2}{2\sqrt{20-x}} = 0$.

So, we have $x = 16$ or $x = 0$ (rejected)

1M

x	$(0, 16)$	16	$(16, 20)$
$\frac{dy}{dx}$	$+$	0	$-$
y	↗	512	↘

1M for testing

Thus, the maximum point of C is $(16, 512)$.

1A

For $\frac{dy}{dx} = 0$, we have $\frac{80x-5x^2}{2\sqrt{20-x}} = 0$.

So, we have $x = 16$ or $x = 0$ (rejected)

1M

$\frac{d^2y}{dx^2}$

$= \frac{15x^2 - 480x + 3200}{4\sqrt{(20-x)^3}}$

$\left.\frac{d^2y}{dx^2}\right|_{x=16}$

$= -20$

< 0

1M for testing

Thus, the maximum point of C is $(16, 512)$.

1A

(c) $y = 512$

1M

----------(7)

Solution	Marks	Remarks
8. (a) $\frac{\mathrm{d}}{\mathrm{d}x}(x\ln x)$	1M	
$=x\left(\frac{1}{x}\right)+\ln x$		
$=1+\ln x$		
So, we have $\ln x=\frac{\mathrm{d}}{\mathrm{d}x}(x\ln x)-1$.		
$\int \ln x\,\mathrm{d}x$	1A	
$=x\ln x-x+\text{constant}$		
(b) $\int \frac{\ln x}{x}\mathrm{d}x$		
$=\frac{(\ln x)^2}{2}+\text{constant}$	1A	
(c) $y=0$		
$\frac{(x-1)(\ln x-1)}{x}=0$		
$x=1$ or $x=e$	1A	can be absorbed
The required area		
$=-\int_1^e \frac{(x-1)(\ln x-1)}{x}\mathrm{d}x$	1M	
$=-\int_1^e \frac{x\ln x-x-\ln x+1}{x}\mathrm{d}x$		
$=-\int_1^e \left(\ln x-1-\frac{\ln x}{x}+\frac{1}{x}\right)\mathrm{d}x$		
$=-\left[x\ln x-x-x-\frac{(\ln x)^2}{2}+\ln x\right]_1^e$	1M	
$=e-\frac{5}{2}$	1A	
	----------(7)	

Solution	Marks	Remarks
(a) $P(W > 260) = 0.1056$		
$P(\mu < W \le 260) = 0.5 - 0.1056$		
$P\left(0 < Z \le \frac{260-\mu}{16}\right) = 0.3944$		
$\frac{260-\mu}{16} = 1.25$	1M	for either one
$\mu = 240$	1A	
$P(a < W \le 240) = 0.7357 - 0.3944$		
$P\left(\frac{a-240}{16} < Z \le 0\right) = 0.3413$		
$\frac{a-240}{16} = -1$		
$a = 224$	1A	
	----------(3)	
(b) The required probability		
$= C_6^8(0.7357)^6(1-0.7357)^2 + C_7^8(0.7357)^7(1-0.7357) + C_8^8(0.7357)^8$	1M	
≈ 0.642619261		
≈ 0.6426	1A	r.t. 0.6426
	----------(2)	
(c) (i) The required probability		
$= (C_7^8(0.7357)^7(0.1587))^3 + 6(C_6^8(0.7357)^6(0.1587)^2)(C_7^8(0.7357)^7(0.1587))(0.7357)^8$	1M	
$= 1856(0.7357)^{21}(0.1587)^3$		
≈ 0.011776727		
≈ 0.0118	1A	r.t. 0.0118
(ii) The required probability		
$\approx \frac{1856(0.7357)^{21}(0.1587)^3}{(0.642619261)^3}$	1M+ 1M	
≈ 0.044377559		
≈ 0.0444	1A	r.t. 0.0444
(iii) The required probability		
$= \frac{1856(0.7357)^{21}(0.1587)^3}{c_{21}^{24}(0.7357)^{21}(0.1587)^3}$	1M	
$= \frac{232}{253}$	1A	r.t. 0.9170
≈ 0.916996047		
≈ 0.9170		
The required probability		
$= \frac{1856(0.7357)^{21}(0.1587)^3}{1856(0.7357)^{21}(0.1587)^3 + 3(C_5^8(0.7357)^5(0.1587)^3)((0.7357)^8)^2}$	1M	
$= \frac{232}{253}$	1A	r.t. 0.9170
≈ 0.916996047		
≈ 0.9170		
	----------(7)	

Solution	Marks	Remarks
10. (a) The required probability	1M	
$=e^{-1.8}\left(\frac{1.8^0}{0!}+\frac{1.8}{1!}\right)$	1A	r.t. 0.4628
$=2.8e^{-1.8}$		
≈ 0.462836887	----------(2)	
≈ 0.4628		
(b) Let $p=2.8e^{-1.8}$.		
The expected bonus according to Suggestion I		
$=5000p^4+2500C_1^4p^3(1-p)+1500C_2^4p^2(1-p)^2+600C_3^4p(1-p)^3$	1M+1M	
$\approx \$1490.505464$		
$\approx \$1490.5055$		
The probability that Albert is late for fewer than 5 times in four months		
$=e^{-7.2}\left(\frac{7.2^0}{0!}+\frac{7.2^1}{1!}+\frac{7.2^2}{2!}+\frac{7.2^3}{3!}+\frac{7.2^4}{4!}\right)$	1M+1M	
$=208.3024e^{-7.2}$		
≈ 0.155515616		
The expected bonus according to Suggestion II	1M	
$=(8000)(208.3024e^{-7.2})$		
$=\$1666419.2e^{-7.2}$		
$\approx \$1244.1249$		
$<\$1490.5055$	1A	f.t.
Thus, Suggstion I is more favourable to Albert.	----------(6)	
(c) (i) The required probability		
$=\left(\frac{1.8^2}{2!}e^{-1.8}\right)\left(\frac{\lambda^0}{0!}e^{-\lambda}\right)$	1M	
$=1.62e^{-1.8-\lambda}$	1A	
(ii) $\frac{1.62e^{-1.8-\lambda}}{\left(\frac{1.8^2}{2!}e^{-1.8}\right)\left(\frac{\lambda^0}{0!}e^{-\lambda}\right)+\left(\frac{1.8}{1!}e^{-1.8}\right)\left(\frac{\lambda^1}{1!}e^{-\lambda}\right)+\left(\frac{1.8^0}{0!}e^{-1.8}\right)\left(\frac{\lambda^2}{2!}e^{-\lambda}\right)}=0.36$	1M+1M	1M for using (c)(i) in numerator+1M for deno
$\frac{1.62}{1.62+1.8\lambda+0.5\lambda^2}=0.36$		
$\lambda^2+3.6\lambda-5.76=0$		
$\lambda=1.2$ or $\lambda=-4.8$ (rejected)		
Thus, we have $\lambda=1.2$.	1A	
	----------(5)	

Solution	Marks	Remarks
11. (a) (i) D_1		
$=\frac{1}{2}\left(\frac{0.5-0.1}{4}\right)(\mathrm{A}(0.1)+\mathrm{A}(0.5)+2(\mathrm{A}(0.2)+\mathrm{A}(0.3)+\mathrm{A}(0.4)))$	1M	
≈ 50.25132348		
≈ 50.2513	1A	r.t. 50.2513
(ii) $\frac{\mathrm{d}A}{\mathrm{d}t}$		
$=60(e^{-2t}(10)+(1+10t)e^{-2t}(-2))$	1A	
$=480e^{-2t}-1200te^{-2t}$		
For all $t\in[0.1,0.5]$,		
$\frac{\mathrm{d}^2A}{\mathrm{d}t^2}$		
$=480e^{-2t}(-2)-1200e^{-2t}-1200te^{-2t}(-2)$	1M	
$=2400te^{-2t}-2160e^{-2t}$	1A	
$=240e^{-2t}(10t-9)$		
<0		
Thus, D_1 is an under-estimate of D .	1A	f.t.
	----------(6)	
(b) (i) Let $u=1+2t$.	1M	
Then, we have $\frac{\mathrm{d}u}{\mathrm{d}t}=2$.		
D_2		
$=\int_{0.1}^{0.5}\mathrm{B}(t)\,\mathrm{d}t$	1M	
$=25\int_{1.2}^{2}\frac{5u-4}{u}\mathrm{d}u$		
$=25\left[5u-4\ln u\right]_{1.2}^{2}$	1M	
$=100-100\ln\frac{5}{3}$	1A	r.t. 48.9174
≈ 48.91743762		
≈ 48.9174		
Note that $\frac{1+10t}{1+2t}=\frac{-4}{1+2t}+5$.	1M	
D_2		
$=\int_{0.1}^{0.5}\mathrm{B}(t)\,\mathrm{d}t$	1M	
$=50\left[-2\ln(1+2t)+5t\right]_{0.1}^{0.5}$	1M	
$=100-100\ln\frac{5}{3}$	1A	r.t. 48.9174
≈ 48.91743762		
≈ 48.9174		
(ii) By (a)(ii), D_1 is an under-estimate of D .		
Since $D_2<D_1$, we have $D_2<D_1<D$.	1M	
Thus, the claim is disagreed.	1A	f.t.
	----------(6)	

Solution	Marks	Remarks
12. (a) $r=9$	1A	
$s\ln 3=-0.1\ln 9$ $s\ln 3=-0.2\ln 3$ $s=-0.2$	1A (2)	
(b) (i) $\ln\left(\frac{120-3N}{N}\right)=\ln 9-(0.2\ln 3)t$ $\ln\left(\frac{120-3N}{N}\right)=\ln 9+\ln 3^{-0.2t}$ $\frac{120-3N}{N}=3^{2-0.2t}$	1M	
$120-3N=N(3^{2-0.2t})$ $N=\frac{120}{3^{2-0.2t}+3}$ $N=\frac{40}{3^{1-0.2t}+1}$	1	
(ii) $4=\frac{40}{3^{1-0.2t}+1}$ $3^{1-0.2t}=9$ $t=-5$	1M	
Note that $0\le t\le 20$. Thus, it is not possible that there are 4 million bacteria in the room during the experiment.	1A	f.t.
Note that $N=10$ when $t=0$. $\frac{dN}{dt}$ $=-40(3^{1-0.2t}+1)^{-2}(3^{1-0.2t})(\ln 3)(-0.2)$ $=\frac{8(\ln 3)(3^{1-0.2t})}{(3^{1-0.2t}+1)^2}$ >0	1M	
Note that N is increasing and its least value is 10 for $0\le t\le 20$. Thus, it is not possible that there are 4 million bacteria in the room during the experiment.	1A	f.t.
(iii) $\frac{dN}{dt}$ $=-40(3^{1-0.2t}+1)^{-2}(3^{1-0.2t})(\ln 3)(-0.2)$	1M	for $3^{1-0.2t}(\ln 3)(-0.2$
$=\frac{8(\ln 3)(3^{1-0.2t})}{(3^{1-0.2t}+1)^2}$	1A	
$\frac{d^2N}{dt^2}$ $=\frac{8(\ln 3)((3^{1-0.2t}+1)^2(3^{1-0.2t})(\ln 3)(-0.2)-(3^{1-0.2t})(2)(3^{1-0.2t}+1)(3^{1-0.2t})(\ln 3)(-0.2))}{(3^{1-0.2t}+1)^4}$	1M	for quotient rule
$=\frac{8(\ln 3)^2 3^{1-0.2t}(3^{1-0.2t}-1)}{5(3^{1-0.2t}+1)^3}$	1A	

<table>
<tr><th>Solution</th><th>Marks</th><th>Remarks</th></tr>
<tr><td>(iv) For $\frac{d}{dt}\left(\frac{dN}{dt}\right)=0$, we have $3^{1-0.2t}=1$.
Hence, we have $\frac{d}{dt}\left(\frac{dN}{dt}\right)=0$ when $t=5$.</td><td>1M</td><td></td></tr>
<tr><td><table>
<tr><td>t</td><td>$[0,5)$</td><td>5</td><td>$(5,20]$</td></tr>
<tr><td>$\frac{d}{dt}\left(\frac{dN}{dt}\right)$</td><td>+</td><td>0</td><td>−</td></tr>
</table></td><td>1M</td><td>for testing</td></tr>
<tr><td>Thus, $\frac{dN}{dt}$ increases for $0\le t\le 5$ and
$\frac{dN}{dt}$ decreases for $5\le t\le 20$.</td><td>1A
(11)</td><td>f.t.</td></tr>
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