2016-DSE
MATH EP
M1

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2016

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Candidate Number									

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)
Question-Answer Book

8.30 am – 11.00 am (2½ hour)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of TWO sections, A and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

(7) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

* A 0 3 1 E 0 0 1 *

SECTION A (50 marks)

1. Let X and Y be two events such that $\mathrm{P}(X)=0.4$, $\mathrm{P}(Y)=0.7$ and $\mathrm{P}(Y \mid X)=0.5$.

   (a) Are X and Y independent? Explain your answer.

   (b) Find $\mathrm{P}(X \cup Y)$.

(5 marks)

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2. A box contains six cards numbered 1 , 2 , 3 , 4 , 5 and 6 respectively.

(a) Three cards are drawn randomly from the box one by one with replacement. Given that the sum of the numbers drawn is 7 , find the probability that the number 1 is drawn exactly two times.

(b) If the card numbered 6 is taken away before three cards are drawn, will the probability described in (a) change? Explain your answer.

(6 marks)

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3. A museum opens at 10:00 . The number of visitors entering the museum in a minute follows a Poisson distribution with a mean of 1.8 .

(a) Write down the variance of the number of visitors entering the museum in a minute.

(b) Find the probability that 3 visitors entered the museum in the first two minutes after the museum opens.

(c) At 10:00 , only one gate at the entrance of the museum is opened. If in any two consecutive minutes, there are at least 4 visitors entering the museum in each minute, then a second gate will be opened. Find the probability that the second gate is just opened three minutes after the museum opens.

(7 marks)

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4. There are many packs of seeds and each pack contains 100 seeds. Let p be the population proportion of seeds that germinate in a pack.

(a) A pack of seeds is randomly selected, 64 seeds germinate. Find an approximate 95% confidence interval for p.

(b) It is given that the proportion of seeds that germinate in these packs of seeds follows a normal distribution with a mean of p and a standard deviation of 0.05. Find the least sample size to be taken such that the width of a 90% confidence interval for p is less than 0.04.

(7 marks)

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5. Let k be a constant.

(a) Expand e^{kx} in ascending powers of x as far as the term in x^2 .

(b) If the coefficient of x in the expansion of $(1+2x)^7 e^{kx}$ is 8 , find the coefficient of x^2 .

(5 marks)

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6. Let $f(x) = 3^{2x} - 10(3^x) + 9$.

(a) Find $\int f(x)\,dx$.

(b) The equation of the curve C is $y = f(x)$. Find

(i) the two x-intercepts of C ,

(ii) the exact value of the area of the region bounded by C and the x-axis.

(6 marks)

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7. Consider the curve $C: y=(2x+8)^{\frac{3}{2}}+3x^2$, where $x>-4$.

(a) Find $\frac{dy}{dx}$.

(b) Someone claims that two of the tangents to C are parallel to the straight line $6x+y+4=0$. Do you agree? Explain your answer.

(7 marks)

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8. Define $f(x) = \dfrac{(\ln x)^2}{x}$ for all $x > 0$. Let α and β be the two roots of the equation $f'(x) = 0$, where $\alpha > \beta$.

(a) Express α in terms of e . Also find β .

(b) Using integration by substitution, evaluate $\int_{\beta}^{\alpha} f(x)\,dx$.

(7 marks)

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SECTION B (50 marks)

9. X and Y are two schools with the same number of students. The daily reading times (in minutes) of the students in each school are assumed to be normally distributed. In school X, 0.6% of the students read less than 40 minutes daily while 1.5% read more than 70 minutes. In school Y, 1.5% of the students read less than 48 minutes daily while 1.7% read more than 72 minutes.

   (a) Which school has less students reading more than 60 minutes daily? Explain your answer. (6 marks)

   (b) For the school that has less students reading more than 60 minutes daily, find the probability that the 4th randomly selected student is the 2nd one who reads more than 60 minutes daily. (2 marks)

   (c) Students reading T minutes or more daily will be awarded. What should the least value of T be so that no more than 10% of students are awarded in each school? Give your answer in integral minutes. (4 marks)

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10. Tom arrives at the bus stop at 7:10 . A bus arrives at 7:20 and another bus arrives at 7:30 . The probability that Tom can take the bus is 0.9 each time. If Tom takes the bus at 7:20 , the probability for him to be late is 0.1 . If Tom takes the bus at 7:30 , the probability for him to be late is 0.4 . Tom will be late if he cannot take these two buses.

(a) Find the probability that Tom takes a bus on or before 7:30 on a certain day. (2 marks)

(b) Find the probability that Tom is late on a certain day. (2 marks)

(c) Find the probability that Tom is late 2 times in 6 days. (2 marks)

(d) There are 7 persons, including Tom, waiting for a lift at the lobby. If Tom is late, he will go to the second floor; otherwise he will go to the third floor. The probabilities for each of the other 6 persons to go to the second and third floor are 0.7 and 0.3 respectively. When an empty lift arrives, the 7 persons enter the lift. No person enters the lift afterwards.

(i) Find the probability that the 7 persons are going to the same floor.

(ii) Find the probability that exactly 3 persons are going to the third floor.

(iii) Given that exactly 3 persons are going to the third floor, find the probability that Tom is late.

(7 marks)

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11. An investment consultant, Albert, predicts the total profit made by a factory in the coming year. He models the rate of change of profit (in million dollars per month) made by the factory by

$$A(t) = \ln(t^2 - 8t + 95) ,$$

where $t\ (0 \le t \le 12)$ is the number of months elapsed since the prediction begins. Let P_1 million dollars be the total profit made by the factory in the coming year under Albert's model.

(a) (i) Using the trapezoidal rule with 4 sub-intervals, estimate P_1 .

(ii) Find $\dfrac{d^2A(t)}{dt^2}$.

(4 marks)

(b) The factory manager, Christine, models the rate of change of profit (in million dollars per month) made by the factory in the coming year by

$$B(t) = \frac{t+8}{\sqrt{t+3}} ,$$

where $t\ (0 \le t \le 12)$ is the number of months elapsed since the prediction begins. Let P_2 million dollars be the total profit made by the factory in the coming year under Christine's model.

(i) Find P_2 .

(ii) Albert claims that the difference between P_1 and P_2 does not exceed 2 . Do you agree? Explain your answer.

(9 marks)

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12. The chickens in a farm are infected by a certain bird flu. The number of chickens (in thousand) in the farm is modelled by

$$N = \frac{27}{2+\alpha t e^{\beta t}} ,$$

where $t\,(\geq 0)$ is the number of days elapsed since the start of the spread of the bird flu and α and β are constants.

(a) Express $\ln\left(\frac{27-2N}{Nt}\right)$ as a linear function of t. (2 marks)

(b) It is given that the slope and the intercept on the horizontal axis of the graph of the linear function obtained in (a) are -0.1 and $10\ln 0.03$ respectively.

(i) Find α and β.

(ii) Will the number of chickens in the farm be less than 12 thousand on a certain day after the start of the spread of the bird flu? Explain your answer.

(iii) Describe how the rate of change of the number of chickens in the farm varies during the first 20 days after the start of the spread of the bird flu. Explain your answer.

(10 marks)

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END OF PAPER

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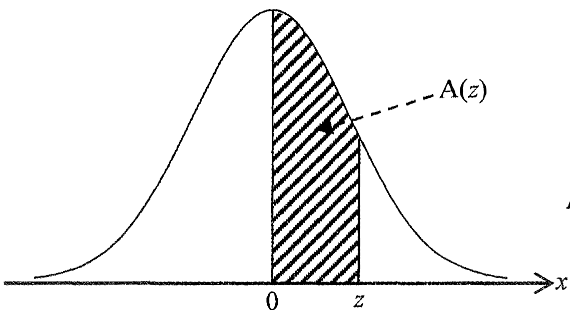
Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \, dx$$