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Candidate Number

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2014

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)

Question-Answer Book

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9, 11, 13 and 15.

2. Answer **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

3. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** this Book.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

6. For definite integrals, answers obtained by using numerical integration functions in calculators are not accepted.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

A031E001

Section A (50 marks)

1. Air is leaking from a spherical balloon at a constant rate of 100 cm^3 per second. Find the rate of change of the radius of the balloon at the instant when the radius is 10 cm .

(3 marks)

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2. Let $f(x) = \dfrac{x^x}{(2x+13)^6}$, where $x > 1$.

(a) By considering $\ln f(x)$, find $f'(x)$.

(b) Show that $f(x)$ is increasing for $x > 1$.

(6 marks)

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3. The slope of the tangent to a curve S at any point (x, y) on S is given by $\dfrac{dy}{dx}=\left(2x-\dfrac{1}{x}\right)^3$, where $x>0$.

A point $P(1, 5)$ lies on S .

(a) Find the equation of the tangent to S at P .

(b) (i) Expand $\left(2x-\dfrac{1}{x}\right)^3$.

(ii) Find the equation of S for $x>0$.

(7 marks)

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4. Evaluate the following definite integrals:

(a) $\int_1^3 \frac{t+2}{t^2+4t+11}\, dt$,

(b) $\int_1^3 \frac{t^2+3t+9}{t^2+4t+11}\, dt$.

[Note: For definite integrals, answers obtained by using numerical integration functions in calculators are not accepted.]

(6 marks)

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5. The government of a country is going to announce a new immigration policy which will last for 3 years. At the moment of the announcement, the population of the country is 8 million. After the announcement, the rate of change of the population can be modelled by

$$\frac{dx}{dt}=\frac{t\sqrt{9-t^2}}{3} \qquad (0 \le t \le 3) ,$$

where x is the population (in million) of the country and t is the time (in years) which has elapsed since the announcement. Find x in terms of t.

(5 marks)

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6. Let X be a discrete random variable with probability function as shown in the following table.

x	k	0	4	6
$P(X=x)$	0.1	0.2	0.3	0.4

It is given that $E(X)=3.4$.

(a) Find the value of k .

(b) Find $\mathrm{Var}(3-4X)$.

(c) Let G be the event that $X<4$ and H be the event that $X \geq -1$. Find $P(G \cap H)$.

(5 marks)

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7. Let A and B be two events such that $P(A \mid B) = 0.4$, $P(A \cup B) = 0.45$ and $P(B') = 0.75$, where B' is the complementary event of B .

(a) Find $P(A \cap B)$ and $P(A)$.

(b) Are events A and B independent? Justify your answer.

(6 marks)

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8. A company produces microwave ovens by production lines A and B. It is known that 4% of all microwave ovens fail to function properly and that 2% of microwave ovens produced by line A fail to function properly. Among the microwave ovens which function properly, $\frac{2}{3}$ of them are produced by line B. Suppose a microwave oven is randomly selected.

(a) What is the probability that the microwave oven is produced by line B and functions properly?

(b) What is the probability that the microwave oven is produced by line A?

(c) If the microwave oven is produced by line B, what is the probability that it functions properly?

(5 marks)

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9. The manager of a fitness centre wants to promote aerobic classes.

(a) The manager randomly selected 200 Hong Kong residents and found out that 80 of them had taken part in aerobic classes. Let p be the proportion of Hong Kong residents who had taken part in aerobic classes. Find an approximate 95% confidence interval for p.

(b) The manager wants to randomly select n Hong Kong residents and invite them to take part in a free aerobic class. The probability that an invited resident will show up is 0.85. Let X be the proportion of the n invited residents who will show up. Assume that X can be modelled by a normal distribution with mean 0.85 and variance $\frac{0.85(1-0.85)}{n}$. Find the maximum number of n such that the probability that more than 100 invited residents will show up is less than 0.05.

(7 marks)

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Section B (50 marks)

10. (a) (i) Find $\frac{d}{dv}(ve^{-v})$.

(ii) Using (a)(i), or otherwise, show that $\int ve^{-v}dv = -e^{-v}(1+v)+C$, where C is a constant.

(3 marks)

(b)

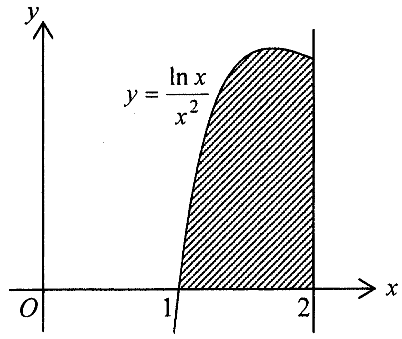

Figure 1

Figure 1 shows a shaded region bounded by the curve $y=\frac{\ln x}{x^2}$, the line $x=2$ and the x-axis. Using a suitable substitution and the result of (a), show that the area of the shaded region is $\frac{1-\ln 2}{2}$.

(5 marks)

(c) (i) Find $\frac{d^2}{dx^2}\left(\frac{\ln x}{x^2}\right)$.

(ii) Using (b) and (c)(i), show that

$$\frac{\ln 1.1}{1.1^2}+\frac{\ln 1.2}{1.2^2}+\frac{\ln 1.3}{1.3^2}+\cdots+\frac{\ln 1.9}{1.9^2}<5-\frac{41}{8}\ln 2 .$$

(6 marks)

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11. Let y be the amount (in suitable units) of suspended particulate in a laboratory. It is given that

$$(E): \quad y=\frac{340}{2+e^{-t}-2e^{-2t}} \qquad (t\geq 0),$$

where t is the time (in hours) which has elapsed since an experiment started.

(a) Will the value of y exceed 171 in the long run? Justify your answer.

(2 marks)

(b) Find the greatest value and least value of y.

(6 marks)

(c) (i) Rewrite (E) as a quadratic equation in e^{-t}.

(ii) It is known that the amounts of suspended particulate are the same at the time $t=\alpha$ and $t=3-\alpha$. Given that $0\leq\alpha<3-\alpha$, find α.

(4 marks)

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12. The delivery time X (in minutes) of an order received by a pizza restaurant follows a normal distribution with mean μ and standard deviation σ. It is known that 27.43% of the delivery times are longer than 25 minutes and 51.60% of the delivery times fall within 3.5 minutes of μ.

(a) Find μ and σ.

(4 marks)

(b) If the delivery time of an order is longer than k minutes, then a coupon will be given as a compensation to the customer who has made the order. Suppose that a total of 200 orders are received in a day. Assuming independence among delivery times of different orders, find the minimum integral value of k such that the expected number of coupons given out is at most 5 in that day.

(3 marks)

(c) The employees of the pizza restaurant recently received training to improve their efficiency. After training, the delivery time Y (in minutes) of an order follows a normal distribution with mean θ and standard deviation 4.7.

(i) Manager A draws a random sample of 12 orders and the delivery times (in minutes) are recorded as follows:

22	15	18	21	22	31
20	16	21	19	23	24

Construct a 90% confidence interval for θ.

(ii) Manager B is going to draw another random sample of n orders. He requires that the probability that the mean delivery time of the n orders falls within 3 minutes of θ be greater than 0.99. Find the minimum value of n to meet his requirement.

(6 marks)

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13. The number of delays in a day of a railway system follows the Poisson distribution with mean 4.8. Assume that the daily numbers of delays are independent.

(a) Find the probability that there are not more than 3 delays in a day.

(2 marks)

(b) Find the probability that, in 3 consecutive days, there are at most 2 days with not more than 3 delays in each day.

(2 marks)

(c) A day is called a *bad day* if there are more than 5 delays in that day; otherwise it is called a *good day*.

(i) Suppose today is a *bad day*. Find the mean number of *good days* between today and next *bad day*.

(ii) Find the probability that the last day of a week is the third *bad day* in that week.

(iii) Find the probability that there are at least 4 consecutive *bad days* in a week.

(7 marks)

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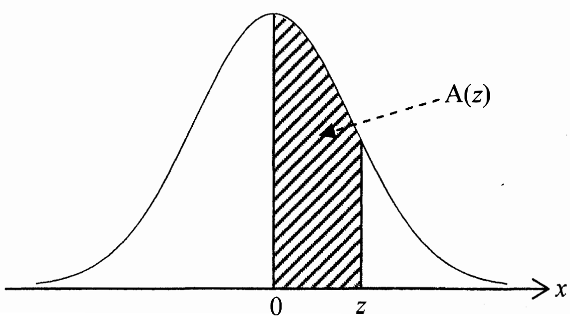
Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$