2013-DSE
MATH EP
M1

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HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2013

Candidate Number

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)

Question-Answer Book

8.30 am – 11.00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

1. After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9, 11, 13 and 15.

2. Answer **ALL** questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

3. Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string **INSIDE** this Book.

4. Unless otherwise specified, all working must be clearly shown.

5. Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

6. For definite integrals, answers obtained by using numerical integration functions in calculators are not accepted.

7. The diagrams in this paper are not necessarily drawn to scale.

8. No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

Section A (50 marks)

1. (a) Expand $\left(u+\frac{1}{u}\right)^4$ in descending powers of u.

(b) Express $(e^{ax}+e^{-ax})^4$ in ascending powers of x up to the term in x^2.

(c) Suppose the coefficient of x^2 in the result of (b) is 2. Find all possible values of a.

(5 marks)

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2. The population p (in million) of a city at time t (in years) can be modelled by

$$p = 8 - \frac{2.1}{\sqrt{t+4}} \quad \text{for } t \geq 0 .$$

An environment study indicates that, when the population is p million, the concentration of carbon dioxide in the air is given by

$$C = 2^p \text{ units.}$$

Find the rate of change of the concentration of carbon dioxide in the air at $t = 5$.

(4 marks)

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3. Consider the curve $C: \quad y = x(x-2)^{\frac{1}{3}}$ and the straight line L that passes through the origin and is parallel to the tangent to C at $x = 3$.

(a) Find the equation of L .

(b) Find the x-coordinates of the two intersecting points of C and L .

(c) Find the area bounded by C and L .

[Note: For definite integrals, answers obtained by using numerical integration functions in calculators are not accepted.]

(8 marks)

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4. After launching an advertisement for x weeks, the number y (in thousand) of members of a club can be modelled by

$$y = \frac{8(1 - ae^{-bx})}{1 + ae^{-bx}}$$, where a and b are positive #constants and $x \geq 0$.

The values of y when $x = 2, 4, 6, 8, 10$ were recorded as follows:

x	2	4	6	8	10
y	5.97	6.26	6.75	7.11	7.37

(a) Let $u = ae^{-bx}$.

(i) Express $\ln u$ as a linear function of x .

(ii) Find u in terms of y .

(b) It is known that one of the values of y in the above table is incorrect.

(i) Using the graph paper on page 9 to determine which value of y is incorrect.

(ii) By removing the incorrect value of y , estimate the values of a and b . Correct your answers to 2 decimal places.

(7 marks)

#In the live paper, the word 'integers' was wrongly used instead of 'constants'. Candidates were found to be able to understand the gist of the question and there was no evidence that they were misled. No special consideration was therefore necessary when marking candidates' work. The question has been rectified in this 'Examination Report and Question papers'.

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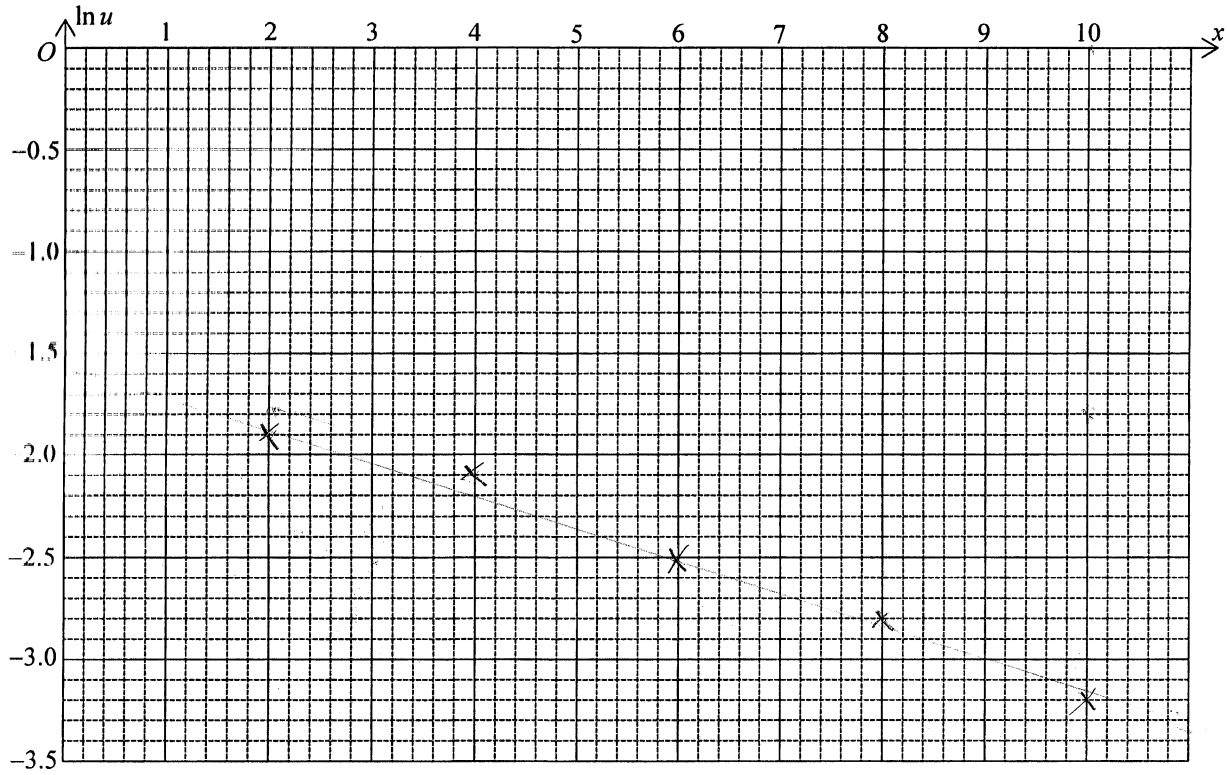

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5. (a) Find $\frac{d}{dx}(x \ln x)$.

(b) Use (a) to evaluate $\int_1^e \ln x \, dx$.

(4 marks)

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6. In a random sample of 120 swimmers in a certain beach, 75 of them are not satisfied with the water quality of the beach. Let p be the population proportion of the swimmers in this beach who are not satisfied with the water quality of the beach. Find an approximate 90% confidence interval for p.

(4 marks)

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7. Let X and Y be two independent discrete random variables with their respective probability distributions shown as follows:

x	0	1	3	5	7
$P(X = x)$	0.2	0.3	0.3	0.	0.1

y	1	2	4	m
$P(Y = y)$	0.4	0.3	0.2	0.1

Suppose that $E(Y) = 2.4$.

(a) Find the value of m .

(b) Let A be the event that $X + Y \leq 2$ and B be the event that $X = 0$.

(i) Find $P(A)$.

(ii) Are events A and B independent? Justify your answer.

(5 marks)

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8. In a shooting game, one member from each team will be selected to shoot a target three times. The team will get a prize if the target is hit at least once. Team A consists of Mabel and Owen, with the probability that Mabel is selected to shoot being 0.7. Suppose that the probabilities of Mabel and Owen to hit the target in each shot are 0.6 and 0.5 respectively.

(a) Find the probability that Team A will get a prize if Mabel is selected.

(b) Find the probability that Team A will get a prize.

(c) Given that Team A does not get a prize, find the probability that Owen is selected.

(6 marks)

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9. The lifetime of a randomly selected LED bulb produced by a manufacturer is assumed to be normally distributed with mean μ hours and standard deviation 5000 hours. It is known that 96.41% of the bulbs will have a lifetime shorter than 39000 hours.

(a) Find the value of μ.

(b) Suppose a random sample of 100 bulbs is drawn. Find the probability that the mean lifetime of the sample lies between 30200 hours and 30800 hours.

(c) The manufacturer wants to select another random sample of n bulbs such that the probability that the mean lifetime of the sample exceeding 28500 hours is at least 0.985. Find the least value of n.

(7 marks)

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Section B (50 marks)

10. (a) Consider the function $f(x) = \ln(x^2 + 16) - \ln(3x + 20)$ for $x > \frac{-20}{3}$.

(i) Find the range of values of x such that $f(x) < 0$.

(ii) Consider the integral $I = \int_0^4 f(x)\,dx$.

(1) Using the trapezoidal rule with 4 subintervals, find an estimate for I.

(2) Determine whether the estimate in (1) is an over-estimate or under-estimate. Justify your answer.

(8 marks)

(b) A certain species of insects lives in a certain environment. Let $N(t)$ (in thousand) be the number of the insects at time t (in months). Assume that $N(t)$ can be treated as a differentiable function when $N(t) > 0$. The birth rate and death rate of the insects at time t are equal to $10\ln(t^2 + 16)$ and $10\ln(3t + 20)$ respectively when $N(t) > 0$. It is given that $N(0) = 8$.

(i) Express $N'(t)$ in terms of t when $N(t) > 0$.

(ii) Jane claims that the species will not die out until $t = 4$. Do you agree? Justify your answer.

(4 marks)

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11. Let $P(t)$ and $C(t)$ (in suitable units) be the electric energy produced and consumed respectively in a city during the time period $[0, t]$, where t is in years and $t \geq 0$. It is known that $P'(t) = 4(4 - e^{\frac{-t}{5}})$ and $C'(t) = 9(2 - e^{\frac{-t}{10}})$. The redundant electric energy being generated during the time period $[0, t]$ is $R(t)$, where $R(t) = P(t) - C(t)$ and $t \geq 0$.

(a) Find t such that $R'(t) = 0$.

(3 marks)

(b) Show that $R'(t)$ decreases with t.

(3 marks)

(c) Find the total redundant electric energy generated during the period when $R'(t) > 0$.

(3 marks)

(d) The electric energy production is improved at $t = 5$. Let $Q(t)$ be the electric energy produced during the period $[5, t]$, where $t \geq 5$, and

$$Q'(t) = \frac{(t+1)[\ln(t^2 + 2t + 3)]^3}{t^2 + 2t + 3} + 9 .$$

Find the total electric energy produced for the first 3 years after the improvement.

(5 marks)

[Note: For definite integrals, answers obtained by using numerical integration functions in calculators are not accepted.]

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12. The cholesterol levels (in suitable units) of the adults in a city are assumed to be normally distributed with mean μ and variance σ^2. From a random sample of 49 adults, a 95% confidence interval for μ is found to be $(4.596, 5.044)$.

(a) (i) Find the value of σ.

(ii) Find the mean of the sample.

(3 marks)

(b) Another sample of 15 adults is randomly selected and their cholesterol levels are recorded as follows:

3.6	3.8	3.9	4.3	4.3	4.5	4.8	5.0
5.1	5.2	5.3	5.5	5.8	6.0	6.4	

The two samples are then combined. Construct a 99% confidence interval for μ using the combined sample.

(4 marks)

(c) A health organisation classifies the cholesterol level of an adult to be low, medium and high if his/her cholesterol value is respectively at most 5.2, between 5.2 and 6.2, and at least 6.2. Suppose $\mu = 4.8$.

(i) Find the probability that the cholesterol level of a randomly selected adult in the city is low.

(ii) A sample of 20 adults is randomly selected in the city. Find the probability that there are more than 17 adults with low cholesterol level and at least 1 adult with medium cholesterol level in this sample.

(5 marks)

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13. A lift company provides a regular maintenance service for every lift in an estate at the beginning of each month. Assume that the number of breakdowns of a lift in a month follows the Poisson distribution with mean 1.9. Suppose there are totally 15 lifts in the estate, and the regular maintenance service of a lift in a month is regarded as unacceptable if there are more than 2 breakdowns in that month after the regular maintenance. Assume that the monthly numbers of breakdowns of lifts are independent.

(a) Find the probability that the regular maintenance service of a randomly selected lift in a certain month in the estate is unacceptable.

(2 marks)

(b) For a certain lift, find the probability that June of 2014 is the 3rd month in 2014 such that the regular maintenance service of that lift is unacceptable.

(2 marks)

(c) Find the expected total number of unacceptable regular maintenance services of all lifts in the estate for one year.

(2 marks)

(d) In order to assure the quality of the maintenance service provided by the lift company, the estate management office introduces the following term in the new maintenance contract for the 15 lifts, which will be effective on 1st January 2015.

> For each lift in the estate, if the regular maintenance services is unacceptable for 3 consecutive months in the new contract period, one warning letter will be immediately issued to the lift company, provided that no warning letter has been issued for that lift before.

(i) For a randomly selected lift, find the probability that a warning letter will be issued to the lift company on or before 30th April 2015.

(ii) Find the probability that 3 or more warning letters will be issued to the lift company on or before 30th April 2015.

(6 marks)

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END OF PAPER

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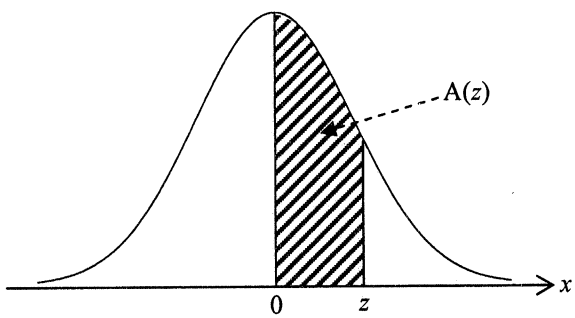
Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$