9. Binomial, Geometric and Poisson Distributions

Learning Unit	Learning Objective
Statistics Area	
Binomial, Geometric and Poisson Distributions	
14. Binomial distribution	14.1 recognise the concept and properties of the binomial distribution 14.2 calculate probabilities involving the binomial distribution
15. Geometric distribution	15.1 recognise the concept and properties of the geometric distribution 15.2 calculate probabilities involving the geometric distribution
16. Poisson distribution	16.1 recognise the concept and properties of the Poisson distribution 16.2 calculate probabilities involving the Poisson distribution
17. Applications of binomial, geometric and Poisson distributions	17.1 use binomial, geometric and Poisson distributions to solve problems

Section A

1. Susan plays a game. In each trial of the game, her probability of winning a doll is 0.6 . Susan plays the game until she wins a doll.
   (a) Find the probability that Susan wins a doll at the 4th trial in the game.
   (b) If Susan cannot win a doll in k trials, then the probability that she wins a doll within 10 trials in the game is greater than 0.95 . Find the greatest value of k .
   (c) In each trial of the game, Susan has to pay \$15. Find the expected amount of money she has to pay to win a doll in the game.

(7 marks) (2017 DSE-MATH-M1 Q4)

2. A museum opens at 10:00 . The number of visitors entering the museum in a minute follows a Poisson distribution with a mean of 1.8 .
   (a) Write down the variance of the number of visitors entering the museum in a minute.
   (b) Find the probability that 3 visitors entered the museum in the first two minutes after the museum opens.
   (c) At 10:00 , only one gate at the entrance of the museum is opened. If in any two consecutive minutes, there are at least 4 visitors entering the museum in each minute, then a second gate will be opened. Find the probability that the second gate is just opened three minutes after the museum opens.

(7 marks) (2016 DSE-MATH-M1 Q3)

3. A manufacturer of brand B biscuits starts a promotion plan by giving one reward points card in each packet of biscuits. It is found that 75% of the packets of brand B biscuits contain 3-point cards and the rest contain 7-point cards. A total of 20 points or more can be exchanged for a gift coupon. John buys 4 packets of brand B biscuits and he opens them one by one.
   (a) Find the probability that John gets the first 7-point card when the 4th packet of brand B biscuits has been opened.
   (b) Find the probability that John can exchange for a gift coupon.
   (c) Given that John can exchange for a gift coupon, find the probability that he gets a 7-point card when the 4th packet of brand B biscuits has been opened.

(7 marks) (2015 DSE-MATH-M1 Q4)

4. The number of goals scored in a randomly selected match by a football team follows a Poisson distribution with mean λ. The probability that the team scores no goals in a match is 0.1653.
   (a) Find the value of λ correct to 1 decimal place.
   (b) Find the probability that the team scores less than 3 goals in a match.
   (c) It is known that the numbers of goals scored by the team in any two matches are independent. Find the probability that the team totally scores less than 3 goals in two randomly selected matches.

(5 marks) (2012 DSE-MATH-M1 Q7)

5. Eggs from a farm are packed in boxes of 30. The probability that a randomly selected egg is rotten is 0.04 .
   (a) Find the probability that a box contains more than 1 rotten egg.
   (b) Boxes of eggs are inspected one by one.
      (i) Find the probability that the 1st box containing more than 1 rotten egg is the 6th box inspected.
      (ii) What is the expected number of boxes inspected until a box containing more than 1 rotten egg is found?

(7 marks) (PP DSE-MATH-M1 Q8)

6. The monthly number of traffic accidents occurred in a certain highway follows a Poisson distribution with mean 1.7. Assume that the monthly numbers of traffic accidents occurred in this highway are independent.
   (a) Find the probability that at least four traffic accidents will occur in this highway in the first quarter of a certain year.
   (b) Find the probability that there is exactly one quarter with at least four traffic accidents in a certain year.

(6 marks) (SAMPLE DSE-MATH-M1 Q8)

7. Let \$$X$ be the amount of money won in playing a certain game. It is known that $X \sim B(10, p)$. Two plans are proposed for calculating the game fee (\$$F$) .

Plan 1: $F = (1+\theta)\mathrm{E}(X)$,
Plan 2: $F = \mathrm{E}(X) + 0.1\mathrm{Var}(X)$,

where θ is a constant, E(X) is the expected value of X and Var(X) is the variance of X .

It is known that the game fees are same for both plans if $p = \frac{1}{4}$.

   (a) Find θ .
   (b) Show that the variance of X is the greatest when $p = \frac{1}{2}$.
   (c) Determine which plan will give a lower game fee when $p = \frac{1}{2}$.

(8 marks) (2013 ASL-M&S Q6)

8. Soft drinks are produced in packs by a production line in a company. Assume that the number of defective packs in a day follows a Poisson distribution with mean λ . The company has decided to inspect the production line whenever 4 or more defective packs are found in a day. It is known that the probability that at least 1 defective pack found in a day is $1-e^{-2}$.
   (a) Find the value of λ .
   (b) Find the probability that the company will have to inspect the production line in a given day.
   (c) It is given that the probability that the production line will not be inspected for n consecutive days is greater than 0.5 . Find the greatest integral value of n.

(6 marks) (2012 ASL-M&S Q4)

9. It is known that 36% of the customers of a certain supermarket will bring their own shopping bags. There are 3 cashiers and each cashier has 5 customers in queue.
   (a) Find the probability that among all the customers in queue, at least 4 of them have brought their own shopping bags.
   (b) If exactly 4 customers in queue have brought their own shopping bags, what is the probability that each cashier will have at least 1 customer who has brought his/her own shopping bag?

(6 marks) (2009 ASL-M&S Q5)

10. Assume that the number of passengers arriving at a bus stop per hour follows a Poisson distribution with mean 5. The probability that a passenger arriving at the bus strop is male is 0.65.
    (a) Find the probability that 4 passengers arrive at the bus stop in an hour.
    (b) Find the probability that 4 passengers arrive at the bus stop in an hour and exactly 2 of them are male.

(5 marks) (2002 ASL-M&S Q6)

11. The number of people killed in a traffic accident follows a Poisson distribution with mean 0.1. There are 5 traffic accidents on a given day, find the probability that there is at most 1 accident in which some people are killed.

(5 marks) (2000 ASL-M&S Q7)

12. 60% of passengers who travel by train use Octopus. A certain train has 12 compartments and there are 10 passengers in each compartment.
    (a) What is the probability that exactly 5 of the passengers in a compartment use Octopus?
    (b) What is the mean number of passengers using Octopus in a compartment?
    (c) What is the probability that the third compartment is the first one to have exactly 5 passengers using Octopus?

(6 marks) (1999 ASL-M&S Q5)

13. On the average, 5 cars pass through an auto-toll every minute. Assuming that the cars pass through the auto-toll independently, find the probability that more than 5 cars will pass through the auto-toll
    (a) in 1 minute,
    (b) in any 3 of the next 4 minutes.

(6 marks) (1997 ASL-M&S Q6)

14. A brewery has a backup motor for its bottling machine. The backup motor will be automatically turned on if the original motor breaks down during operating hours. The probability that the original motor breaks down during operating hours is 0.15 and when the backup motor is turned on, it has a probability of 0.24 of breaking down. Only when both the original and backup motors break down is the machine not able to work.
    (a) What is probability that the machine is not working during operating hours?
    (b) If the machine is working, what is the probability that it is operated by the original motor?
    (c) The machine is working today. Find the probability that the first break down of the machine occurs on the 10th day after today.

(7 marks) (1997 ASL-M&S Q7)

15. 5000 children are divided into 100 groups, each consisting of 50 children. The number of "over-weight" children are counted in each group and the numbers of groups having 0 , 1 , 2 , … "over-weight" children are recorded. The distributions, Poisson(λ) and Binomial(n , p), are respectively used to approximate the number of "over-weight" children in each group and some of expected frequencies are shown in the table below.

Expected frequencies of the number of groups by number of "over-weight" children

Number of "over-weight" children	Expected frequency *	
	Poisson(λ)	Binomial(n, p)
3	19.5	19.9
4	19.5	20.4
5	15.6	16.3

* Correct to 1 decimal place

It is known that λ is an integer.
    (a) Find λ .
    (b) If the mean of the two distributions are equal, find p .

(5 marks) (*modified from* 1998 ASL-M&S Q7)

Section B – Binomial and Geometric distribution

16. Tom arrives at the bus stop at 7:10 . A bus arrives at 7:20 and another bus arrives at 7:30 . The probability that Tom can take the bus is 0.9 each time. If Tom takes the bus at 7:20 , the probability for him to be late is 0.1 . If Tom takes the bus at 7:30 , the probability for him to be late is 0.4 . Tom will be late if he cannot take these two buses.

(a) Find the probability that Tom takes a bus on or before 7:30 on a certain day.

(2 marks)

(b) Find the probability that Tom is late on a certain day.

(2 marks)

(c) Find the probability that Tom is late 2 times in 6 days.

(2 marks)

(d) There are 7 persons, including Tom, waiting for a lift at the lobby. If Tom is late, he will go to the second floor; otherwise he will go to the third floor. The probabilities for each of the other 6 persons to go to the second and third floor are 0.7 and 0.3 respectively. When an empty lift arrives, the 7 persons enter the lift. No person enters the lift afterwards.

(i) Find the probability that the 7 persons are going to the same floor.

(ii) Find the probability that exactly 3 persons are going to the third floor.

(iii) Given that exactly 3 persons are going to the third floor, find the probability that Tom is late.

(7 marks)

(2016 DSE-MATH-M1 Q10)

17. According to the school regulation, air-conditioners can only be switched on if the temperature at 8 am exceeds 26 °C . From past experience, the probability that the temperature at 8 am does NOT exceed 26 °C is q ($q > 0$). Assume that there are five school days in a week. For two consecutive school days, the probability that the air-conditioners are switched on for not more than one day is $\frac{7}{16}$.

(a) (i) Show that the probability that the air-conditioners are switched on for not more than one day on two consecutive school days is $2q - q^2$.

(ii) Find the value of q .

(2 marks)

(b) The air-conditioners are said to be *fully engaged* in a week if the air-conditioners are switched on for all five school days in a week.

(i) Find the probability that the fifth week is the second week that the air-conditioners are *fully engaged*.

(ii) What is the expected number of consecutive weeks when the air-conditioners are not *fully engaged*?

(5 marks)

(c) On a certain day, the temperature at 8 am exceeds 26 °C and all the 5 classrooms on the first floor are reserved for class activities after school. There are 2 air-conditioners in each classroom. The number of air-conditioners being switched off in the classroom after school depends on the number of students staying in the classroom. Assume that the number of students in each classroom is independent.

Case	I	II	III
Number of air-conditioners being switched off	2	1	0
Probability	0.25	0.3	0.45

(i) What is the probability that all air-conditioners are switched off on the first floor after school?

(ii) Find the probability that there are exactly 2 classrooms with no air-conditioners being switched off and at most 1 classroom with exactly 1 air-conditioner being switched off on the first floor after school.

(iii) Given that there are 6 air-conditioners being switched off on the first floor after school, find the probability that at least 1 classroom has no air-conditioners being switched off.

(8 marks)

(2013 ASL-M&S Q11)

18. A fitness centre has 8 certified personal trainers providing personal training programmes to its customers in evenings. A trainer can only train one customer each evening.

The customers have to book the service in advance. Assume all bookings are made independently. Past data revealed that 'no show' bookings account for one-third of the bookings and therefore the fitness centre accepts over-bookings every evening. Trainers are assigned based on a first-come-first-serve basis. If a customer has made a booking but cannot get training due to over-booking, the customer will be given a coupon for compensation.

(a) Suppose there are 12 bookings in a particular evening.

(i) Find the probability that the fitness centre needs to give out 2 or 3 coupons.

(ii) Find the probability that every customer with booking who shows up can be assigned a trainer.

(4 marks)

(b) Find the largest number of bookings the fitness centre can accept for an evening so that at least 80% of customers who have made a booking can be assigned a trainer.

(3 marks)

(c) The centre provides three kinds of personal training programmes for its customers in each evening as follows:

Personal training programmes	Fee per programme
Diamond	\$ 3800
Platinum	\$ 2800
Jade	\$ 1800

It is known that 50%, 30% and 20% of the customers select Diamond, Platinum and Jade programmes respectively. In a particular evening, all trainers are assigned customers.

(i) Find the expected income of the centre in that evening.

(ii) Find the probability that the 8th customer is the first one to select Jade programme.

(iii) Find the probability that all programmes are selected and exactly 3 are Diamond programmes.

(iv) It is given that all programmes are selected and exactly 3 are Diamond programmes. Find the probability that more than 2 customers select Platinum programmes.

(8 marks)

(2012 ASL-M&S Q12)

19. A manufacturer produces a specific kind of tablets. He uses one machine to produce ingredient A and ingredient B, and then one mixer to mix the ingredients to produce the tablets and pack them in bags. The bags of tablets are then delivered to a hospital.

Past records indicate that 0.6% of ingredients A and B respectively are contaminated during the ingredient production process, while 0.1% of the tablets are contaminated during the mixing and packing process. A tablet is regarded as a *contaminated tablet* if

- the ingredient A in the tablet is contaminated, or
- the ingredient B in the tablet is contaminated, or
- the tablet is contaminated during the mixing and packing process.

The pharmacist of the hospital draws a random sample of 20 tablets from each bag to test for contamination. A bag is considered *unsafe* if it contains more than 1 tablet tested positive as a *contaminated tablet*.

(a) Find the probability that a randomly selected tablet from a certain bag is a *contaminated tablet*.

(3 marks)

(b) Find the probability that a bag of tablets is regarded *unsafe*.

(2 marks)

(c) In a certain week, 100 bags of such tablets are delivered to the hospital. The hospital will suspend the supply of the tablets from the manufacturer if more than 4 bags are found *unsafe* within a week.

(i) Find the probability that the 10th bag will be the first one which is regarded *unsafe*.

(ii) Find the probability that the supply from the manufacturer will be suspended in a certain week.

(5 marks)

(d) The manufacturer wants to increase the production and requires the probability of a tablet being contaminated to be less than 1% . To achieve this, he plans to add new machines for producing the ingredients A and B which has contamination probability of 0.4% respectively. Suppose equal amount of ingredients A and B are produced by the original machine and each of the n new machines.

(i) Express the probability that the ingredient A is contaminated in terms of n .

(ii) What is the least value of n?

(5 marks)

(2010 ASL-M&S Q12)

20. Officials of the Food Safety Centre of a city inspect the imported "Choy Sum" by selecting 40 samples of "Choy Sum" from each lorry and testing for an unregistered insecticide. A lorry of "Choy Sum" is classified as *risky* if more than 2 samples show positive results in the test.

Farm A supplies "Choy Sum" to the city. Past data indicated that 1% of the Farm A "Choy Sum" showed positive results in the test. On a certain day, "Choy Sum" supplied by Farm A is transported by a number of lorries to the city.

(a) Find the probability that a lorry of "Choy Sum" is *risky*.

(3 marks)

(b) Find the probability that the 5th lorry is the first lorry transporting *risky* "Choy Sum".

(2 marks)

(c) If k lorries of "Choy Sum" are inspected, find the least value of k such that the probability of finding at least one lorry of *risky* "Choy Sum" is greater than 0.05 .

(3 marks)

(d) Farm B also supplies "Choy Sum" to the city. It is known that 1.5% of the Farm B "Choy Sum" showed positive results in the test. On a certain day, "Choy Sum" supplied by Farm A and Farm B is transported by 8 and 12 lorries respectively to the city.

(i) Find the probability that a lorry of "Choy Sum" supplied by Farm B is *risky*.

(ii) Find the probability that exactly 2 of these 20 lorries of "Choy Sum" are *risky*.

(iii) It is given that exactly 2 of these 20 lorries of "Choy Sum" are *risky*. Find the probability that these 2 lorries transport "Choy Sum" from Farm B .

(7 marks)

(2008 ASL-M&S Q12)

21. In game A, two players take turns to draw a ball randomly, with replacement, from a bag containing 4 green balls and 1 red ball. The first player who draws the red ball wins the game. Christine and Donald play the game until one of them wins. Christine draws a ball first.

(a) Find the probability that Donald wins game A before his 4th draw.

(2 marks)

(b) Find the probability that Donald wins game A.

(3 marks)

(c) Given that Donald wins game A, find the probability that Donald does not win game A before his 4th draw.

(3 marks)

(d) After game A, Christine and Donald play game B. In game B, there are box X and box Y. Box X contains 2 cards which are numbered 4 and 8 respectively while box Y contains 7 cards which are numbered 1, 2, ... , 7 respectively. A player randomly draws one card from each box without replacement. If the number drawn from box X is greater than that from box Y, then the player wins game B. Christine and Donald take turns to draw cards until one of them wins game B. Donald draws cards first.

(i) Find the probability that Donald wins game B in his 1st draw.

(ii) Find the probability that Christine wins game B.

(iii) Given that Christine and Donald win one game each, find the probability that Donald wins game A.

(7 marks)

(2007 ASL-M&S Q12)

22. A manufacturer of brand E grape juice starts a marketing campaign by issuing points which can be exchanged for gifts. The number of points is shown on the back of the lid of each can of brand E grape juice. The probabilities for a customer to get a can of brand E grape juice with a 2-point lid and 5-point lid are 0.8 and 0.2 respectively. A total of 15 points or more can be exchanged for a packet of potato chips while a total of 20 points or more can be exchanged for a radio.

(a) Find the probability that a customer can exchange for a packet of potato chip in buying 5 cans of brand E grape juice.

(3 marks)

(b) A customer, Peter, buys 7 cans of brand E grape juice.

(i) Find the probability that only when the 7th can of brand E grape juice has been opened, Peter gets a 5-point lid.

(ii) Find the probability that only when the 7th can of brand E grape juice has been opened, Peter can exchange for a radio.

(iii) Given that Peter can exchange for a radio only when the 7th can of brand E grape juice has been opened, find the probability that the 7th can of brand E grape juice has a 5-point lid.

(iv) Given that Peter cannot get a packet of potato chip after opening 5 cans of brand E grape juice, find the probability that he can exchange for a radio only when the 7th can of brand E grape juice has been opened.

(12 marks)

(2006 ASL-M&S Q11)

23. A certain test gives a positive result in 94% of the people who have disease S . The test gives a positive result in 14% of the people who do not have disease S . In a city, 7.5% of the citizens have disease S .

(a) Find the probability that the test gives a positive result for a randomly selected citizen.

(3 marks)

(b) Given that the test gives a positive result for a randomly selected citizen, find the probability that the citizen does not have disease S .

(3 marks)

(c) The test is applied to a group of citizens one by one. Let M be the number of tests carried out when the first positive result is obtained. Denote the mean and the standard deviation of M by μ and σ respectively.

(i) Find $P(M=3)$.

(ii) Find the exact values of μ and σ .

(iii) Using the fact that $P(-k\sigma \le M-\mu \le k\sigma) \ge 1-\frac{1}{k^2}$ for any positive constant k , prove that $P(1 \le M \le 25) \ge 0.95$.

(9 marks)

(2004 ASL-M&S Q10)

24. A manufacturer of brand C potato chips runs a promotion plan. Each packet of brand C potato chips contains either a red coupon or a blue coupon. Four red coupons can be exchanged for a toy. Five blue coupons can be exchanged for a lottery ticket. It is known that 30% of the packets contain red coupons and the rest contain blue coupons.

(a) Find the probability that a lottery ticket can be exchanged only when the 6th packet of brand C potato chips has been opened.

(3 marks)

(b) A person buys 10 packets of brand C potato chips.

(i) Find the probability that at least 1 toy can be exchanged.

(ii) Find the probability that exactly 1 toy and exactly 1 lottery ticket can be exchanged.

(iii) Given that at least 1 toy can be exchanged, find the probability that exactly 1 lottery ticket can also be exchanged.

(8 marks)

(c) Two persons buy 10 packets of brand C potato chips each. Assume that they do not share coupons or exchange coupons with each other.

(i) Find the probability that they can each get at least 1 toy.

(ii) Find the probability that one of them can get at least 1 toy and the other can get 2 lottery tickets.

(4 marks)

(2004 ASL-M&S Q11)

25. In a game, two boxes A and B each contains n balls which are numbered $1, 2, \ldots, n$. A player is asked to draw a ball randomly from each box. If the number drawn from box A is greater than that from box B, the player wins a prize.

(a) Find the probability that the two numbers drawn are the same.

(1 mark)

(b) Let p be the probability that a player wins the prize.

(i) Find, in terms of p only, the probability that the number drawn from box B is greater than that from box A .

(ii) Using the result of (i), express p in terms of n .

(iii) If the above game is designed so that at least 46% of the players win the prize, find the least value of n .

(6 marks)

(c) Two winners, John and Mary, are selected to play another gamer. They take turns to throw a fair six-sided die. The first player who gets a number '6' wins the game. John will throw the die first.

(i) Find the probability that John will win the game on his third throw.

(ii) Find the probability that John will win the game.

(iii) Given that Mary has won the game, find the probability that Mary did not win the game before her third throw.

(8 marks)

(2003 ASL-M&S Q11)

26. You may use the probabilities list in the table to answer this question.

A salesman is promoting a new fertilizer which will improve the growth of potatoes. He claims that using the fertilizer, farmers will produce 65% of Grade A and 35% of Grade B potatoes (referred as *the claim* below). A farmer uses the fertilizer on his potatoes. In order to test the effectiveness of the fertilizer, he randomly selects 8 potatoes as a sample for testing.

(a) If *the claim* is valid, find the probability that there is at most 1 Grade A potato in the sample.

(2 marks)

(b) The farmer will reject the claim if there are not more than 3 Grade A potatoes in the sample.

(i) If *the claim* is valid, find the probability that the farmer will reject *the claim*.

(ii) If the fertilizer can only produce 20% Grade A and 80% Grade B potatoes, find the probability that the farmer will not *reject the claim*.

(5 marks)

(c) The farmer's wife takes 3 independent samples of 8 potatoes each to check *the claim*. She will reject *the claim* if not more than 3 Grade A potatoes are found in 2 or more of the 3 samples. If *the claim* is valid, find the probability that the farmer's wife will reject *the claim*.

(4 marks)

(d) Suppose *the claim* is valid. By comparing the methods described in (b) and (c), determine who, the farmer or his wife, will have a bigger chance of rejecting the claim wrongly.

(1 mark)

(e) The farmer's son will reject *the claim* if there are not more than k Grade A potatoes in a sample of 8 potatoes. Find the greatest value of k such that the probability of rejecting *the claim* is less than 0.05 given that *the claim* is valid.

(3 marks)

Table: Probabilities of two binomial distributions

Number of success	Probability *	
	B(8, 0.65)	B(8, 0.2)
0	0.0002	0.1678
1	0.0033	0.3355
2	0.0217	0.2936
3	0.0808	0.1468
4	0.1875	0.0459
5	0.2786	0.0092
6	0.2587	0.0011
7	0.1373	0.0001
8	0.0319	0.0000

* Correct to 4 decimal places.

(2001 ASL-M&S Q13)

27. Madam Wong purchases cartons of oranges from a supplier every day. Her buying policy is to randomly select five oranges from a carton and accept the carton if all five are not rotten. Under usual circumstance, 2% of the oranges are rotten.

(a) Find the probability that a carton of oranges will be rejected by Madam Wong.

(3 marks)

(b) Every day, Madam Wong keeps on buying all the accepted cartons of oranges and stops the buying exercise when she has to reject a carton. What is the mean, correct to 1 decimal place, of the number of cartons inspected by Madam Wong in a day?

(3 marks)

(c) Today, Madam Wong has a target of buying 20 acceptable cartons of oranges from the supplier. Instead of applying the stopping rule in (b), she will keep on inspecting the cartons until her target is achieved. Unfortunately, the supplier has a stock of 22 cartons only.

(i) Find the probability that she can achieve her target.

(ii) Assuming she can achieve her target, find the probability that she needs to inspect 20 cartons only.

(7 marks)

(d) The supplier would like to import oranges of better quality so that each carton will have at least a 95% probability of being accepted by Madam Wong. If r% of these oranges are rotten, find the greatest acceptable value of r .

(2 marks)

(1995 ASL-M&S Q11)

28. A day is regarded as humid if the relative humidity is over 80 % and is regarded as dry otherwise. In city K, the probability of having a humid day is 0.7.

(a) Assume that whether a day is dry or humid is independent from day to day.

(i) Find the probability of having exactly three dry days in a week (7 days).

(ii) What is the mean number of dry days before the next humid day? Give your answer correct to 3 decimal places.

(iii) Today is dry. What is the probability of having two or more humid days before the next dry day?

(8 marks)

(b) After some research, it is known that the relative humidity in city K depends solely on that of the previous day. Given a dry day, the probability that the following day is dry is 0.9 and given a humid day, the probability that the following day is humid is 0.8.

(i) If it is dry on March 19, what is the probability that it will be humid on March 20 and dry on March 21?

(ii) If it is dry on March 19, what is the probability that it will be dry on March 21?

(iii) Suppose it is dry on both March 19 and March 21. What is the probability that it is humid on March 20?

(7 marks)

(1994 ASL-M&S Q11)

Section B - Poisson distribution

29. A company records the numbers of lateness of its staff monthly. The performance of a staff member in a month is regarded as *good* if the staff member is late for fewer than 2 times in that month. Albert is a staff member of the company. The number of lateness of Albert in a month follows a Poisson distribution with a mean of 1.8 .

(a) Find the probability that Albert's performance in a certain month is *good*.

(2 marks)

(b) To improve the performance of the staff, the company launches a bonus scheme on staff performance in the coming four months. Two suggestions for the bonus scheme are listed below:

Suggestion I

Number of month with *good* performance	4	3	2	1	0
Bonus	\$ 5000	\$ 2500	\$ 1500	\$ 600	\$ 0

Suggestion II

Total number of lateness in these four months	Fewer than 5	Otherwise
Bonus	\$ 8000	\$ 0

Which one of the above suggestions is more favourable to Albert? Explain your answer.

(6 marks)

(c) The company also records the numbers of early leaves of its staff monthly. The number of early leaves of Albert in a month follows a Poisson distribution with a mean of λ. It is assumed that whether Albert is late and whether he leaves early are independent events.

(i) Express, in terms of e and λ, the probability that Albert is late for 2 times and does not leave early in a certain month.

(ii) Given that the sum of the number of lateness and the number of early leaves of Albert in a certain month is 2, the probability that Albert is late for 2 times and does not leave early in that month is 0.36 . Find λ.

(5 marks)

(2018 DSE-MATH-M1 Q10)

30. A department store issues a cash coupon to a customer spending at least \$500 in a transaction. The details are given in the following table:

Transaction amount (\$ x)	Cash coupon
$500 \le x < 1000$	\$50
$1000 \le x < 2000$	\$100
$x \ge 2000$	\$200

At the department store, 45%, 20% and 10% of the customers each gets one cash coupon of \$50 , \$100 and \$200 respectively in a transaction. Assume that the number of transactions per minute follows a Poisson distribution with a mean of 2 .

(a) Find the probability that there are at most 4 transactions at the department store in a certain minute.

(3 marks)

(b) Find the probability that there are exactly 3 transactions at the department store in a certain minute and cash coupons of total value \$200 are issued.

(3 marks)

(c) If there are exactly 4 transactions at the department store in a certain minute, find the probability that cash coupons of total value \$200 are issued by the department store in this minute.

(3 marks)

(d) Given that there are at most 4 transactions at the department store in a certain minute, find the probability that cash coupons of total value \$200 are issued by the department store in this minute.

(3 marks)

(2017 DSE-MATH-M1 Q10)

31. The number of customers buying tickets at cinema A in a minute can be modelled by a Poisson distribution with a mean of 3.2. The probability distribution of the number of tickets bought by a customer at cinema A is shown in the following table:

Number of tickets bought	1	2	3	4	5	6	≥ 7
Probability	0.12	0.7	0.08	0.04	0.03	0.02	0.01

(a) Find the probability that fewer than 4 customers buy tickets at cinema A in a certain minute.

(3 marks)

(b) Find the probability that the 8th customer buying tickets at cinema A is the 3rd customer who buys 2 tickets.

(2 marks)

(c) Find the probability that exactly 3 customers buy tickets at cinema A in a certain minute and each of them buys 2 tickets.

(2 marks)

(d) Find the probability that exactly 3 customers buy tickets at cinema A in a certain minute and they buy a total of 6 tickets.

(3 marks)

(e) Given that fewer than 4 customers buy tickets at cinema A in a certain minute, find the probability that they buy a total of 6 tickets.

(3 marks)

(2015 DSE-MATH-M1 Q10)

32. The number of delays in a day of a railway system follows the Poisson distribution with mean 4.8. Assume that the daily numbers of delays are independent.

(a) Find the probability that there are not more than 3 delays in a day.

(2 marks)

(b) Find the probability that, in 3 consecutive days, there are at most 2 days with not more than 3 delays in each day.

(2 marks)

(c) A day is called a *bad day* if there are more than 5 delays in that day; otherwise it is called a *good day*.

(i) Suppose today is a *bad day*. Find the mean number of *good days* between today and next *bad day*.

(ii) Find the probability that the last day of a week is the third *bad day* in that week.

(iii) Find the probability that there are at least 4 consecutive *bad days* in a week.

(7 marks)

(2014 DSE-MATH-M1 Q13)

33. A lift company provides a regular maintenance service for every lift in an estate at the beginning of each month. Assume that the number of breakdowns of a lift in a month follows the Poisson distribution with mean 1.9. Suppose there are totally 15 lifts in the estate, and the regular maintenance service of a lift in a month is regarded as unacceptable if there are more than 2 breakdowns in that month after the regular maintenance. Assume that the monthly numbers of breakdowns of lifts are independent.

(a) Find the probability that the regular maintenance service of a randomly selected lift in a certain month in the estate is unacceptable.

(2 marks)

(b) For a certain lift, find the probability that June of 2014 is the 3rd month in 2014 such that the regular maintenance service of that lift is unacceptable.

(2 marks)

(c) Find the expected total number of unacceptable regular maintenance services of all lifts in the estate for one year. **expected value**

(2 marks)

(d) In order to assure the quality of the maintenance service provided by the lift company, the

estate management office introduces the following term in the new maintenance contract for the 15 lifts, which will be effective on 1st January 2015.

> For each lift in the estate, if the regular maintenance services is unacceptable for 3 consecutive months in the new contract period, one warning letter will be immediately issued to the lift company, provided that no warning letter has been issued for that lift before.

(i) For a randomly selected lift, find the probability that a warning letter will be issued to the lift company on or before 30th April 2015.

(ii) Find the probability that 3 or more warning letters will be issued to the lift company on or before 30th April 2015.

(6 marks)

(2013 DSE-MATH-M1 Q13)

34. Drunk driving is against the law in a city. The police set up an inspection block at the entrance of a certain highway at night in order to arrest drunk drivers. From past experience, the number of drunk drivers arrested follows a Poisson distribution with mean 2.3 per hour.

(a) Find the probability that at least 2 drunk drivers are arrested in a certain hour.

(2 marks)

(b) Given that at least 2 drunk drivers are arrested in a certain hour, find the probability that not more than 4 drunk drivers are arrested.

(3 marks)

(c) In a certain week, the police sets up an inspection block for three nights, all at the same period from 1:00 am to 2:00 am. It is known that the numbers of drunk drivers arrested in different nights are independent.

(i) Find the probability that the third night is the first night to have at least 2 drunk drivers arrested.

(ii) Find the probability that at least 2 drunk drivers are arrested in each of the 3 nights and there are totally 10 drunk drivers arrested.

(5 marks)

(2012 DSE-MATH-M1 Q13)

35. There are 80 operators in an emergency hotline centre. Assume that the number of incoming calls for the operators are independent and the number of incoming calls for each operator is distributed as Poisson with mean 6.2 in a ten-minute time interval (TMTI). An operator is said to be *idle* if the number of incoming calls received is less than three in a certain TMTI.

(a) Find the probability that a certain operator is *idle* in a TMTI.

(3 marks)

(b) Find the probability that there are at most two *idle* operators in a TMTI.

(3 marks)

(c) A manager, Calvin, checks the numbers of incoming calls of the operators one by one in a TMTI. What is the least number of operators to be checked so that the probability of finding an *idle* operator is greater than 0.9 ?

(4 marks)

(SAMPLE DSE-MATH-M1 Q13)

36. A group of 5 members is waiting for a mini-bus to Mong Kok at a mini-bus station. It is known that there is one mini-bus every fifteen minutes and the number of empty seats on a mini-bus can be modelled by a Poisson distribution with mean λ . The probability that each of three consecutive mini-buses has at least one empty seat is 0.6465 . Assume the number of empty seats for each mini-bus is independent and the 5 members want to travel together.

(a) Find λ . Correct your answer to the nearest integer.

(2 marks)

(b) By using the λ corrected to the nearest integer, find the probability that

(i) the 5 members cannot get on the first arriving mini-bus together,

(ii) the 5 members will have to wait for more than two mini-buses.

(4 marks)

(c) After waiting for a long time, the 5 members decided to break up into a group of 2 members and a group of 3 members.

- All the 5 members will wait for the coming mini-buses if the mini-bus has less than two empty seats.
- The group of 2 members will get on a mini-bus if the mini-bus has exactly two empty seats and the group of 3 members will wait for the coming mini-buses.
- The group of 3 members will get on a mini-bus if the mini-bus has three or four empty seats and the group of 2 members will wait for the coming mini-buses.
- All the 5 members will get on a mini-bus if the mini-bus has at least five empty seats.

By using the λ corrected to the nearest integer, find the probability that

(i) the group of 2 members gets on the first arriving mini-bus and the group of 3 members gets on the next mini-bus,

(ii) none of the members have to wait for more than two mini-buses,

(iii) the group of 2 members will go first given that some members have to wait for more than two mini-buses.

(9 marks)

(2013 ASL-M&S Q12)

37.

64/F
63/F
62/F
61/F
…
G/F

In a multi-storey office building as shown in Figure 4, there is a lift with maximum capacity of 6 persons that only serves G/F, 61/F – 64/F and operates on demand. The lift is said to be full when there are 6 persons in the lift. People waiting for the lift will enter the lift until it is full.

(a) In the morning, the lift only allows passengers from G/F to enter and travel to upper floors. The number of persons waiting at G/F can be modelled by a Poisson distribution with a mean of 4 persons. The probability that a person goes to each floor (61/F – 64/F) is the same.

(i) Find the probability that the lift is full at G/F.

(ii) Find the probability that there are exactly 4 persons getting into the lift at G/F and they will get off the lift at different floors.

(iii) Find the probability that at least 1 person will get off the lift at each floor (61/F – 64/F) in a single trip.

(7 marks)

(b) In the evening, the lift only takes passengers from upper floors (61/F – 64/F) to G/F and passengers are only allowed to exit the lift at G/F. The number of persons waiting at each floor (61/F – 64/F) can be modelled by a Poisson distribution with a mean of 3 persons.

(i) Find the probability that there are exactly 3 persons waiting for the lift and they are all from different floors.

(ii) Find the probability that there are exactly 2 persons waiting for the lift.

(iii) If there are exactly 3 persons waiting at 62/F, find the probability that all of them can get into the lift.

(8 marks)

(2011 ASL-M&S Q11)

38. Assume that the number of visitors arriving at each counter in an immigration hall is independent and follows a Poisson distribution with a mean of 3.9 visitors per minute. A counter is classified as busy if at least 4 visitors arriving at it in one minute.

(a) Find the probability that a counter is *busy* in a certain minute.

(3 marks)

(b) An officer checks 4 counters in a certain minute. Find the probability that at least one *busy* counter is found.

(2 marks)

(c) If 10 counters are open, find the probability that more than 7 of them are *busy* in a certain minute.

(3 marks)

(d) Suppose 10 counters are open and one of them is randomly selected. Find the probability that more than 7 of them are *busy* and the randomly selected counter is not busy in a certain minute.

(3 marks)

(e) The immigration hall is called *congested* if more than 90% of the open counters are *busy* in a minute. Suppose 15 counters in the hall are open. A senior officer checks the counters in a certain minute. It is given that more than 7 of the first 10 checked counters are *busy*. Find the probability that the hall is *congested*.

(4 marks)

(2008 ASL-M&S Q10)

39. There are many plants in a greenhouse and all of them are of the same species. Assume that the numbers of infected leaves on the plants in the greenhouse are independent and the number of infected leaves on each plant follows a Poisson distribution with a mean of 2.6 . A plant with at least 4 infected leaves is classified as *unhealthy*.

(a) Find the probability that a certain plant in the greenhouse is *unhealthy*.

(3 marks)

(b) A researcher, Teresa, inspects the plants one by one in the greenhouse. She finds that the Mth inspected plant is the first *unhealthy* plant.

(i) Find the probability that M is less than 5 .

(ii) Given that M is less than 5, find the probability that M is greater than 2 .

(iii) If Teresa inspects m plants in the greenhouse, find the least value of m so that the probability of finding an *unhealthy* plant is greater than 0.95 .

(9 marks)

(c) It is given that there are 150 plants in the greenhouse. The number of unhealthy plants in the greenhouse is recorded every Friday. Let N be the number of unhealthy plants recorded on a Friday. Find the mean and the variance of N.

(3 marks)

(2006 ASL-M&S Q12)

40. A bank customer service center records the number of incoming telephone calls in five-minute time intervals (FMTIs). The following table lists the number of calls in a sample of 50 FMTIs.

Number of calls	0	1	2	3	4	5	6	7 or more
Frequency	5	12	14	10	6	2	1	0

(a) Find the sample mean and the sample standard deviation of the data in the table.

(2 marks)

(b) The manager of the bank believes that the number of calls in a FMTI follows a Poisson distribution and its mean can be estimated by the sample mean obtained in (a).

(i) Find the probability that there are fewer than 4 calls in a FMTI.

(ii) Find the probability that there are fewer than 4 calls each in exactly 2 FMTIs out of 6 consecutive FMTIs.

(6 marks)

(c) Assume that model in (b) is adopted and it is known that 55% of the calls are from male customers and 45% of the calls are from female customers.

(i) If there are 3 calls in a FMTI, find the probability that exactly 2 calls are from male customers.

(ii) Find the probability that there are 2 calls in a FMTI and they are both from male customer.

(iii) Given that there are fewer than 4 calls in a FMTI, find the probability that there are at least 2 calls and all of there calls are from male customers.

(7 marks)

(2003 ASL-M&S Q10)

41. A building has only two entrances A and B. Within a 15-minute period, the numbers of persons who entered the building by using entrances A and B follow that Poisson distributions with means 3.2 and 2.7 respectively.

(a) Find the probability that, on a given 15-minute period,

(i) no one entered the building by using entrance A;

(ii) no one entered the building by using entrance B;

(iii) at least one person entered the building;

(iv) exactly two persons entered the building.

(7 marks)

(b) Let X be the number of persons who entered the building within a 15-minite period. Suppose X follows a Poisson distribution with mean λ and k is the most probability number of persons who entered the building within a 15-minute period.

(i) By considering $P(X=k-1)$, $P(X=k)$ and $P(X=k+1)$, show that $\lambda-1\le k\le\lambda$.

(ii) Suppose $\lambda=5.9$. For any 5 successive 15-minute periods, find the probability that the third time that exactly k persons entered the building within a 15-minute period will occur during the fifth 15-minute period.

(8 marks)

(2001 ASL-M&S Q11)

42. A bus company finds that the number of complaints received per day follows a Poisson distribution with mean 10. 40% of the complaints involve the time schedule, 35% involve the manner of drivers, 13% involve the routes and 12% involve other things. These four kinds of complaints are mutually exclusive and can be resolved to the passenger's satisfaction with probabilities 0.6 , 0.2 , 0.7 and 0.5 respectively.

(a) If a complaint cannot be resolved to the passenger's satisfaction, find the probability that this complaint involves the manner of drivers.

(4 marks)

(b) Find the probability that on a given day,

(i) there are 5 complaints,

(ii) there are 5 complaints and 3 of them involve the time schedule.

(4 marks)

(c) Find the probability that on a given day, there are n complaints and 9 of them involve the time schedule.

(2 marks)

(d) (i) Show that $\sum_{k=9}^{\infty}\frac{x^k}{(k-9)!}=x^9e^x$.

(ii) Find the probability that, on a given day, there are 9 complaints involving the time schedule

(5 marks)

(1999 ASL-M&S Q12

43. Suppose that the number of printing mistakes on each page of a 200-page Mathematics book is independent of that on other pages, and it follows a Poisson distribution with mean 0.2.

(a) Find the probability that there is no printing mistake on page 23.

(2 marks)

(b) Let page N be the first page which contains printing mistakes. Find

(i) the probability that N is less than or equal to 3 .

(ii) the mean and variance of N .

(7 marks)

(c) Let M be the number of pages which contain printing mistakes. Find the mean and variance of M .

(2 marks)

(d) Suppose there is another 200-page Statistics book and there are 40 printing mistakes randomly and independently scattered through it. Let Y be the number of printing mistakes on page 23.

(i) Which of the distributions – Bernoulli, binomial, geometric, Poisson or normal, does Y follow? Write down the parameter(s) of the distribution.

(ii) Find the probability that there is no printing mistake on page 23 .

(4 marks)

(1998 ASL-M&S Q11)

44. In city A, the occurrences of rainstorms are assumed to be independent. The number of occurrences may be modelled by a Poisson distribution with mean occurrence rate of 2 rainstorms per year.

(a) Find the probability of having more than two rainstorms in a particular year.

(3 marks)

(b) Last year, more than two rainstorms occurred. Estimate the number of years which will elapse before the next occurrence of more than two rainstorms in a year. Give the answer correct to the nearest integer.

(3 marks)

(c) Past experience suggests that the probability of having at least one serious landslide in a year depends on the number of rainstorms in that year as follows:

Number of rainstorms	0	1 or 2	3 or more
Probability of having at least one serious landslide	0.2	0.3	0.5

Find the probability that, in city A ,

(i) there is no serious landslide in a particular year;

(ii) no rainstorm has occurred if there is no serious landslide in a particular year;

(iii) there is no serious landslide for at most 2 out of 5 years.

(9 marks)

(1996 ASL-M&S Q13)

9. Binomial, Geometric and Poisson Distribution

1. (2017 DSE-MATH-M1 Q4)

(a)	The required probability $=(1-0.6)^3(0.6)$ $=0.0384$	1M 1A	for $(1-p)^3 p$, $0<p<1$
(b)	$1-(1-0.6)^{10-k}>0.95$ $0.4^{10-k}<0.05$ $\log(0.4^{10-k})<\log 0.05$ $k<6.730587608$ Thus, the greatest value of k is 6 .	1M 1M 1A	for $1-(1-q)^{10-k}$, $0<q<1$
(c)	The expected amount of money $=15\left(\frac{1}{0.6}\right)$ $=\$25$	1M 1A ----------(7)	for $15\left(\frac{1}{r}\right)$, $0<r<1$

(a)	Very good. Most candidates were able to write down a probability of geometric distribution but a few candidates wrongly wrote down $(0.6)^3(1-0.6)$ instead of $(1-0.6)^3(0.6)$.
(b)	Poor. Less than 10% of the candidates were able to set up the correct inequality $1-(1-0.6)^{10-k}>0.95$.
(c)	Good. Only some candidates were unable to find the expected amount of money correctly.

2. (2016 DSE-MATH-M1 Q3)

(a)	The variance of the number of visitors entering the museum in a minute is 1.8 .	1A	
(b)	The required probability $=\frac{e^{-3.6}3.6^3}{3!}$ $=\frac{7.776}{e^{3.6}}$ ≈ 0.212469265 ≈ 0.2125	 1M+1M 1A	 1M for Poisson probability + 1M using mean 3.6 r.t. 0.2125
	The required probability $=2\left(\frac{e^{-1.8}1.8^0}{0!}\right)\left(\frac{e^{-1.8}1.8^3}{3!}\right)+2\left(\frac{e^{-1.8}1.8^1}{1!}\right)\left(\frac{e^{-1.8}1.8^2}{2!}\right)$ $=\frac{7.776}{e^{3.6}}$ ≈ 0.212469265 ≈ 0.2125	 1M + 1M 1A	 1M for 4 cases + 1M for Poisson probability using mean 1.8 r.t. 0.2125
(c)	P(at most 3 visitors in a minute) $=\frac{e^{-1.8}1.8^0}{0!}+\frac{e^{-1.8}1.8^1}{1!}+\frac{e^{-1.8}1.8^2}{2!}+\frac{e^{-1.8}1.8^3}{3!}$ ≈ 0.891291605 ≈ 0.8913 The required probability $\approx(0.891291605)(1-0.891291605)^2$ ≈ 0.010532851 ≈ 0.0105	 1M 1M 1A ----------(7)	 r.t. 0.0105

(a)	Very good. A very high proportion of the candidates were able to write down the required variance.
(b)	Very good. More than 70% of the candidates were able to find the answer using a Poisson probability with a mean of 3.6 instead of a mean of 1.8 .
(c)	Good. Only a number of candidates made careless mistakes in finding the required probability.

3. (2015 DSE-MATH-M1 Q4)

(a) The required probability		
$=(0.75)^3(1-0.75)$	1M	for $p^3(1-p)$, $0<p<1$
$=\frac{27}{256}$	1A	
$=0.10546875$		
≈ 0.1055		r.t. 0.1055
(b) The required probability		
$=1-\left((0.75)^4+4\left(\frac{27}{256}\right)\right)$	1M+1M	1M for $1-p+$ 1M for using (a)
$=\frac{67}{256}$	1A	
$=0.26171875$		
≈ 0.2617		r.t. 0.2617
The required probability		
$=(1-0.75)^4+C_1^4(1-0.75)^3(0.75)+C_2^4(1-0.75)^2(0.75)^2$	1M+1M	1M for the 3 cases + 1M for binomial probability
$=\frac{67}{256}$	1A	
$=0.26171875$		
≈ 0.2617		r.t. 0.2617
(c) The required probability		
$=\frac{(1-(0.75)^3)(1-0.75)}{0.26171875}$	1M	for denominator using (b)
$=\frac{37}{67}$	1A	
≈ 0.552238806		
≈ 0.5522		r.t. 0.5522
The required probability		
$=\frac{((1-0.75)^3+C_1^3(1-0.75)^2(0.75)+C_2^3(1-0.75)(0.75)^2)(1-0.75)}{0.26171875}$	1M	for denominator using (b)
$=\frac{37}{67}$	1A	
≈ 0.552238806		
≈ 0.5522		r.t. 0.5522
	----------(7)	

(a)	Very good. Most candidates were able to write a binomial probability while a few candidates wrongly wrote $(0.25)^3(1-0.25)$ instead of $(0.75)^3(1-0.75)$.
(b)	Very good. Most candidates were able to use the result of (a) while a few candidates wrongly wrote $1-\left((0.75)^4+\left(\frac{27}{256}\right)\right)$ instead of $1-\left((0.75)^4+4\left(\frac{27}{256}\right)\right)$.
(c)	Good. Some candidates failed to get the correct answer because they made a mistake in (b).

4. (2012 DSE-MATH-M1 Q7)

(a) $\frac{e^{-\lambda}}{0!}=0.1653$	
$\lambda=-\ln 0.1653$	
≈ 1.8	1A
(b) $P(\text{no. of goals in a match}<3)=\frac{e^{-1.8}}{0!}+\frac{e^{-1.8}(1.8)}{1!}+\frac{e^{-1.8}(1.8)^2}{2!}$	1M
≈ 0.7306	1A
(c) The number of goals scored in two matches by the team $\sim \text{Po}(3.6)$.	
$\therefore$ P(no. of goals in two matches < 3)	
$=\frac{e^{-3.6}}{0!}+\frac{e^{-3.6}(3.6)}{1!}+\frac{e^{-3.6}(3.6)^2}{2!}$	1M
<u>Alternative Solution</u>	
P(no. of goals in two matches < 3)	
$=P(0,0)+P(0,1)+P(1,0)+P(1,1)+P(0,2)+P(2,0)$	
$=\left(\frac{e^{-1.8}}{0!}\right)^2+2\left(\frac{e^{-1.8}}{0!}\right)\left[\frac{e^{-1.8}(1.8)}{1!}\right]+\left[\frac{e^{-1.8}(1.8)}{1!}\right]^2+2\left(\frac{e^{-1.8}}{0!}\right)\left[\frac{e^{-1.8}(1.8)^2}{2!}\right]$	1M
≈ 0.3027	1A
	(5)

(a)	Excellent.
(b)	Very good.
(c)	Poor. A few candidates used the Poisson distribution with mean 2λ . Many failed to consider all the events related to the required probability when using the Poisson distribution with mean λ .

5. (PP DSE-MATH-M1 Q8)

(a) P(a box contains more than 1 rotten eggs)		
$=1-(0.96)^{30}-C_1^{30}(0.96)^{29}(0.04)$	1M+1M	1M for binomial prob 1M for correct cases
≈ 0.338820302		
≈ 0.3388	1A	
(b) (i) P(the 1st box containing more than 1 rotten egg is the 6th box inspected)		
$=(1-0.338820302)^5(0.338820302)$	1M	
≈ 0.0428	1A	
(ii) E(no. of boxes inspected until a box containing more than 1 rotten egg is found)		
$=\frac{1}{0.338820302}$	1M	
≈ 2.9514	1A	
	(7)	

(a)	良好。少部分學生誤以爲所求概率是 $1-(0.96)^{30}-(0.96)^{29}(0.04)$ 。
(b) (i)	平平。部分學生誤以爲所求概率是 $1-(0.3388)(0.3388)^5$ 。
(ii)	平平。部分學生忘記期望值的公式。

6. (SAMPLE DSE-MATH-M1 Q8)

(a) Let X be the number of traffic accidents occurred in this highway in the first quarter of this year. Therefore $X \sim \text{Po}(5.1)$.

The required probability

$= P(X \geq 4)$

$= 1 - \left(\frac{e^{-5.1}5.1^0}{0!} + \frac{e^{-5.1}5.1^1}{1!} + \frac{e^{-5.1}5.1^2}{2!} + \frac{e^{-5.1}5.1^3}{3!}\right)$ — 1M+1M (1M for correct cases; 1M for $\frac{e^{-5.1}5.1^x}{x!}$)

≈ 0.748731735

≈ 0.7487 — 1A

(b) The required probability

$\approx C_1^4(0.748731735)(1-0.748731735)^3$ — 1M+1M (1M for $C_r^n p^r(1-p)^{n-r}$; 1M for using (a))

≈ 0.047511545

≈ 0.0475 — 1A

(6)

7. (2013 ASL-M&S Q6)

(a) $E(X) = 10\left(\frac{1}{4}\right) = 2.5$ — 1A

$\text{Var}(X) = 10\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = 1.875$ — 1A

$\therefore\ (1+\theta)(2.5) = 2.5 + (0.1)(1.875)$

$\theta = 0.075$ — 1A

(b) $\text{Var}(X) = 10p(1-p)$

$= -10\left[p^2 - p + \left(\frac{1}{2}\right)^2 - \frac{1}{4}\right]$ — 1M

$= -10\left(p - \frac{1}{2}\right)^2 + \frac{5}{2}$ — 1A

Alternative Solution

$\frac{d}{dp}\text{Var}(X) = 10(1-2p)$ — 1A

$\therefore\ \frac{d}{dp}\text{Var}(X) = 0$ when $p = \frac{1}{2}$

$\frac{d^2}{dp^2}\text{Var}(X) = -20 < 0$ — 1M

Hence $\text{Var}(X)$ is greatest when $p = \frac{1}{2}$. — 1

(c) For Plan 1, $F = (1+0.075)\cdot 10\left(\frac{1}{2}\right) = 5.375$.

For Plan 2, $F = 10\left(\frac{1}{2}\right) + 0.1 \times 10\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 5.25$. — 1M For both

Hence Plan 2 will give a lower game fee. — 1

(8)

Good.
In (b), some candidates were not able to present the proof well.

8. (2012 ASL-M&S Q4)

(a) Let X be the number of defective packs in a day.

$P(X \geq 1) = 1 - \frac{e^{-\lambda}\lambda^0}{0!}$ — 1A

$\therefore\ 1 - e^{-2} = 1 - e^{-\lambda}$

i.e. $\lambda = 2$ — 1A

(b) P(the company will have to inspect the production line in a given day)

$= P(X \geq 4)$

$= 1 - e^{-2} - \frac{e^{-2}2^1}{1!} - \frac{e^{-2}2^2}{2!} - \frac{e^{-2}2^3}{3!}$ — 1M

≈ 0.142876539

≈ 0.1429 — 1A

(c) $(1-0.142876539)^n > 0.5$ — 1M

$n\ln(1-0.142876539) > \ln 0.5$

$n < 4.495896098$

i.e. the greatest integral value of n is 4 . — 1A

(6)

Very good.
Nevertheless, some candidates were still not very competent in handling inequalities.

9. (2009 ASL-M&S Q5)

(a) The required probability

$= 1 - (0.64)^{15} - C_1^{15}(0.36)(0.64)^{14} - C_2^{15}(0.36)^2(0.64)^{13} - C_3^{15}(0.36)^3(0.64)^{12}$ — 1M+1A

≈ 0.8469 — 1A

(b) The required probability

$= \frac{3 \times C_1^5(0.36)(0.64)^4 \times C_1^5(0.36)(0.64)^4 \times C_2^5(0.36)^2(0.64)^3}{C_4^{15}(0.36)^4(0.64)^{11}}$ — 1M+1A OR $= \frac{C_1^3 \cdot \frac{4!}{2!1!1!} \cdot \frac{11!}{3!4!4!}}{\frac{15!}{5!5!5!}}$

$= \frac{50}{91}$ — 1A OR 0.5495

(6)

Part (a) was well attempted although a number of candidates mistook the total number of customers to be 5 instead of 15. Many candidates still had difficulty in analysing the situation and hence could not exhaust and count the number of relevant outcomes.

10. (2002 ASL-M&S Q6)

Let N be the number of passengers arriving the bus stop in an hour and M be the number of male passengers.

(a) $P(N=4) = \frac{5^4}{4!}e^{-5}$ — 1A

$\approx 0.17547 \approx 0.1755$ — 1A a–1 for r.t. 0.175

(b) $P(M=2$ and $N=4)$

$= C_2^4(0.65)^2(1-0.65)^2 \cdot 0.17547$ — 1M for binomial distribution; 1M for multiplication rule

≈ 0.0545 — 1A a–1 for r.t. 0.055

----------(5)

11. (2000 ASL-M&S Q7)

The probability that there is no people killed in a traffic accident

$= e^{-0.1}$ (p)

≈ 0.904837418

The required probability

$= p^5 + 5p^4(1-p)$

≈ 0.925477591

≈ 0.9255

Alternatively,

The probability that there is at least 1 people killed in a traffic accident

$= 1 - e^{-0.1}$ (q) — 2A

≈ 0.095162582

The required probability

$= (1-q)^5 + 5(1-q)^4 q$ — 1M binomial (at least 2 terms); 1M cases 0 and 1

≈ 0.925477591

≈ 0.9255 — 1A $a-1$ for r.t. 0.925

(5)

12. (1999 ASL-M&S Q5)

Let X be the no. of passengers using Octopus in a compartment.

(a) $P(X=5) = C_5^{10}(0.6)^5(1-0.6)^5$ — 1A

≈ 0.200658

≈ 0.2007 (p_1) — 1A $a-1$ for r.t. 0.201

(b) $E(X) = np = 10 \times 0.6 = 6$ — 1A+1A

The mean number of passengers using Octopus in a compartment is 6.

(c) The probability that the third compartment is the first one to have exactly 5 passengers using Octopus

$\approx (1-0.200658)^2(0.200658)$ — 1M $(1-p_1)^2 p_1$

≈ 0.1282 — 1A $a-1$ for r.t. 0.128

(6)

13. (1997 ASL-M&S Q6)

(a) Let X be the number of cars passing through the auto-toll in a minute, then $X \sim Po(5)$. — 1M

$P(X>5)$

$= 1 - \sum_{x=0}^{5} \frac{5^x e^{-5}}{x!}$ — 1A

≈ 0.3840 — 1A a−1 for r.t. 0.384

(b) Out of the next 4 minutes, let Y be the number of minutes in which more than 5 cars will pass through the auto-toll, then $Y \sim B(4, 0.3840)$. — 1M

$P(Y=3)$

$\approx C_3^4(0.3840)^3(1-0.3840)$ — 1M For binomial formula

≈ 0.1395 (or 0.1396) — 1A a−1 for r.t. 0.140

(6)

14. (1997 ASL-M&S Q7)

Let A_1 be the event that the original motor breaks down,
A_2 be the event that the backup motor breaks down and
W be the even that the machine is working.

(a) $P(A_1 A_2)$

$= 0.15 \times 0.24$ — 1A

$= 0.036$ — 1A

(b) $P(W) = 1 - P(A_1 A_2)$

$= 1 - 0.036$ — 1M

$= 0.964$

Alternatively,

$P(W) = P(\overline{A_1}) + P(A_1\overline{A_2})$

$= 0.85 + 0.15 \times 0.76$ — 1M

$= 0.964$

The probability that the machine is operated by the original motor

$= \frac{P(\overline{A_1})}{P(W)}$

$= \frac{0.85}{0.964}$ — 1M

≈ 0.8817 — 1A a−1 for r.t. 0.882

(c) The prob. that the 1st break down of the machine occurs on the 10th day

$= (0.036)(1-0.036)^{10-1}$ — 1M

≈ 0.0259 — 1A a−1 for r.t. 0.026

(7)

15. (1998 ASL-M&S Q7)

(a) Under Poisson (λ), $\frac{100\lambda^3 e^{-\lambda}}{3!} \approx 19.5$ — 1A

and $\frac{100\lambda^4 e^{-\lambda}}{4!} \approx 19.5$

Therefore $\frac{100\lambda^3 e^{-\lambda}}{3!} \approx \frac{100\lambda^4 e^{-\lambda}}{4!}$ — 1M can be omitted

$\lambda \approx 4$

Since λ is an integer, $\lambda = 4$. — 1A

Alternatively,

By calculating the expected frequencies under $Po(\lambda)$ when $\lambda = 1, 2, 3, \ldots$, — 1M

Number of "over-weight" children	Expected frequency Po(1)	Po(2)	Po(3)	Po(4)
3	6.1	18.0	22.4	19.5
4	1.5	9.0	16.8	19.5
5	0.3	3.6	10.1	15.6

From the table above, $\lambda = 4$. — 2A 1A for just writing $\lambda = 4$

(b) If $\lambda = np$, then $p = \frac{\lambda}{n}$

$= \frac{4}{50}$ — 1M

$= 0.08$ — 1A

(5)

Section B – Binomial and Geometric distribution

16. (2016 DSE-MATH-M1 Q10)

Solution	Marks	Remarks
(a) The required probability		
$=0.9+(1-0.9)(0.9)$	1M	
$=0.99$	1A	
	----------(2)	
(b) The required probability		
$=(0.9)(0.1)+(1-0.9)(0.9)(0.4)+(1-0.9)^2(1)$	1M	
$=0.136$	1A	
	----------(2)	
(c) The required probability		
$=C_2^6(1-0.136)^4(0.136)^2$	1M	
≈ 0.154605181		
≈ 0.1546	1A	r.t. 0.1546
	----------(2)	
(d) (i) The required probability		
$=(0.136)(0.7)^6+(1-0.136)(0.3)^6$	1M	
≈ 0.01663012		
≈ 0.0166	1A	r.t. 0.0166
(ii) The required probability		
$=(0.136)C_3^6(0.7)^3(0.3)^3+(1-0.136)C_2^6(0.7)^4(0.3)^2$	1M+1M	
≈ 0.30524256		
≈ 0.3052	1A	r.t. 0.3052
(iii) The required probability		
$=\dfrac{(0.136)C_3^6(0.7)^3(0.3)^3}{(0.136)C_3^6(0.7)^3(0.3)^3+(1-0.136)C_2^6(0.7)^4(0.3)^2}$	1M	1M for denominator using (d)(ii)
≈ 0.082524271		
≈ 0.0825	1A	r.t. 0.0825
	----------(7)	

(a)	Very good. More than 70% of the candidates were able to find the required probability.
(b)	Good. Some candidates missed the term $(1-0.9)^2(1)$ when finding the required probability.
(c)	Very good. Most candidates were able to formulate the required probability using binomial distribution.
(d) (i)	Good. About half of the candidates were able to find the required probability by using the result of (b). However, some candidates wrongly used 0.7 and 0.3 instead of $(0.7)^6$ and $(0.3)^6$ respectively in the required probability.
(ii)	Good. Many candidates were able to formulate the required probability by using an appropriate binomial probability
(iii)	Good. Many candidates were able to formulate the required conditional probability by using the result in (d)(ii).

17. (2013 ASL-M&S Q11)

Solution	Marks	Remarks
(a) (i) P(the air-conditioners are switched on for not more than one day on two consecutive school days) $=q^2+C_1^2q(1-q)$		OR $1-(1-q)^2$
$=2q-q^2$	1	
(ii) $2q-q^2=\dfrac{7}{16}$		
$16q^2-32q+7=0$		
$q=0.25$ or 1.75 (rejected)	1A	
	(2)	
(b) (i) P(the fifth week is the second week that the air-conditioners are *fully engaged*)		
$=C_1^4(0.75^5)(1-0.75^5)^3\cdot(0.75^5)$	1M+1M	1M for Binomial prob 1M for Geometric prob
≈ 0.0999	1A	
(ii) Expected number of consecutive weeks $=\dfrac{1}{0.75^5}-1$	1M	For $\dfrac{1}{0.75^5}$
$=3\dfrac{52}{243}$	1A	OR 3.2140
	(5)	
(c) (i) P(all conditioners are switched off) $=0.25^5$		
$=\dfrac{1}{1024}$	1A	OR 0.0010
(ii) P(exactly 2 classrooms with no air-conditioners being switched off and at most 1 classroom with exactly 1 air-conditioner being switched off)		
$=C_2^5(0.45)^2[0.25^3+C_1^3(0.25)^2(0.3)]$	1M+1A	
$=\dfrac{1863}{12800}$	1A	OR 0.1455
(iii) P(at least 1 classroom has no air-conditioners being switched off)		
$=\dfrac{\frac{5!}{2!2!1!}(0.25)^2(0.3)^2(0.45)+C_2^5(0.45)^2(0.25)^3}{C_1^5(0.25)(0.3)^4+\frac{5!}{2!2!1!}(0.25)^2(0.3)^2(0.45)+C_2^5(0.45)^2(0.25)^3}$	1M+1M+1A	1M for conditional prob 1M for cases in numerator 1A for numerator
$=\dfrac{85}{93}$	1A	OR 0.9140
	(8)	

(a)			Very good.
(b)	(i)		Good.
	(ii)		Satisfactory. Some candidates did not know the expression of the expected value of a geometric distribution, while some others did not minus one from the expected value, which would be the first occurrence of a *fully engaged* week.
(c)	(i)		Good.
	(ii)		Fair.
	(iii)		Poor. Many candidates were not able to analyse the given situation correctly and they were confused by the number of classrooms and the number of air-conditioners being switched off.

18. (2012 ASL-M&S Q12)

Solution	Marks	Remarks
(a) (i) P(the centre needs to give out 2 or 3 coupons)		
= P(10 or 11 customers show up)		
$=C_{10}^{12}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^2+C_{11}^{12}\left(\frac{2}{3}\right)^{11}\left(\frac{1}{3}\right)$	1M	
$=\frac{10240}{59049}$	1A	OR 0.1734
(ii) P(every customer with booking who shows up can be assigned a trainer)		
= P(at most 8 customers show up)		
$=1-C_9^{12}\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)^3-C_{10}^{12}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^2-C_{11}^{12}\left(\frac{2}{3}\right)^{11}\left(\frac{1}{3}\right)-\left(\frac{2}{3}\right)^{12}$	1M	
$=\frac{107515}{177147}$	1A	OR 0.6069
	(4)	
(b) If the centre accepts 10 bookings, then		
P(every customer who have made a booking can be assigned a trainer)		
$=1-C_9^{10}\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)-\left(\frac{2}{3}\right)^{10}$		
≈ 0.8960	1A	
>0.8		
If the centre accepts 11 bookings, then		
P(every customer who have made a booking can be assigned a trainer)		
$=1-C_9^{11}\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)^2-C_{10}^{11}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)-\left(\frac{2}{3}\right)^{11}$		
≈ 0.7659	1A	
<0.8		
Hence the centre can accepts 10 bookings at most.	1A	
	(3)	
(c) (i) The expected income in that evening		
$=\$(0.5\times 3800+0.3\times 2800+0.2\times 1800)\times 8$	1M	
$=\$24800$	1A	
(ii) P(the 8th customer is the first one to select Jade programs)		
$=(0.8)^7(0.2)$		
$=\frac{16384}{390625}$	1A	OR 0.0419
(iii) P(all programs are selected and exactly 3 are Diamond programs)		
$=\frac{8!}{3!4!1!}(0.5)^3(0.3)^4(0.2)^1+\frac{8!}{3!3!2!}(0.5)^3(0.3)^3(0.2)^2$		OR $C_3^8(0.5)^3[(0.5)^5-(0.3)^5-(0.2)^5]$
$+\frac{8!}{3!2!3!}(0.5)^3(0.3)^2(0.2)^3+\frac{8!}{3!1!4!}(0.5)^3(0.3)^1(0.2)^4$	1M+1A	
$=0.1995$	1A	
(iv) The required probability		
$=\frac{1}{0.1995}\left[\frac{8!}{3!4!1!}(0.5)^3(0.3)^4(0.2)^1+\frac{8!}{3!3!2!}(0.5)^3(0.3)^3(0.2)^2\right]$	1M	OR $\frac{C_3^8(0.5)^3[C_1^5(0.3)^4(0.2)+C_2^5(0.3)^3(0.2)^2]}{0.1995}$
≈ 0.6632	1A	
	(8)	

(a)	(i)		Satisfactory. Some candidates had difficulties in analysing the scenarios.
	(ii)		Poor. Many candidates were not able to come up with all the possible outcomes.
(b)			Very poor. Candidates were weak in calculating probabilities by counting the number of relevant outcomes followed by comparing with a given value.
(c)	(i)(ii)		Good.
	(iii)(iv)		Poor. The weakness of candidates was similar to that stated in (a)(ii).

19. (2010 ASL-M&S Q12)

(a) P(a tablet is contaminated)

$= 1-(1-0.6\%)(1-0.6\%)(1-0.1\%)$ — 1M+1M

≈ 0.012952036

≈ 0.0130 — 1A

(3)

(b) P(a bag is *unsafe*)

$= 1-(1-0.012952036)^{20}-20(1-0.012952036)^{19}(0.012952036)$ — 1M

≈ 0.027306899

≈ 0.0273 — 1A

(2)

(c) (i) P(the 10th bag is the first *unsafe* bag)

$\approx (1-0.027306899)^{10-1}(0.027306899)$ — 1M

≈ 0.0213 — 1A

(ii) P(the supply will be suspended in a certain week)

$\approx 1-(1-0.027306899)^{100}-C_1^{100}(1-0.027306899)^{99}(0.027306899)$

$-C_2^{100}(1-0.027306899)^{98}(0.027306899)^2$

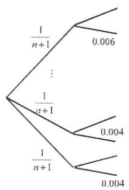
$-C_3^{100}(1-0.027306899)^{97}(0.027306899)^3-C_4^{100}(1-0.027306899)^{96}(0.027306899)^4$ — 1M+1A

≈ 0.1390 — 1A

(5)

(d) (i) P(the ingredient A is contaminated)

$= \dfrac{0.006+0.004n}{n+1}$ — 1M



(ii) P(the ingredient B is contaminated) $= \dfrac{0.006+0.004n}{n+1}$

$\therefore\ 1-\left(1-\dfrac{0.006+0.004n}{n+1}\right)\left(1-\dfrac{0.006+0.004n}{n+1}\right)(1-0.001)<0.01$ — 1A

$\left(1-\dfrac{0.006+0.004n}{n+1}\right)^2>\dfrac{110}{111}$

$1-\dfrac{0.006+0.004n}{n+1}>\sqrt{\dfrac{110}{111}}$ or $1-\dfrac{0.006+0.004n}{n+1}<-\sqrt{\dfrac{110}{111}}$ (rejected) — 1M

$n>2.885790831$ — 1A — OR $n>2.8858$

Hence the least number of n is 3. — 1A

(5)

(a)		Unsatisfactory. There were three sources of contamination and many candidates had difficulty in sorting out the situation. Many could not see that the required event was the complement of "a tablet completely free from contamination".
(b)		Very good.
(c)		Very good.
(d) (i)		Poor. Many candidates seemed to have difficulty in understanding the question.
(ii)		Very poor. Very few candidates were able to use the concept in (a) and most were weak in handling inequalities.

20. (2008 ASL-M&S Q12)

(a) The required probability

$= 1-[(1-0.01)^{40}+C_1^{40}(1-0.01)^{39}(0.01)+C_2^{40}(1-0.01)^{38}(0.01)^2]$ — 1M+1M (1M for cases correct; 1M for Binomial probability (Can be awarded in (d)(i)))

≈ 0.007497363

≈ 0.0075 — 1A

(3)

(b) The required probability

$\approx (1-0.007497363)^4(0.007497363)$ — 1M

≈ 0.0073 — 1A

(2)

(c) $C_1^k(0.007497363)(1-0.007497363)^{k-1}+C_2^k(0.007497363)^2(1-0.007497363)^{k-2}$

$+\cdots+C_{k-1}^k(0.007497363)^{k-1}(1-0.007497363)+(0.007497363)^k>0.05$

Alternative Solution

$0.007497363+(1-0.007497363)(0.007497363)+(1-0.007497363)^2(0.007497363)$

$+\ldots+(1-0.007497363)^{k-1}(0.007497363)>0.05$

$1-(1-0.007497363)^k>0.05$ — 1M

$0.992502636^k<0.95$

$k\ln 0.992502636<\ln 0.95$ — 1M

$k>\dfrac{\ln 0.95}{\ln 0.992502636}\approx 6.815832223$

Hence the least value of k is 7. — 1A

(3)

(d) (i) The required probability

$= 1-[(1-0.015)^{40}+C_1^{40}(1-0.015)^{39}(0.015)+C_2^{40}(1-0.015)^{38}(0.015)^2]$

≈ 0.022069897

≈ 0.0221 — 1A

(ii) The required probability

$= [C_0^8(1-0.007497363)^8(0.007497363)^0][C_2^{12}(1-0.022069897)^{10}(0.022069897)^2]$

$+[C_1^8(1-0.007497363)^7(0.007497363)][C_1^{12}(1-0.022069897)^{11}(0.022069897)]$

$+[C_2^8(1-0.007497363)^6(0.007497363)^2][C_0^{12}(1-0.022069897)^{12}(0.022069897)^0]$ — 1M+1M (1M for any 1 case correct; 1M for all cases correct)

≈ 0.037154780

≈ 0.0372 — 1A

(iii) The required probability

$\approx \dfrac{[C_0^8(1-0.007497363)^8(0.007497363)^0][C_2^{12}(1-0.022069897)^{10}(0.022069897)^2]}{0.037154780}$ — 1M+1M (1M for form correct; 1M for denominator using (ii))

≈ 0.6517 — 1A

(7)

(a)		Very good.
(b)		Very good.
(c)		Fair. Candidates were less skilful in handling inequalities.
(d)(i)		Good. Some candidates encountered difficulty in counting the number of relevant events.
(ii)(iii)		Fair. Some candidates had difficulty in counting the events.

21. (2007 ASL-M&S Q12)

Solution	Marks
(a) The required probability	
$=\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)$	1M for geometric probability
$=\frac{5124}{15625}$	1A
≈ 0.327936	
≈ 0.3279	a–1 for r.t. 0.328 ---------(2)
(b) The required probability	
$=\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)+\cdots$	1M must indicate infinite series and have at least 3 terms
$=\frac{\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)}{1-\left(\frac{4}{5}\right)^2}$	1M for summing geometric sequence
$=\frac{4}{9}$	1A
≈ 0.444444444	
≈ 0.4444	a–1 for r.t. 0.444 ---------(3)
(c) The required probability	
$=\frac{\frac{4}{9}-\frac{5124}{15625}}{\frac{4}{9}}$	1M for numerator = (b) – (a) + 1M for denominator using (b)
$=\frac{4096}{15625}$	1A
≈ 0.262144	
≈ 0.2621	a–1 for r.t. 0.262
The required probability	
$=1-\frac{\frac{5124}{15625}}{\frac{4}{9}}$	1M for complementary probability + 1M for denominator using (b)
$=\frac{4096}{15625}$	1A
≈ 0.262144	
≈ 0.2621	a–1 for r.t. 0.262
	---------(3)

Solution	Marks
(d) (i) The required probability	
$=(\frac{1}{2})(\frac{3}{7})+(\frac{1}{2})(1)$	1M for either case
$=\frac{5}{7}$	1A
≈ 0.714285714	
≈ 0.7143	a–1 for r.t. 0.714
The required probability	
$=1-(\frac{1}{2})(\frac{4}{7})$	1M for complementary probability
$=\frac{5}{7}$	1A
≈ 0.714285714	
≈ 0.7143	a–1 for r.t. 0.714
(ii) The required probability	
$=1-\frac{5}{7}$	1M for 1 – (d)(i)
$=\frac{2}{7}$	1A
≈ 0.285714286	
≈ 0.2857	a–1 for r.t. 0.286
The required probability	
$=(\frac{1}{2})(\frac{4}{7})(1)(1)$	1M for denominator = (2)(7)
$=\frac{2}{7}$	1A
≈ 0.285714286	
≈ 0.2857	a–1 for r.t. 0.286
(iii) The required probability	
$=\frac{(\frac{4}{9})(\frac{2}{7})}{(\frac{4}{9})(\frac{2}{7})+(1-\frac{4}{9})(1-\frac{2}{7})}$	1M for $\frac{pq}{pq+(1-p)(1-q)}$ + 1M for $\begin{cases} p=\text{(b)} \\ q=\text{(d)(ii)} \end{cases}$ or $\begin{cases} p=\text{(d)(ii)} \\ q=\text{(b)} \end{cases}$
$=\frac{8}{33}$	1A
≈ 0.242424242	
≈ 0.2424	a–1 for r.t. 0.242 ---------(7)

(a)		Good.
(b)		Good. Some candidates were not able to sum the infinite geometric series.
(c)		Fair. Some candidates could not work out the complementary probability.
(d)(i)		Good. Some candidates encountered difficulty in counting the number of relevant events.
(ii)		Fair. Many candidates did not realise this is the complementary event of d(i).
(iii)		Poor. Very few candidates attempted this part.

22. (2006 ASL-M&S Q11)

(a) The required probability	
$=1-\left((0.8)^5+C_1^5(0.8)^4(0.2)\right)$	1M for cases correct + 1M for binomial probability
$=\frac{821}{3125}$	1A
$=0.26272$	
≈ 0.2627	a-1 for r.t. 0.263
The required probability	
$=(0.2)^5+C_1^5(0.2)^4(0.8)+C_2^5(0.2)^3(0.8)^2+C_3^5(0.2)^2(0.8)^3$	1M for the 4 cases + 1M for binomial probability
$=\frac{821}{3125}$	1A
$=0.26272$	
≈ 0.2627	a-1 for r.t. 0.263
	----------(3)
(b) (i) The required probability	
$=(0.8)^6(0.2)$	1M for $p^6(1-p)$, where $0<p<1$
$=\frac{4096}{78125}$	1A
$=0.0524288$	
≈ 0.0524	a-1 for r.t. 0.052
(ii) The required probability	
$=\left(C_2^6(0.8)^4(0.2)^2\right)(0.8)+\left(C_2^6(0.8)^4(0.2)^2\right)(0.2)+\left(C_1^6(0.8)^5(0.2)\right)(0.2)$	1M for the 3 cases + 1M for binomial probability
$=\frac{25344}{78125}$	1A
$=0.3244032$	
≈ 0.3244	a-1 for r.t. 0.324
The required probability	
$=C_2^6(0.8)^4(0.2)^2+\left(C_1^6(0.8)^5(0.2)\right)(0.2)$	1M for the 2 cases + 1M for binomial probability
$=\frac{25344}{78125}$	1A
$=0.3244032$	
≈ 0.3244	a-1 for r.t. 0.324
The required probability	
$=C_2^7(0.8)^5(0.2)^2+\left(C_2^6(0.8)^4(0.2)^2\right)(0.2)$	1M for the 2 cases + 1M for binomial probability
$=\frac{25344}{78125}$	1A
$=0.3244032$	
≈ 0.3244	a-1 for r.t. 0.324
(iii) The required probability	
$=\frac{\left(C_2^6(0.8)^4(0.2)^2\right)(0.2)+\left(C_1^6(0.8)^5(0.2)\right)(0.2)}{0.3244032}$	1A for numerator 1M for denominator using (b)(ii)
$=\frac{13}{33}$	1A
≈ 0.3939393939	
≈ 0.3939	a-1 for r.t. 0.394

The required probability	
$=1-\frac{\left(C_2^6(0.8)^4(0.2)^2\right)(0.8)}{0.3244032}$	1A for numerator 1M for denominator using (b)(ii)
$=\frac{13}{33}$	1A
≈ 0.3939393939	
≈ 0.3939	a-1 for r.t. 0.394
(iv) The required probability	
$=\frac{(0.8)^5(0.2)^2+C_1^5(0.8)^4(0.2)\left((0.2)^2+C_1^2(0.8)(0.2)\right)}{1-0.26272}$	1M (one term) + 1A for numerator 1M for denominator using (a)
$=\frac{49}{225}$	1A
≈ 0.2177777777	
≈ 0.2178	a-1 for r.t. 0.218
	----------(12)

(a)		Very good.
(b) (i)		Very good.
(ii)		Fair. Some candidates encountered difficulty in counting the number of relevant events.
(iii)		Fair. Some candidates encountered difficulty in counting the number of relevant events.
(iv)		Not satisfactory. Very few candidates attempted this part.

23. (2004 ASL-M&S Q10)

(a) The required probability
$= (0.075)(0.94) + (1-0.075)(0.14)$
$= 0.2$

1M for $(p(0.94)+(1-p)(0.14))$ +1A
1A
---------(3)

(b) The required probability
$= \dfrac{(1-0.075)(0.14)}{0.2}$

1M for denominator using (a) +1A

$= 0.6475$

1A (accept $\dfrac{259}{400}$) a–1 for r.t. 0.648
---------(3)

(c) (i) $P(M=3)$
$= (1-0.2)^2(0.2)$
$= 0.128$

1M for $(1-(a))^2(a)$
1A

(ii) μ
$= \dfrac{1}{0.2}$
$= 5$

1M for $\dfrac{1}{(a)}$

σ
$= \sqrt{\dfrac{1-0.2}{0.2^2}}$
$= \sqrt{20}$
$= 2\sqrt{5}$

1M for $\sqrt{\dfrac{1-(a)}{(a)^2}}$
1A for both correct

(iii) Putting $k = 2\sqrt{5}$ in $P(-k\sigma \le M-\mu \le k\sigma) \ge 1-\dfrac{1}{k^2}$, we have
$P(-2\sqrt{5}\sigma \le M-\mu \le 2\sqrt{5}\sigma) \ge 1-(\dfrac{1}{2\sqrt{5}})^2$.
By (c)(ii), we have $P(-20 \le M-5 \le 20) \ge 0.95$.
So, we have $P(-15 \le M \le 25) \ge 0.95$.
Note that $P(-15 \le M < 1) = 0$.
Thus, we have
$P(1 \le M \le 25)$
$= P(-15 \le M \le 25) - P(-15 \le M < 1)$
$= P(-15 \le M \le 25)$
≥ 0.95

1A for $k = 2\sqrt{5}$ or $k = \sqrt{20}$

1M
1M for using $P(-l \le M < 1) = 0$ for any $l > 0$

1 do not accept finding the value of $P(1 \le M \le 25)$ directly
---------(9)

(a/b)		Very good.
(c) (i)		Good.
(ii)		Fair. Some candidates confused δ with δ^2.
(iii)		Poor. Many candidates did not have the confidence to try this unfamiliar question.

24. (2004 ASL-M&S Q11)

(a) The required probability
$= (C_4^5(0.7)^4(0.3))(0.7)$
$= 0.252105$
≈ 0.2521

1M for binomial probability + 1M for multiplication rule
1A
a–1 for r.t. 0.252
---------(3)

(b) Let X be the number of red coupons in the 10 packets of brand C potato chips.

(i) The required probability
$= P(X \ge 4)$
$= 1-(0.7)^{10} - C_1^{10}(0.7)^9(0.3) - C_2^{10}(0.7)^8(0.3)^2 - C_3^{10}(0.7)^7(0.3)^3$
[illegible]
≈ 0.3504

1M
1A
1A a–1 for r.t. 0.350

(ii) The required probability
$= P(4 \le X \le 5)$
$= C_4^{10}(0.7)^6(0.3)^4 + C_5^{10}(0.7)^5(0.3)^5$
[illegible]
≈ 0.3030

1M
1A a–1 for r.t. 0.303

(iii) The required probability
$= P(4 \le X \le 5 \mid X \ge 4)$
$= \dfrac{P(4 \le X \le 5)}{P(X \ge 4)}$
$\approx \dfrac{0.3030402942}{0.3503892816}$
[illegible]
≈ 0.8649

1M for numerator using (b)(ii) +
1M for denominator using (b)(i)
1A (accept 0.8647 and 0.8648)
a–1 for r.t. 0.865
---------(8)

(c) (i) The required probability
$= (P(X \ge 4))^2$
$\approx (0.3503892816)^2$ (by (b)(i))
[illegible]
≈ 0.1228

1M for $((b)(i))^2$
1A a–1 for r.t. 0.123

(ii) The required probability
$= 2P(X \ge 4)\,P(X=0)$
$\approx 2(0.3503892816)(0.0282475249)$ (by (b)(i))
[illegible]
≈ 0.0198

1M
1A a–1 for r.t. 0.020
---------(4)

(a)		Fair. Many candidates wrongly adopted the geometric distribution.
(b/c)		Good.

25. (2003 ASL-M&S Q11)

Solution	Marks
(a) The required probability	
$= (1)\left(\frac{1}{n}\right)$	
$= \frac{1}{n}$	1A
[Alternative] The required probability	
$= (n)\left(\frac{1}{n}\right)\left(\frac{1}{n}\right)$	
$= \frac{1}{n}$	1A
	----------(1)
(b) (i) The required probability	
$= p$	1A
(ii) $p + p + \frac{1}{n} = 1$	1M
$p = \frac{1}{2}(1 - \frac{1}{n})$	1A
(iii) $p \geq 0.46$	1A can be absorbed
$\frac{1}{2}(1 - \frac{1}{n}) \geq 0.46$	1M
$n \geq 12.5$	
$\because$ n is a positive integer.	
$\therefore$ the least value of n is 13 .	1A
	----------(6)
(c) (i) The required probability	
$= \left(\frac{5}{6}\right)^4 \frac{1}{6}$	1M for $\left(\frac{5}{6}\right)^k \frac{1}{6}$
$= \frac{625}{7776}$	1A
≈ 0.080375514	
≈ 0.0804	a−1 for r.t. 0.080
(ii) The required probability	
$= \frac{1}{6} + \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^6\left(\frac{1}{6}\right) + \cdots$	1A must indicate infinite series and have at least 3 terms
$= \frac{\frac{1}{6}}{1 - \frac{25}{36}}$	1M for sum of GP
$= \frac{6}{11}$	1A
≈ 0.545454545	
≈ 0.5455	a−1 for r.t. 0.545

Solution	Marks
(iii) The required probability	
$= \frac{\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^9\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^{11}\left(\frac{1}{6}\right) + \cdots}{1 - \frac{6}{11}}$	1M for denominator using 1−(c)(ii) + 1A for numerator
$= \frac{\frac{\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^5}{1 - \frac{25}{36}}}{\frac{5}{11}}$	
$= \frac{625}{1296}$	1A
≈ 0.482253086	
≈ 0.4823	a−1 for r.t. 0.482
[Alternative] The required probability	
$= \frac{\frac{5}{11} - \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) - \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)}{1 - \frac{6}{11}}$	1M for denominator using 1−(c)(ii) + 1A for numerator
$= \frac{\frac{5}{11} - \frac{5}{36} - \frac{125}{1296}}{\frac{5}{11}}$	
$= \frac{625}{1296}$	1A
≈ 0.482253086	
≈ 0.4823	a−1 for r.t. 0.482
	----------(8)

(a/b)		Not satisfactory. Many candidates were not able to identify the symmetry nature of '$P(A > B) = P(A < B)$'.
(c)		Satisfactory. When applying a geometric distribution, some candidates miscounted the number of dice throwing. Part (iii) was performed poorly.

26. (2001 ASL-M&S Q13)

Let X be the number of Grade A potatoes in the 8 selected potatoes.

(a) $P(X \le 1 \mid p = 0.65) \approx 0.0002 + 0.0033$	1M
≈ 0.0035 $\boxed{0.0036}$	1A
	----------(2)
(b) (i) $P(X \le 3 \mid p = 0.65) \approx 0.0002 + 0.0033 + 0.0217 + 0.0808$	1M
≈ 0.1060 $\boxed{0.1061}$ (q)	1A
(ii) $P(X > 3 \mid p = 0.2)$	1M
$\approx 0.0459 + 0.0092 + 0.0011 + 0.0001 + 0.0000$	1M
$\boxed{\approx 1 - (0.1678 + 0.3355 + 0.2936 + 0.1468)}$	
≈ 0.0563	1A
	----------(5)
(c) The required probability	
$= C_2^3 q^2 (1-q) + C_3^3 q^3$	1M for the 2 cases 1M for 1st term 1M for 2nd term
$\approx C_2^3 (0.1060)^2 (1-0.1060) + C_3^3 (0.1060)^3$	1M+1M+1M
≈ 0.0313 $\boxed{0.0314}$	1A
	----------(4)
(d) The probability that the farmer will wrongly reject *the claim* is 0.1060 whereas the probability that his wife will wrongly reject *the claim* is 0.0313. Therefore the farmer will have a bigger chance of rejecting *the claim* wrongly.	1M
	----------(1)
(e) $P(X \le 2 \mid p = 0.65) \approx 0.0252$	
$P(X \le 3 \mid p = 0.65) \approx 0.0252 + 0.0808 \approx 0.1060$	1M+1A 1M for 0.05 as a value between
Since $P(X \le 2 \mid p = 0.65) < 0.05 < P(X \le 3 \mid p = 0.65)$	
$\therefore$ $k = 2$.	1A independent
	----------(3)

27. (1995 ASL-M&S Q11)

(a) Probability of acceptance, $p_a = (1-0.02)^5$	1A	
$= 0.9039$		
Probability of rejection, $p_r = 1 - p_a$	1M	
$= 0.0961$	1A	
(b) Let X be the number of cartons inspected by Madam Wong in a day, then $X \sim \text{Geom}(p_r)$.	1M	
$\therefore$ mean $= \dfrac{1}{p_r}$	1A	
≈ 10.4	1A	
(c) (i) Prob. that Madam Wong can achieve her target,		
p_i = P(All cartons are acceptable) + P(exactly 1 carton is not acceptable) + P(exactly 2 cartons are not acceptable)	1M	
$= (p_a)^{22} + \binom{22}{1} p_r (p_a)^{21} + \binom{22}{2} (p_r)^2 (p_a)^{20}$	1M + 1A	
≈ 0.6445	1A	Accept 0.6444 - 0.6445
Alternatively,		
p_i = P(the 1st 20 cartons are accepted) + P(1 is rejected in the 1st 20 cartons and the 21st carton is accepted) + P(2 is rejected in the 1st 21 cartons and the 22nd carton is accepted)	1M	
$= (p_a)^{20} + \binom{20}{1} p_r (p_a)^{20} + \binom{21}{2} (p_r)^2 (p_a)^{20}$	1M + 1A	
≈ 0.6445	1A	Accept 0.6444 - 0.6445
(ii) If Madam Wong can achieve her target, the prob. that she needs to inspect 20 cartons only		
$= \dfrac{(p_a)^{20}}{p_i}$	1M + 1A	
≈ 0.2058	1A	Accept 0.2057
(d) $(1 - r\%)^5 \ge 0.95$	1M	
$r\% \le 0.010206$		
$r \le 1.0206$		
$\therefore$ The greatest acceptable value of r is 1.0206 .	1A	

28. (1994 ASL-M&S Q11)

Solution	Marks	Remarks
(a) (i) Let X be the number of dry days in a week.		
$X \sim \text{Bin}(7, 0.3)$	1M	
$f_X(x) = \binom{7}{x}(0.3)^x(0.7)^{7-x}$ for $x=0,1,2,\ldots,7$	1A	
The prob. of having exactly 3 dry days in a week is		
$f_X(3) = \binom{7}{3}(0.3)^3(0.7)^4 = 0.2269$	1A	
(ii) Let Y be the no. of days elapsed until the 1st humid day.		
$Y \sim \text{Geom}(0.7)$	1M	
$E(Y) = \frac{1}{0.7}$	1A	
Henca the mean no. of dry days before the next humid day is		
$E(Y) - 1 = \frac{1}{0.7} - 1 = 0.429$	1A	
(iii) The prob. of having 2 or more humid days before the next dry day is		
$1 - 0.3 - (0.7)(0.3)$	1A	
$= 1 - 0.51$		
$= 0.49$	1A	
Alternatively		
$\sum_{k=2}^{\infty}(0.3)(0.7)^k$	1A	
$= (0.3)(0.7)^2[1+0.7+(0.7)^2+\ldots]$		
$= (0.3)\frac{(0.7)^2}{1-0.7}$		
$= 0.49$	1A	
(b) Let a dry day and a humid day be denoted by D and H respectively.		
(i) 19th-20th-21st : D-H-D		
P(H on 20th, D on 21st | D on 19th)		
$= (1-0.9)(1-0.8)$	1M	
$= 0.02$	1A	
(ii) 19th-20th-21st : D-H-D or D-D-D		
P(D on 21st | D on 19th)		
$= 0.02 + (0.9)(0.9)$	1M	
$= 0.83$	1A	
(iii) P(H on 20th | D on 19th and 21st)		
$= \frac{0.02}{0.83}$	2M	1 for nominator, 1 for denominator
$= 0.02410$	1A	

Section B – Poisson distribution

29. (2017 DSE-MATH-M1 Q10)

Solution	Marks	Remarks
(a) The required probability		
$=\frac{2^0e^{-2}}{0!}+\frac{2^1e^{-2}}{1!}+\frac{2^2e^{-2}}{2!}+\frac{2^3e^{-2}}{3!}+\frac{2^4e^{-2}}{4!}$	1M+1M	1M for the 5 cases + 1M for Poisson probability
[redacted]		
≈ 0.9473	1A	r.t. 0.9473
	----------(3)	
(b) The required probability		
$=\frac{2^3e^{-2}}{3!}(3(0.25)^2(0.1)+3(0.25)(0.2)^2+3(0.45)^2(0.2))$	1M+1M	1M for Poisson probability + 1M for any one correct
[redacted]		
≈ 0.0307	1A	r.t. 0.0307
	----------(3)	
(c) The required probability		
$=4(0.25)^3(0.1)+6(0.25)^2(0.2)^2+(4)(3)(0.45)^2(0.2)(0.25)+(0.45)^4$	1M+1M	1M for any one correct + 1M for any three correct
[redacted]		
≈ 0.1838	1A	r.t. 0.1838
	----------(3)	
(d) The required probability		
$\approx \frac{\left(\frac{2e^{-2}}{1!}\right)(0.1)+\left(\frac{2^2e^{-2}}{2!}\right)(2(0.25)(0.1)+(0.2)^2)+0.030721109+\left(\frac{2^4e^{-2}}{4!}\right)(0.18375625)}{0.947346982}$	1M+1M	1M for numerator using (b) or (c) +1M for denominator using (a)
[redacted]		
≈ 0.1042	1A	r.t. 0.1042
	----------(3)	

(a)	Very good. Over 85% of the candidates were able to write down all the five Poisson probabilities.
(b)	Very good. A few candidates were unable to use correct combinations in counting.
(c)	Good. Some candidates wrongly multiplied the Poisson probability to the required probability.
(d)	Good. Only some candidates were unable to consider all the possible cases that cash coupons of total value $200 are issued in a minute.

30. (2015 DSE-MATH-M1 Q10)

Solution	Marks	Remarks
(a) The required probability		
$=\frac{3.2^0e^{-3.2}}{0!}+\frac{3.2^1e^{-3.2}}{1!}+\frac{3.2^2e^{-3.2}}{2!}+\frac{3.2^3e^{-3.2}}{3!}$	1M+1M	1M for the 4 cases + 1M for Poisson probability
≈ 0.602519724		
≈ 0.6025	1A	r.t. 0.6025
	----------(3)	
(b) The required probability		
$=C_2^7(0.7)^2(1-0.7)^5(0.7)$	1M	for binomial probability
≈ 0.01750329		
≈ 0.0175	1A	r.t. 0.0175
	----------(2)	
(c) The required probability		
$=\frac{3.2^3e^{-3.2}}{3!}(0.7)^3$	1M	
≈ 0.076357282		
≈ 0.0764	1A	r.t. 0.0764
	----------(2)	
(d) The required probability		
$\approx 0.076357282+\frac{3.2^3e^{-3.2}}{3!}\left(3(0.12)^2(0.04)+3!(0.12)(0.7)(0.08)\right)$	1M+1A	1M for using (c) + 1A for any one correct
≈ 0.085717839		
≈ 0.0857	1A	r.t. 0.0857
	----------(3)	
(e) The required probability		
$\approx\frac{\left(\frac{3.2e^{-3.2}}{1!}\right)(0.02)+\left(\frac{3.2^2e^{-3.2}}{2!}\right)\left(2(0.12)(0.03)+2(0.7)(0.04)+(0.08)^2\right)+0.085717839}{0.602519724}$	1M+1M	1M for numerator using (d) +1M for denominator using (a)
≈ 0.170703644		
≈ 0.1707	1A	r.t. 0.1707
	----------(3)	

(a)	Very good. A few candidates missed the first case in the required sum of the Poisson probabilities.
(b)	Very good. A few candidates unnecessarily multiplied the Poisson probability to the required probability form.
(c)	Very good. A few candidates wrongly used $\frac{3.2^3e^{-3.2}}{3!}(0.7)^2$ instead of $\frac{3.2^3e^{-3.2}}{3!}(0.7)^3$ in the calculation.
(d)	Good. Some candidates failed to count the number of cases correctly, such as they wrongly multiplied 3 instead of 3! to the term $(0.12)(0.7)(0.08)$.
(e)	Good. Some candidates did not realize that a conditional probability is considered here. Some candidates did not consider the Poisson probabilities as a part of the joint probability in the numerator of the required conditional probability.

31. (2014 DSE-MATH-M1 Q13)

Solution	Marks	Remarks
(a) P(not more than 3 delays in a day)		
$=e^{-4.8}\left(1+4.8+\frac{4.8^2}{2!}+\frac{4.8^3}{3!}\right)$	1M	
≈ 0.294229916		
≈ 0.2942	1A	
	(2)	
(b) P(at most 2 days with not more than 3 delays in a day in 3 consecutive days)		
$\approx 1-0.294229916^3$	1M	OR $\sum_{r=0}^{2}C_r^3p^r(1-p)^{3-r}$, where $p\approx 0.294229916$
≈ 0.9745	1A	
	(2)	
(c) Denote P(*bad day*) by k.		
(i) $k=1-e^{-4.8}\left(1+4.8+\frac{4.8^2}{2!}+\frac{4.8^3}{3!}+\frac{4.8^4}{4!}+\frac{4.8^5}{5!}\right)$		
≈ 0.348993562	1A	
$\therefore$ the mean number of *good days* between today and next *bad day*		
$=\frac{1}{k}-1$	1M	
≈ 1.8654	1A	
(ii) P(the last day in a week is the third *bad day* in that week)		
$=C_2^6k^2(1-k)^4k$	1M	
≈ 0.1145	1A	
(iii) P(there are at least 4 consecutive bad days in a week)		
$=k^4\cdot 1^3+(1-k)k^4\cdot 1^2+1(1-k)k^4\cdot 1+1^2(1-k)k^4$	1M	
Alternative Solution 1: $=[2k^4(1-k)+2k^4(1-k)^2]+[2k^5(1-k)+k^5(1-k)^2]+2k^6(1-k)+k^7$	1M	
$=2(k^4-k^5+k^4-2k^5+k^6)+2k^5-2k^6+k^5-2k^6+k^7+2k^6-2k^7+k^7$		
Alternative Solution 2: $=4k^4(1-k)^3+9k^5(1-k)^2+6k^6(1-k)+k^7$	1M	
$=4(k^4-3k^5+3k^6-k^7)+9(k^5-2k^6+k^7)+6k^6-6k^7+k^7$		
$=4k^4-3k^5$		
≈ 0.0438	1A	
	(7)	

(c)(iii)

S	M	T	W	T	F	S
B	B	B	B	/	/	/
G	B	B	B	B	/	/
/	G	B	B	B	B	/
/	/	G	B	B	B	B

(c)(iii) Alt Sol 1

S	M	T	W	T	F	S
B	B	B	B	G	/	/
G	B	B	B	B	G	/
/	G	B	B	B	B	G
/	/	G	B	B	B	B
B	B	B	B	B	G	/
G	B	B	B	B	B	G
/	G	B	B	B	B	B
B	B	B	B	B	B	G
G	B	B	B	B	B	B
B	B	B	B	B	B	B

(c)(iii) Alt Sol 2

S	M	T	W	T	F	S
B	B	B	B	G	G	G
G	B	B	B	B	G	G
G	G	B	B	B	B	G
G	G	G	B	B	B	B
B	B	B	B	B	G	G
G	B	B	B	B	B	G
G	G	B	B	B	B	B
B	B	B	B	G	B	G
B	B	B	B	G	G	B
G	B	B	B	B	G	B
B	G	B	B	B	B	G
B	G	G	B	B	B	B
G	B	G	B	B	B	B
B	B	B	B	B	B	G
B	B	B	B	B	G	B
B	B	B	B	G	B	B
B	B	G	B	B	B	B
B	G	B	B	B	B	B
G	B	B	B	B	B	B
B	B	B	B	B	B	B

(a)		Excellent. Some candidates missed the case of 3 delays in a day.
(b)		Good. Some candidates used incorrect expressions such as $1-(1-0.2942)^3$ to find the required probability.
(c)	(i)	Poor. Quite a number of candidates wrongly used $\frac{1}{\text{P(bad day)}}$ to find the required mean number.
	(ii)	Satisfactory. Some candidates used C_2^7 instead of C_2^6 in the calculation.
	(iii)	Very poor. Many candidates were able to write the related terms for the required probability, but assigned wrong coefficients to them.

32. (2013 DSE-MATH-M1 Q13)

Solution	Marks
(a) P(the regular maintenance service of a lift in a certain month in the estate is unacceptable)	
$=1-e^{-1.9}\left(1+\frac{1.9^1}{1!}+\frac{1.9^2}{2!}\right)$	1M
≈ 0.296279646	
≈ 0.2963	1A
	(2)
(b) P(the maintenance service of a lift in June of 2014 is the 3rd month unacceptable)	
$\approx C_2^5(0.296279646)^2(1-0.296279646)^3\cdot(0.296279646)$	1M
≈ 0.0906	1A
	(2)
(c) The expected total number of unacceptable maintenance services of all lifts for one year	
$\approx 15\times 12\times 0.296279646$	1M
≈ 53.3303	1A
	(2)
(d) (i) P(a warning letter will be issued for a lift on or before 30th April 2015)	
$\approx(0.296279646)^3+(1-0.296279646)\cdot(0.296279646)^3$	1M+1M
≈ 0.044310205	
≈ 0.0443	1A
(ii) P(3 or more warning letters will be issued on or before 30th April 2015)	
$\approx 1-(1-0.044310205)^{15}-C_1^{15}(0.044310205)(1-0.044310205)^{14}$ $-C_2^{15}(0.044310205)^2(1-0.044310205)^{13}$	1M+1M
≈ 0.0265	1A
	(6)

(a)		Good. Some candidates missed out the term $e^{-1.9}\frac{1.9^2}{2!}$ in the expression $1-e^{-1.9}\left(1+\frac{1.9^1}{1!}+\frac{1.9^2}{2!}\right)$, while some others missed out the factor $e^{-1.9}$.
(b)		Satisfactory. Mistakes found were missing the factor C_2^5 or replacing it by C_3^6.
(c)		Poor. Some candidates missed out the factor 15 or 12, while some others used 1.9, the mean of the Poisson distribution given, instead of the probability found in (a).
(d)		Poor. Most candidates were not able to analyse the events correctly to calculate the probabilities.
	(i)	Some candidates multiplied factors such as C_3^4, 15 or $\frac{1}{15}$ to the probability $(1-0.296279646)\cdot 0.296279646^3$, some multiplied 2 to 0.296279646^3, while some others wrote $2(1-0.296279646)\cdot 0.296279646^3$ without adding 0.296279646^4 to it.
	(ii)	Some candidates used the probability found in (a) instead of that in (d)(i).

33. (2012 DSE-MATH-M1 Q13)

Solution	Marks	
(a) P(at least 2 drunk drivers are prosecuted)		
$=1-e^{-2.3}-e^{-2.3}(2.3)$	1A	
≈ 0.669145815		
≈ 0.6691	1A	
	(2)	
(b) P(≤ 4 drunk drivers are prosecuted | at least 2 drunk drivers are prosecuted)		
$\approx\dfrac{e^{-2.3}\left(\frac{2.3^2}{2!}+\frac{2.3^3}{3!}+\frac{2.3^4}{4!}\right)}{0.669145815}$	1M+1M	1M for Poisson 1M for conditional prob
≈ 0.8748	1A	
	(3)	
(c) (i) P(the third night was the 1st night to have ≥ 2 drunk drivers prosecuted)		
$\approx(1-0.669145815)^2(0.669145815)$	1M	
≈ 0.0732	1A	
(ii) P(≥ 2 drunk drivers prosecuted in each night and totally 10 prosecuted)		
$=C_2^3\left(e^{-2.3}\frac{2.3^2}{2!}\right)^2\left(e^{-2.3}\frac{2.3^6}{6!}\right)+3!\left(e^{-2.3}\frac{2.3^2}{2!}\right)\left(e^{-2.3}\frac{2.3^3}{3!}\right)\left(e^{-2.3}\frac{2.3^5}{5!}\right)$ $+C_2^3\left(e^{-2.3}\frac{2.3^2}{2!}\right)\left(e^{-2.3}\frac{2.3^4}{4!}\right)^2+C_2^3\left(e^{-2.3}\frac{2.3^3}{3!}\right)^2\left(e^{-2.3}\frac{2.3^4}{4!}\right)$	1M+1M	1M for any one case 1M for all cases
≈ 0.0471	1A	
	(5)	

(a)		Excellent. However, a small number of candidates forgot the formula of Poisson probabilities.
(b)		Satisfactory. Some candidates failed to write all the terms needed in the numerator.
(c)	(i)	Satisfactory. Many candidates were able to apply the correct method, although some got wrong numerical answers.
	(ii)	Poor. Most candidates failed to identify all the events related to the probability required and some even used 4.6 instead of 2.3 as the mean of the Poisson distribution.

34. (SAMPLE DSE-MATH-M1 Q13)

(a) The required probability

$$= \frac{6.2^0 e^{-6.2}}{0!} + \frac{6.2^1 e^{-6.2}}{1!} + \frac{6.2^2 e^{-6.2}}{2!}$$ — 1M+1M (1M for correct cases; 1M for Poisson prob)

≈ 0.053617557

≈ 0.0536 — 1A

(3)

Let p be the probability obtained in (a).

(b) The required probability

$= (1-p)^{80} + {}_{80}C_1(1-p)^{79}p + {}_{80}C_2(1-p)^{78}p^2$

$\approx (1-0.053617557)^{80} + 80(1-0.053617557)^{79}(0.053617557)$

$\quad + 3160(1-0.053617557)^{78}(0.053617557)^2$ — 1M+1M (1M for correct cases; 1M for binomial prob)

≈ 0.1908 — 1A

(3)

(c) $p + (1-p)p + (1-p)^2 p + \ldots + (1-p)^{m-1} p > 0.9$ — 1M+1A (1M for geometric prob)

$1-(1-p)^m > 0.9$

$(1-p)^m < 0.1$

$m \ln(1-0.053617557) < \ln(0.1)$ — 1M

$m > 41.78274367$

Thus, the least number of operators to be checked is 42. — 1A

(4)

35. (2013 ASL-M&S Q12)

(a) P(three consecutive mini-buses with at least one empty seat) = 0.6465

$(1-e^{-\lambda})^3 = 0.6465$ — 1M

$\lambda = -\ln(1-\sqrt[3]{0.6465})$

≈ 2 (correct to the nearest integer) — 1A

(2)

(b) (i) P(the 5 members cannot get on the first arriving mini-bus together)

$= e^{-2} + \frac{2e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!} + \frac{2^3 e^{-2}}{3!} + \frac{2^4 e^{-2}}{4!}$ — 1M

$= 7e^{-2}$ — 1A — OR 0.9473

(ii) P(the 5 members will have to wait for more than two mini-buses)

$= (7e^{-2})^2$ — 1M

$= 49e^{-4}$ — 1A — OR 0.8975

(4)

(c) (i) P(the group of 2 gets on the first mini-bus and the group of 3 gets on the next mini-bus)

$= \frac{2^2 e^{-2}}{2!}\left[1-\left(e^{-2} + \frac{2e^{-2}}{1!} + \frac{2^2 e^{-2}}{2!}\right)\right]$ — 1M

$= 2e^{-2}(1-5e^{-2})$ — 1A — OR 0.0875

(ii) P(none of the members have to wait for more than two mini-buses)

$= \left(e^{-2} + \frac{2e^{-2}}{1!}\right)(1-7e^{-2}) + 2e^{-2}(1-5e^{-2})$

$\quad + \left(\frac{2^3 e^{-2}}{3!} + \frac{2^4 e^{-2}}{4!}\right)\left[1-\left(e^{-2} + \frac{2e^{-2}}{1!}\right)\right] + 1 - 7e^{-2}$ by (b)(i) & (c)(i) — 1M+1M (1M for using (c)(i); 1M for any other one case)

$= 1-37e^{-4}$ — 1A — OR 0.3223

(iii) P(the group of 2 go first | some members have to wait for more than two mini-buses)

$$= \frac{\frac{2^2 e^{-2}}{2!} \cdot e^{-2}\left(1+2+\frac{2^2}{2!}\right) + e^{-2}(1+2)\cdot\frac{2^2 e^{-2}}{2!} + [e^{-2}(1+2)]^2 \cdot \frac{2^2 e^{-2}}{2!} + \cdots}{1-(1-37e^{-4})}$$ — 1M+1A (1A for any one case)

$$= \frac{10e^{-4} + \frac{6e^{-4}}{1-3e^{-2}}}{37e^{-4}}$$ — 1M (For sum of geometric series)

$$= \frac{2(8e^2-15)}{37(e^2-3)}$$ — 1A — OR 0.5433

(9)

(a)		Satisfactory. Some candidates overlooked that the given probability is for three consecutive arriving mini-buses rather than for one only.
(b)		Good.
(c)	(i)	Satisfactory.
	(ii)	Fair. Many candidates had difficulty in counting and exhausting all the relevant outcomes.
	(iii)	Poor. Some candidates had difficulties in analysing the outcomes and combining different situations, while some failed to recognise that a conditional probability should be considered.

36. (2011 ASL-M&S Q11)

(a) (i) P(lift is full at G/F)

$=1-e^{-4}\left(1+4+\frac{4^2}{2!}+\frac{4^3}{3!}+\frac{4^4}{4!}+\frac{4^5}{5!}\right)$ — 1M

≈ 0.214869613

≈ 0.2149 — 1A

(ii) P(4 persons gets into the lift and it stops at each floor)

$=\frac{e^{-4}4^4}{4!}\cdot 4!\left(\frac{1}{4}\right)^4$ — 1M

≈ 0.0183 — 1A

(iii) P(lift stops at each floor)

$=\frac{e^{-4}4^4}{4!}\cdot\frac{4!}{4^4}+\frac{e^{-4}4^5}{5!}\cdot\frac{C_2^5\cdot 4!}{4^5}+0.214869613\cdot\frac{C_3^6\cdot 4!+C_2^6C_2^4\cdot\frac{4!}{2}}{4^6}$ — 1M+1M (1M for using (i) and (ii); 1M for correct cases)

≈ 0.1368 — 1A

(7)

(b) (i) P(3 persons from different floor waits for the lift)

$=C_3^4(e^{-3}3)^3(e^{-3})$ — 1M

≈ 0.0007 — 1A

(ii) P(2 persons waits for the lift)

$=C_1^4\left(\frac{e^{-3}3^2}{2!}\right)(e^{-3})^3+C_2^4(e^{-3}3)^2(e^{-3})^2$ — 1M (OR $\frac{e^{-12}12^2}{2!}$)

≈ 0.0004 — 1A

(iii) Let the number of persons waiting above 62/F be X.

P(3 persons get into the lift at the 62/F | 3 persons wait at the 62/F)

$=P(X=0)+P(X=1)+P(X=2)+P(X=3)$

$=(e^{-3})^2+C_1^2(e^{-3}3)(e^{-3})+\left[C_1^2\left(\frac{e^{-3}3^2}{2!}\right)(e^{-3})+(e^{-3}3)(e^{-3}3)\right]$

$+\left[C_1^2\left(\frac{e^{-3}3^3}{3!}\right)(e^{-3})+C_1^2\left(\frac{e^{-3}3^2}{2!}\right)(e^{-3}3)\right]$ — 1M+1M+1M (1M for 4 cases; 1M for the case $X=2$; 1M for the case $X=3$; OR $\sum_{k=0}^{3}\frac{e^{-6}6^k}{k!}$)

≈ 0.1512 — 1A

(8)

(a) (i)		Satisfactory. Candidates seemed to have difficulty in understanding the situation described.
(ii)		Fair. Candidates were unable to master the rules of joint probabilities.
(iii)		Poor. Very few candidates were able to get through this part.
(b) (i) (ii)		Satisfactory.
(iii)		Fair. Candidates had difficulty in exhausting all relevant cases.

37. (2008 ASL-M&S Q10)

(a) The required probability

$=1-\left(\frac{3.9^0e^{-3.9}}{0!}+\frac{3.9^1e^{-3.9}}{1!}+\frac{3.9^2e^{-3.9}}{2!}+\frac{3.9^3e^{-3.9}}{3!}\right)$ — 1M+1M (1M for cases correct; 1M for Poisson probability)

≈ 0.546753239

≈ 0.5468 — 1A

(3)

(b) The required probability

$=1-$P(no *busy* counters are found after the 4th counter is checked)

$\approx 1-(1-0.546753239)^4$ — 1M

≈ 0.9578 — 1A

Alternative Solution 1

The required probability

$\approx C_1^4(0.546753239)(1-0.546753239)^3+C_2^4(0.546753239)^2(1-0.546753239)^2$

$+C_3^4(0.546753239)^3(1-0.546753239)+(0.546753239)^4$ — 1M (1M for Binomial probability)

≈ 0.9578 — 1A

Alternative Solution 2

The required probability

$\approx(0.546753239)+(1-0.546753239)(0.546753239)$

$+(1-0.546753239)^2(0.546753239)+(1-0.546753239)^3(0.546753239)$ — 1M (1M for Geometric probability)

≈ 0.9578 — 1A

(2)

(c) The required probability

$\approx(0.546753239)^{10}+C_9^{10}(0.546753239)^9(1-0.546753239)$

$+C_8^{10}(0.546753239)^8(1-0.546753239)^2$ — 1M+1M (1M for cases correct; 1M for Binomial probability)

≈ 0.096004444

≈ 0.0960 — 1A

(3)

(d) The required probability

$\approx C_8^{10}(0.546753239)^8(1-0.546753239)^2\times\frac{2}{10}$

$+C_9^{10}(0.546753239)^9(1-0.546753239)\times\frac{1}{10}+0$ — 1M+1A (1M for form correct)

≈ 0.0167 — 1A

(3)

(e) The required probability

$\approx\frac{(0.546753239)^{10}\times[(0.546753239)^5+C_4^5(0.546753239)^4(1-0.546753239)]+C_9^{10}(0.546753239)^9(1-0.546753239)\times(0.546753239)^5}{0.096004444}$ — 1M+1M+1A (1M for denominator using (c); 1M for numerator form correct; 1A for numerator correct)

Alternative Solution

The required probability

$\approx\frac{(0.546753239)^{15}+C_{14}^{15}(0.546753239)^{14}(1-0.546753239)}{0.096004444}$ — 1M+1M+1A (1M for denominator using (c); 1M for numerator form correct; 1A for numerator correct)

≈ 0.0163 — 1A

(4)

(a)		Very good.
(b)		Very good.
(c)		Good.
(d)		Poor. Many candidates overlooked that a joint probability should be considered.
(e)		Fair. Many candidates were able to handle conditional probabilities but some were careless.

38. (2006 ASL-M&S Q12)

(a) The required probability

$= 1 - \left(\frac{2.6^0 e^{-2.6}}{0!} + \frac{2.6^1 e^{-2.6}}{1!} + \frac{2.6^2 e^{-2.6}}{2!} + \frac{2.6^3 e^{-2.6}}{3!} \right)$ — 1M for cases correct + 1M for Poisson probability

≈ 0.263998355

≈ 0.2640 — 1A a–1 for r.t. 0.264 ---------(3)

Let p be the probability described in (a).

(b) (i) The required probability

$= p + (1-p)p + (1-p)^2 p + (1-p)^3 p$ — 1M for the 4 cases + 1M for geometric probability

$= 1 - (1-p)^4$

$\approx 1 - (1 - 0.263998355)^4$

≈ 0.70656282

≈ 0.7066 — 1A a–1 for r.t. 0.707

(ii) The required probability

$\approx \frac{(1-0.263998355)^2(0.263998355) + (1-0.263998355)^3(0.263998355)}{0.70656282}$ — 1M for numerator using (a); 1M for denominator using (b)(i)

≈ 0.351864770

≈ 0.3514 — 1A (accept 0.3513) a–1 for r.t. 0.351

(iii) The integer m satisfies $P(M \le m) > 0.95$.

$p + (1-p)p + (1-p)^2 p + \cdots + (1-p)^{m-1} p > 0.95$ — 1M withhold 1M for bearing an equality sig

$1 - (1-p)^m > 0.95$

$(1-p)^m < 0.05$

$(1 - 0.263998355)^m < 0.05$

$m \ln(0.736001645) < \ln(0.05)$ — 1M for using log or trial and error

$m > 9.773273146$

Thus, the least value of m is 10 . — 1A ---------(9)

(c) Note that $N \sim B(150, p)$.

The mean of N

$= 150p$

$\approx (150)(0.263998355)$ — 1M

≈ 39.59975325

≈ 39.5998 — 1A (accept 39.6) a–1 for r.t. 39.600 (either one)

The variance of N

$= 150p(1-p)$

$\approx (150)(0.263998355)(1 - 0.263998355)$

≈ 29.14548553

≈ 29.1455 — 1A (accept 29.1456) a–1 for r.t. 29.145 ---------(3)

(a)		Good. Some candidates overlooked the case of a plant without infected leaves.
(b) (i)		Good. Some candidates overlooked the case of $M = 0$.
(ii)		Good. Many candidates could tackle this part on conditional probability.
(iii)		Not satisfactory. Only a few candidates were able to formulate the inequality correctly and simplify the expression to arrive at the conclusion.
(c)		Good. Many candidates could apply the binomial distribution although some candidates forgot the formulas for the mean and the variance of the distribution.

39. (2003 ASL-M&S Q10)

(a) Sample mean

$= \frac{12(1) + 14(2) + 10(3) + 6(4) + 2(5) + 1(6)}{5 + 12 + 14 + 10 + 6 + 2 + 1}$

$= 2.2$ — 1A

Sample Standard deviation

$= \sqrt{\frac{12(1^2) + 14(2^2) + 10(3^2) + 6(4^2) + 2(5^2) + 1(6)^2 - (50)(2.2)^2}{5 + 12 + 14 + 10 + 6 + 2 + 1 - 1}}$

$= \sqrt{2}$ — 1A

≈ 1.414213562

≈ 1.4142 — a–1 for r.t. 1.414 ---------(2)

(b) (i) The required probability

$= \frac{2.2^0 e^{-2.2}}{0!} + \frac{2.2^1 e^{-2.2}}{1!} + \frac{2.2^2 e^{-2.2}}{2!} + \frac{2.2^3 e^{-2.2}}{3!}$ — 1M for the 4 cases + 1M for Poisson probability

≈ 0.819352421

≈ 0.8194 — 1A (accept 0.8193) a–1 for r.t. 0.819

(ii) The required probability

$\approx C_2^6 (0.819352421)^2 (1 - 0.819352421)^4$ — 1M for Binomial probability + 1M for using (b)(i)

≈ 0.010724111

≈ 0.0107 — 1A a–1 for r.t. 0.011 ---------(6)

(c) (i) The required probability

$= C_2^3 (0.55)^2 (0.45)$ — 1M for $(0.55)^2(0.45)$

$= 0.408375$ — 1A

≈ 0.4084 — a–1 for r.t. 0.408

(ii) The required probability

$= \frac{2.2^2 e^{-2.2}}{2} (0.55^2)$ — 1M

≈ 0.081113452

≈ 0.0811 — 1A a–1 for r.t. 0.081

(iii) The required probability

$\approx \frac{\left(\frac{2.2^2 e^{-2.2}}{2!} \right)(0.55)^2 + \left(\frac{2.2^3 e^{-2.2}}{3!} \right)(0.55)^3}{0.819352421}$ — 1M for numerator + 1M for denominator using (b)(i)

≈ 0.138925825

≈ 0.1389 — 1A a–1 for r.t. 0.139 ---------(7)

(a)		Good. Candidates should have used '$n-1$' rather than 'n' when finding the sample standard deviation.
(b)		Good. Most candidates were able to apply the binomial distribution.
(c) (i)		Good. A few candidates forgot the binomial coefficient 'C_2^3'.
(ii)		Fair.
(iii)		Poor. Very few candidates were able to correctly obtain the required conditional probability.

40. (2001 ASL-M&S Q11)

Let X_A and X_B be the numbers of persons entered the building using entrances A and B respectively within a 15-minute period.

(a) (i) $P(X_A=0)=\dfrac{(3.2)^0e^{-3.2}}{0!}=e^{-3.2}$ $\boxed{0.0408}$ (p_1) — 1A $a-1$ for r.t. 0.041

(ii) $P(X_B=0)=\dfrac{(2.7)^0e^{-2.7}}{0!}=e^{-2.7}$ $\boxed{0.0672}$ (p_2) — 1A $a-1$ for r.t. 0.067

(iii) $P(X_A+X_B\ge 1)=1-P(X_A=0 \text{ and } X_B=0)$

$=1-P(X_A=0)P(X_B=0)$

$=1-e^{-3.2}e^{-2.7}$ — 1M $1-(p_1)(p_2)$

$=1-e^{-5.9}$ $\boxed{0.9973}$ — 1A $a-1$ for r.t. 0.997

(iv) $P(X_A+X_B=2)$

$=P(X_A=2)P(X_B=0)+P(X_A=1)P(X_B=1)+P(X_A=0)P(X_B=2)$ — 1M for the 3 cases

$=\dfrac{(3.2)^2e^{-3.2}}{2!}\cdot e^{-2.7}+\dfrac{3.2e^{-3.2}}{1!}\cdot\dfrac{2.7e^{-2.7}}{1!}+e^{-3.2}\cdot\dfrac{(2.7)^2e^{-2.7}}{2!}$ — 1A

$=17.405e^{-5.9}$ $\boxed{0.0477}$ — 1A $a-1$ for r.t. 0.048

----------(7)

(b) (i) Since k is the most probable number of persons entered the building within a 15-minute period,

$\therefore$ $P(X=k-1)\le P(X=k)$ and $P(X=k+1)\le P(X=k)$ — 1M+1M

Hence $\dfrac{\lambda^{k-1}e^{-\lambda}}{(k-1)!}\le\dfrac{\lambda^k e^{-\lambda}}{k!}$

$k\le\lambda$ — 1

and $\dfrac{\lambda^{k+1}e^{-\lambda}}{(k+1)!}\le\dfrac{\lambda^k e^{-\lambda}}{k!}$

$\lambda\le k+1$

$\lambda-1\le k$ — 1

(ii) From (b)(i), $k=5$. — 1A

The probability required

$=C_2^4[P(X=k)]^2[1-P(X=k)]^2[P(X=k)]$

$=C_2^4\left(\dfrac{(5.9)^5e^{-5.9}}{5!}\right)^2\left(1-\dfrac{(5.9)^5e^{-5.9}}{5!}\right)^2\left(\dfrac{(5.9)^5e^{-5.9}}{5!}\right)$ — 1M for binomial, 1M for all

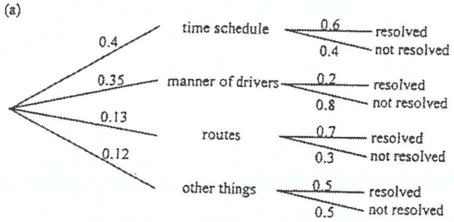
≈ 0.0183 — 1A $a-1$ for r.t. 0.018

----------(8)

41. (1999 ASL-M&S Q12)

Let N be the number of complaints received on a given day and X be the number of complaints involving the time schedule.

(a)



P(manner of drivers | not resolved)

$=\dfrac{0.35\times0.8}{0.4\times0.4+0.35\times0.8+0.13\times0.3+0.12\times0.5}$ $\left(\dfrac{p_1}{p_2}\right)$ — 1M+1A+1A (1A for p_1, 1A for p_2, 1M for $\dfrac{p_1}{p_2}$)

≈ 0.5195 — 1A $a-1$ for r.t. 0.519

(b) (i) $P(N=5)=\dfrac{10^5e^{-10}}{5!}$ — 1A

≈ 0.0378 (p_3) — 1A

(ii) $P(N=5 \text{ and } X=3)=\dfrac{10^5e^{-10}}{5!}\left(C_3^5(0.4)^3(0.6)^2\right)$ — 1M $p_3\left(C_3^5(0.4)^3(0.6)^2\right)$

≈ 0.0087 — 1A $a-1$ for r.t. 0.009

(c) $n\ge 9$. (or $P(N=n \text{ and } X=9)=0$ for $n<9$) — 1M

$P(N=n \text{ and } X=9)=\dfrac{10^ne^{-10}}{n!}C_9^n(0.4)^9(0.6)^{n-9}$ — 1A

(d) (i) $\sum_{k=9}^{\infty}\dfrac{x^k}{(k-9)!}=x^9+\dfrac{x^{10}}{1!}+\dfrac{x^{11}}{2!}+\dfrac{x^{12}}{3!}+\cdots$

$=x^9\left(1+\dfrac{x}{1!}+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\cdots\right)$ — 1A

$=x^9e^x$ — 1

(ii) $P(X=9)=\sum_{n=9}^{\infty}P(N=n \text{ and } X=9)$

$=\sum_{n=9}^{\infty}\dfrac{10^ne^{-10}}{n!}C_9^n(0.4)^9(0.6)^{n-9}$ — 1M

$=\sum_{n=9}^{\infty}\dfrac{10^ne^{-10}}{n!}\cdot\dfrac{n!}{(n-9)!9!}(0.4)^9(0.6)^{n-9}$

$=\dfrac{e^{-10}(0.4)^9}{9!(0.6)^9}\sum_{n=9}^{\infty}\dfrac{6^n}{(n-9)!}$ — 1A

$=\dfrac{e^{-10}(0.4)^9}{9!(0.6)^9}6^9e^6$ (by (b)(i))

$=\dfrac{4^9e^{-4}}{9!}$ (or 0.0132) — 1A $a-1$ for r.t. 0.013

42. (1998 ASL-M&S Q11)

Solution	Marks
(a) Let X be the no. of printing mistakes on P.23, then $X \sim \text{Po}(0.2)$.	
$P(X=0) = e^{-0.2}$	1M+1A
≈ 0.8187	
(b) (i) Let p be the probability that there are printing mistakes on a page, then	
$p = 1 - e^{-0.2}$	1M
Hence $N \sim \text{Geometric}(p)$ and	1M
$P(N \le 3) = P(N=1) + P(N=2) + P(N=3)$	
$= p + p(1-p) + p(1-p)^2$	1M+1A
$= 1-(1-p)^3$	
$= 1-e^{-0.6}$	
≈ 0.4512	1A
(ii) Mean of $N = \dfrac{1}{p} = \dfrac{1}{1-e^{-0.2}} \approx 5.5167$	1A
Variance of $N = \dfrac{1-p}{p^2} = \dfrac{e^{-0.2}}{(1-e^{-0.2})^2} \approx 24.9168$	1A
(c) $M \sim \text{Binomial}(200, p)$ where $p = 1-e^{-0.2}$.	
Mean of $M = np = 200(1-e^{-0.2}) \approx 36.2538$	1A
Variance of $M = np(1-p) = 200e^{-0.2}(1-e^{-0.2}) \approx 29.6821$	1A
(d) (i) $Y \sim \text{Binomial}(40, \frac{1}{200})$.	1A+1A
(ii) $P(Y=0) = \left(1-\dfrac{1}{200}\right)^{40} \approx 0.8183$	1M+1A

43. (1996 ASL-M&S Q13)

Solution	Marks	Remarks
(a) Let X be the number of rainstorms in a year. $X \sim \text{Po}(2)$		
$P(X=x) = \dfrac{e^{-2}2^x}{x!}$, $x = 0, 1, 2, \ldots$		
$P(X \ge 3) = 1 - [P(X=0) + P(X=1) + P(X=2)]$	1M	
$= 1 - e^{-2}\left[1 + 2 + \dfrac{4}{2}\right]$	1A	
$= 1 - 5e^{-2}$		
≈ 0.3233	1A	
(b) Let Y be the number of years which will elapse before the next occurence of more than two rainstorms in a year. $Y \sim \text{Geometric}(p=0.3233)$.	1M	
Number of years which will elapse $= \dfrac{1}{p} - 1$	1M	For $\dfrac{1}{p}$
$= 2.0929$		
≈ 2	1A	
(c) Let A be the event of having at least one serious landslide in city A.		
$P(A \mid X=0) = 0.2$		
$P(A \mid X=1, 2) = 0.3$		
$P(A \mid X \ge 3) = 0.5$		
(i) $P(\bar{A})$		
$= P(\bar{A} \mid X=0)P(X=0) + P(\bar{A} \mid X=1,2)P(X=1,2) + P(\bar{A} \mid X \ge 3)P(X \ge 3)$		
$= 0.8(e^{-2}) + 0.7(4e^{-2}) + 0.5(1-5e^{-2})$	1M+1A	
≈ 0.6489	1A	
<u>Alternatively,</u>		
$P(\bar{A}) = 1 - P(A)$	1M+1A	
$= 1 - [0.2(e^{-2}) + 0.3(4e^{-2}) + 0.5(1-5e^{-2})]$		
≈ 0.6489	1A	
(ii) $P(X=0 \mid \bar{A}) = \dfrac{P(\bar{A} \mid X=0)P(X=0)}{P(\bar{A})}$		
$= \dfrac{0.8(e^{-2})}{0.6489}$	1M+1M	1A for the numerator 1M for the denominator
≈ 0.1669	1A	
(iii) The probability that there is no serious landslide for at most 2 out of 5 years		
$= C_0^5(1-0.6489)^5 + C_1^5(0.6489)(1-0.6489)^4 + C_2^5(0.6489)^2(1-0.6489)^3$	1M+1M	
≈ 0.2369	1A	
<u>Alternatively,</u>		
$1 - [C_3^5(0.6489)^3(1-0.6489)^2 + C_4^5(0.6489)^4(1-0.6489) + C_5^5(0.6489)^5]$	1M+1M	
≈ 0.2369	1A	