7. Further Probability

Learning Unit	Learning Objective
Statistics Area	
Further Probability	
10. Conditional probability and independence	10.1 Understand the concepts of conditional probability and independent events 10.2 use the laws $P(A \cap B) = P(A)\,P(B \mid A)$ and $P(D \mid C) = P(D)$ for independent events C and D to solve problems
11. Bayes' theorem	11.1 use Bayes' theorem to solve simple problems

Set notation

1. Let A and B be two events. Suppose that $P(A)=0.8$, $P(B \mid A)=0.45$ and $P(B \mid A')=0.6$, where A' is the complementary event of A. Find
   (a) $P(B)$,
   (b) $P(A \mid B)$,
   (c) $P(A \cup B)$.

   (5 marks) (2018 DSE-MATH-M1 Q1)

2. Let A and B be two events. Suppose that $P(A)=0.2$, $P(B')=0.7$ and $P(A \mid B)=0.6$, where B' is the complementary event of B.
   (a) $P(B \mid A)$.
   (b) Are A and B mutually exclusive? Explain your answer.
   (c) Are A and B independent? Explain your answer.

   (6 marks) (2017 DSE-MATH-M1 Q2)

3. Let X and Y be two events such that $P(X)=0.4$, $P(Y)=0.7$ and $P(Y \mid X)=0.5$.
   (a) Are X and Y independent? Explain your answer.
   (b) Find $P(X \cup Y)$.

   (5 marks) (2016 DSE-MATH-M1 Q1)

4. A and B are two events. Suppose that $P(A)=0.3$, $P(B)=0.28$ and $P(B' \mid A')=0.6$, where A' and B' are the complementary events of A and B respectively.
   (a) Find $P(A' \cap B')$ and $P(A' \cap B)$.
   (b) Are A and B mutually exclusive? Explain your answer.

   (6 marks) (2015 DSE-MATH-M1 Q2)

5. Let A and B be two events such that $P(A \mid B)=0.4$, $P(A \cup B)=0.45$ and $P(B')=0.75$, where B' is the complementary event of B .
   (a) Find $P(A \cap B)$ and $P(A)$.
   (b) Are events A and B independent? Justify your answer.

   (6 marks) (2014 DSE-MATH-M1 Q7)

6. Suppose A and B are two events. Let A' and B' be the complementary events of A and B respectively. It is given that $P(A \mid B')=0.6$, $P(A \cap B)=0.12$ and $P(A \cap B')=k$, where $k>0$.
   (a) Find $P(A)$, $P(B)$ and $P(A \cup B)$ in terms of k.
   (b) If A and B are independent, find the value of k.

   (6 marks) (PP DSE-MATH-M1 Q9)

7. Let A and B be two events. It is given that $P(A)=a$, $P(B' \mid A)=\frac{27}{32}$ and $P(A \mid B')=\frac{27}{31}$.
   (a) Find $P(A \cap B')$ in terms of a .
   (b) Find $P(B)$ in terms of a .
   (c) It is given that $P(A \cap B)=0.1$.
      (i) Find the value of a .
      (ii) Determine whether A and B are independent or not.

   (7 marks) (2013 ASL-M&S Q5)

8. Let A and B be two events. It is given that $P(A \mid B)=\frac{3}{4}$, $P(B \mid A)=\frac{3}{8}$ and $P(A)=a$.
   (a) Find $P(A \cap B)$ in terms of a .
   (b) Find $P(B)$ in terms of a .
   (c) It is given that $P(A' \cap B')=\frac{7}{16}$.
      (i) Find the value of a .

(ii) Find the value of $P(A|B')$.

(7 marks) (2012 ASL-M&S Q5)

9. Let A and B be two events of a certain sample space such that $P(A \cup B) = 1$. Denote $P(B) = b$ and $P(A \cap B) = c$, where $0 < b < 1$ and $0 < c < 1$.

(a) Express $P(A)$ in terms of b and c .

(b) Suppose that $P(A|B) = \frac{1}{2}$ and $P(B|A) = \frac{2}{3}$.

(i) Find the values of b and c .

(ii) Are the events A and B independent? Explain your answer.

(7 marks) (*modified from* 2010 ASL-M&S Q4)

10. Let A and B be two events. Suppose $P(A \cup B) = \frac{5}{12}$, $P(A) = a$, $P(B) = \frac{1}{4}$ and $P(A \mid B') = k$.

(a) Find $P(A \cap B)$ in terms of a .

(b) Find the value of k .

(c) If A and B are independent, find the value of a .

(8 marks) (2009 ASL-M&S Q4)

11. A and B are two events. A' and B' are the complementary events of A and B respectively. Suppose $P(A) = \frac{1}{5}$, $P(A \cup B) = \frac{9}{20}$, $P(A \mid B) = \frac{1}{6}$ and $P(B) = k$, where $0 < k < 1$.

(a) Using $P(A \mid B)$, express $P(A \cap B)$ in terms of k .

(b) Find the value of k .

(c) Find $P(A' \cap B)$.

(d) Are the two events A' and B' mutually exclusive? Explain your answer.

(7 marks) (2008 ASL-M&S Q4)

12. Let A and B are two events with $P(A) = a$ and $P(B) = b$, where $0 < a < 1$ and $0 < b < 1$. Suppose that $P(A' \mid B) = 0.6$, $P(B \mid A') = 0.3$ and $P(B' \mid A) = 0.7$, where A' and B' are complementary events of A and B respectively.

(a) By considering $P(A' \cap B)$, prove that $a + 2b = 1$.

(b) Using the fact that $A \cup B'$ is the complementary event of $A' \cap B$, or otherwise, find the value of a and b.

(c) Are A and B independent events? Explain your answer.

(7 marks) (2007 ASL-M&S Q5)

13. A and B are two events. Suppose that $P(A \cap B) = 0.2$ and $P(A \mid B') = 0.5$, where B' is the complementary event of B. Let $P(B) = b$, where $b < 1$.

(a) Express $P(A \cap B')$ and $P(A)$ in terms of b.

(b) If A and B are independent events, find the value(s) of b.

(7 marks) (2006 ASL-M&S Q5)

14. A and B are two events. Suppose that $P(A \mid B') = \frac{5}{12}$, $P(B \mid A') = \frac{8}{15}$ and $P(B) = \frac{2}{5}$, where A' and B' are complementary events of A and B respectively. Let $P(A) = a$, where $0 < a < 1$.

(a) Find $P(A \cap B')$.

(b) Express $P(A' \cap B)$ in terms of a .

(c) Using the fact that $A' \cup B$ is the complementary event of $A \cap B'$, or otherwise, find the value of a .

(d) Are A and B mutually exclusive? Explain your answer.

(7 marks) (2005 ASL-M&S Q5)

15. A and B are two events. Suppose that $P(A) = 0.75$ and $P(B) = 0.8$. Let $P(A \cap B) = k$.

(a) Express $P(A \cup B)$ in terms of k .

(b) (i) Prove that $0.55 \le k \le 1$.

(ii) Let A' and B' are complementary events of A and B respectively. Using (b)(i) and $A' \cup B'$ is the complementary event of $A \cap B$, or otherwise, prove that $P(A' \cup B') \le 0.45$

(7 marks) (2004 ASL-M&S Q1)

16. A and B are two events. Suppose that $P(A \mid B) = 0.5$, $P(B \mid A) = 0.4$ and $P(A \cup B) = 0.84$. Let $P(A) = a$, where $a > 0$.

(a) Express $P(A \cap B)$ and $P(B)$ in terms of a .

(b) Using the result of (a), or otherwise, find the value of a .

(c) Are A and B independent events? Explain your answer briefly.

(7 marks) (2003 ASL-M&S Q4)

17. A and B are two independent events. If $P(A) = 0.4$ and $P(A \cup B) = 0.7$, find $P(B)$.

(4 marks) (2001 ASL-M&S Q1)

Tree Diagram, Conditional Probability and Bayes' Theorem

18. A box contains six cards numbered 1 , 2 , 3 , 4 , 5 and 6 respectively.
    (a) Three cards are drawn randomly from the box one by one with replacement. Given that the sum of the numbers drawn is 7, find the probability that the number 1 is drawn exactly two times.
    (b) If the card numbered 6 is taken away before three cards are drawn, will the probability described in (a) change? Explain your answer.

(6 marks) (2015 DSE-MATH-M1 Q2)

19. A bag contains 2 white balls and 5 yellow balls. In a survey, each interviewee draws a ball randomly from the bag. If a white ball is drawn, then the interviewee considers the question 'Are you a smoker?'. If a yellow ball is drawn, then the interviewee considers the question 'Are you a non-smoker?'.
    Finally, the interviewee answers either 'Yes' or 'No'. Let p be the probability that a randomly selected interviewee is a smoker.
    (a) Express, in terms of p , the probability that a randomly selected interviewee answers 'Yes'.
    (b) In this survey, 50 out of 91 interviewees answer 'Yes'.
        (i) Find p .
        (ii) Given that an interviewee answers 'No', find the probability that the interviewee is a non-smoker.

(6 marks) (2015 DSE-MATH-M1 Q3)

20. A company produces microwave ovens by production lines A and B . It is known that 4% of all microwave ovens fail to function properly and that 2% of microwave ovens produced by line A fail to function properly. Among the microwave ovens which function properly, $\frac{2}{3}$ of them are produced by line B . Suppose a microwave oven is randomly selected.
    (a) What is the probability that the microwave oven is produced by line B and functions properly?
    (b) What is the probability that the microwave oven is produced by line A ?
    (c) If the microwave oven is produced by line B , what is the probability that it functions properly?

(5 marks) (2014 DSE-MATH-M1 Q8)

21. In a shooting game, one member from each team will be selected to shoot a target three times. The team will get a prize if the target is hit at least once. Team A consists of Mabel and Owen, with the probability that Mabel is selected to shoot being 0.7. Suppose that the probabilities of Mabel and Owen to hit the target in each shot are 0.6 and 0.5 respectively.
    (a) Find the probability that Team A will get a prize if Mabel is selected.
    (b) Find the probability that Team A will get a prize.
    (c) Given that Team A does not get a prize, find the probability that Owen is selected.

(6 marks) (2013 DSE-MATH-M1 Q8)

22. In a game, there are two bags, A and B, each containing 5 balls. Bag A contains 3 red and 2 blue balls, while bag B contains 4 red and 1 blue balls. A player first chooses a bag at random and then draws a ball randomly from the bag. The player will be rewarded if the ball drawn is blue. The ball is then replaced for the next player's turn.

(a) Find the probability that a player is rewarded in a particular game.

(b) Two players participate in the game. Given that at least one of them is rewarded, find the probability that both of them are rewarded.

(c) If 60 players are rewarded, find the expected number of players among them having drawn a blue ball from bag A.

(5 marks) (PP DSE-MATH-M1 Q7)

23. The percentage of local Year One students in a certain university is 90%, among whom 5% are enrolled with a scholarship. For non-local Year One students, 35% of them are enrolled **without** a scholarship.

(a) If a Year One student is selected at random, find the probability that the student is enrolled with a scholarship.

(b) Given that a selected Year One student is enrolled with a scholarship, find the probability that this student is a non-local student.

(4 marks) (SAMPLE DSE-MATH-M1 Q4)

24. Twelve boys and ten girls in a class are divided into 3 groups as shown in the table below:

	Group A	Group B	Group C
Number of boys	6	4	2
Number of girls	2	3	5

To choose a student as the class representative, a group is selected at random, then a student is chosen at random from the selected group.

(a) Find the probability that a boy is chosen as the class representative.

(b) Suppose that a boy is chosen as the class representative. Find the probability that the boy is from Group A.

(5 marks) (2002 ASL-M&S Q5)

25. In the election of the Legislative Council, 48% of the votes supports Party A, 39% Party B and 13% Party C. Suppose on the polling day, 65%, 58% and 50% of the supporting voters of Parties A, B and C respectively cast their votes.

(a) A voter votes on the polling day. Find the probability that the voter support Party B.

(b) Find the probability that exactly 2 out of 5 voting voters support Party B.

(7 marks) (2001 ASL-M&S Q7)

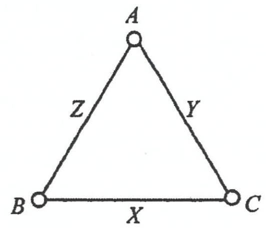
26. A department store uses a machine to offer prizes for customers by playing games A or B. The probability of a customer winning a prize in game A is $\frac{5}{9}$ and that in game B is $\frac{5}{6}$. Suppose each time the machine randomly generates either game A or game B with probabilities 0.3 and 0.7 respectively.

(a) Find the probability of a customer winning a prize in 1 trial.

(b) The department store wants to adjust the probabilities of generating game A and game B so that the probability of a customer winning a prize in 1 trial is $\frac{2}{3}$. Find the probabilities of generating game A and game B respectively.

(6 marks) (2000 ASL-M&S Q8)

27. Three control towers A, B and C are in telecommunication contact by means of three cables X, Y and Z as shown in the figure. A and B remain in contact only if Z is operative or if both cables X and Y are operative. Cables X, Y and Z are subject to failure in anyone day with probabilities 0.015, 0.025 and 0.030 respectively. Such failures occur independently.



(a) Find, to 4 significant figures, the probability that, on a particular day,

(a) both cables X and Z fail to operate,

(b) all cables X, Y and Z fail to operate,

(c) A and B will not be able to make contact.

(b) Given that cable X fails to operate on a particular day, what is the probability that A and B are not able to make contact?

(c) Given that A and B are not able to make contact on a particular day, what is the probability that cable X has failed?

(7 marks)

(7 marks) (1999 ASL-M&S Q7)

28. A factory produced 3 kinds of ice-cream bars A, B and C in the ratio 1 : 2 : 5. It was reported that some ice-cream bars produced on 1 May, 1998 were contaminated. All ice-cream bars produced on that day were withdrawn from sale and a test was carried out. The test results showed hat 0.8% of kind A, 0.2% of kind B and 0% of kind C were contaminated.

(a) An ice-cream bar produced on that day is selected randomly. Find the probability that

(i) the bar is of kind A and is NOT contaminated,

(ii) the bar is NOT contaminated.

(b) If an ice-cream bar produced on that day is contaminated, find the probability that is of kind A.

(6 marks) (1998 ASL-M&S Q6)

29. A company buys equal quantities of fuses, in 100-unit lots, from two suppliers A and B. The company tests two fuses randomly drawn from each lot, and accepts the lot if both fuses are non-defective.
It is known that 4% of the fuses from supplier A and 1% of the fuses from supplier B are defective. Assume that the qualities of the fuses are independent of each other.
(a) What is the probability that a lot will be accepted?
(b) What is the probability that an accepted lot came from supplier A?
(6 marks) (1996 ASL-M&S Q6)

30. An insurance company classifies the aeroplanes it insures into class L (low risk) and class H (high risk), and estimates the corresponding proportions of the aeroplanes as 70% and 30% respectively. The company has also found that 99% of class L and 88% of class H aeroplanes have no accident within a year. If an aeroplane insured by the company has no accident within a year, what is the probability that it belongs to
(a) class H?
(b) class L?
(7 marks) (1995 ASL-M&S Q5)

31. In asking some sensitive question such as "Are you homosexual?", a *randomized response technique* can be applied: The interviewee will be asked to draw a card at random from a box with one red card and two black cards and then consider the statement 'I am homosexual' if the card is red and the statement 'I am not homosexual' otherwise. He will give the response either 'True' or 'False'. The colour of the card drawn is only known to the interviewee so that nobody knows which statement he has responded to. Suppose in a survey, 790 out of 1200 interviewees give the response 'True'.
(a) Estimate the percentage of persons who are homosexual.
(b) For an interviewee who answered 'True', what is the probability that he is really homosexual?
(7 marks) (1994 ASL-M&S Q7)

Section B

32.

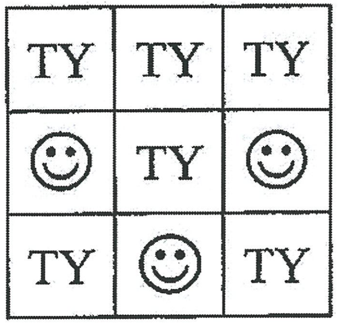


Figure (a)

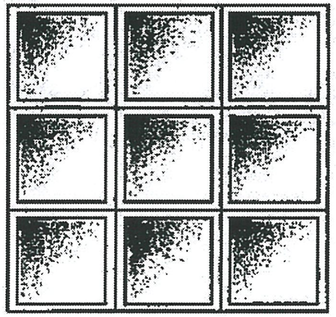


Figure (b)

A soft-drink company proposes a promotion programme by attaching a scratch card to each can of soft drink. Every card has nine squares, with 3 or 4 randomly selected squares each containing a smiley face and in each of the rest a 'TY' denoting 'Thank You'. An example is shown in Figure (a). All squares are covered by metallic films (see Figure (b)).

(a) A customer is asked to rub off the metallic films on 3 squares of a scratch card. If 3 smiley faces are found, the customer will win a prize. Find the probability that the customer can win a prize if the card has
(i) 3 smiley faces,
(ii) 4 smiley faces.
(2 marks)

(b) If the company wants to set the probability of winning a prize to be at most $\frac{1}{60}$, what should be the largest value of the proportion (p) of the cards with 4 smiley faces?
(3 marks)

(c) The company then produces the scratch card according to the proportion p found in (b). The company changes the rule of the game that customers will be asked to rub off the metallic films on 4 squares now and the prizes will be given as follows:
Gold Prize — exactly 4 smiley faces are found on 1 card
Silver Prize — exactly 3 smiley faces are found on 1 card
Bronze Prize — exactly 2 smiley faces are found on each of 2 cards
Find the probability of winning
(i) a Gold Prize with 1 card,
(ii) a Silver Prize with 1 card,
(iii) one or two prizes with 2 cards.
(10 marks)
(2009 ASL-M&S Q10)

33. In a promotion period of an electronic shopping card with spending limit of $3 000 , cardholders who spend over $400 in the maximum amount transaction are classified as VIPs and are eligible for entering an online "click-and-get-point" game once. The rules of the game are detailed in the following table.

Spending ($ x)	VIP Category	Number of clicks allowed
$400 < x \le 800$	Silver	1
$800 < x \le 1000$	Gold	2
$1000 < x \le 3000$	Platinum	3

The probabilities to get 1 , 2 , 3 and 4 points on a single click are 0.4, 0.3, 0.2 and 0.1 respectively. The total number of points got in a game can be exchanged for a cash rebate according to the following table.

Total number of points	Cash rebate
1 to 3	$20
4 to 9	$50
10 to 12	$200

It is known that among the VIPs, 25% belong to Silver, 60% belong to Gold and 15% belong to Platinum.

(a) In a certain completed game, find the probability
 (i) of getting exactly 3 points if the player is a Gold VIP;
 (ii) of getting exactly 3 points;
 (iii) that the player is a Gold VIP given that the player gets exactly 3 points.

(5 marks)

(b) Find the probability that the player gets a cash rebate of exactly $ 20 in a certain completed game.

(2 marks)

(c) In a certain completed game, find the probability that the player gets
 (i) exactly 10 points;
 (ii) a cash rebate of exactly $ 200 .

(3 marks)

(d) Research data reveal that 70% of each category of the VIPs will complete the game. A manager of the card company proposes offering a 4% direct cash rebate of the transaction to all VIPs instead of the online game. However, a senior manager, Winnie, thinks that the cost of that proposal will certainly be higher than the expected cash rebate of the online game.
 (i) Do you agree with Winnie? Explain your answer.
 (ii) Another senior manager, John, thinks that the cost of offering a 2% direct cash rebate to all VIPs will certainly be lower than the expected cash rebate of the online game. Do you agree with John? Explain your answer.

(5 marks)

(2010 ASL-M&S Q11)

34. Boys B_1 , B_2 and girls G_1 , G_2 are students who have qualified to represent their school in a singing contest. One boy and one girl will form one team. The team formed by B_i and G_j is denoted by $B_i G_j$, where $i = 1$, 2 and $j = 1$, 2 . A team can enter the second round of the contest if both team members do not make any mistakes during their performance. Suppose that a student making mistakes in a performance is an independent event, and the probabilities that B_1 , B_2 , G_1 and G_2 do not make any mistakes in a performance are 0.9 , 0.7 , 0.8 and 0.6 respectively.

(a) List all the possible teams that can be formed.

(1 mark)

(b) Find the probability that $B_1 G_1$ can enter the second round of the contest.

(1 mark)

(c) If a team is selected randomly to represent the school, find the probability that the team can enter the second round of the contest.

(2 marks)

(d) If two teams $B_1 G_1$ and $B_2 G_2$ are formed to represent the school, find the probability that
 (i) exactly one team can enter the second round of the contest,
 (ii) at least one team can enter the second round of the contest.

(5 marks)

(e) Suppose that two teams are allowed to represent the school and each student can only join one team.
 (i) If the two teams are formed randomly, find the probability that exactly one team can enter the second round of the contest.
 (ii) How should the teams be formed so that the school has a better chance of having at least one team that can enter the second round of the contest?

(6 marks)

(2000 ASL-M&S Q13)

2021 DSE Q1

The table below shows the probability distribution of a discrete random variable X, where a and b are constants.

x	-1	0	1	2	3	4
$P(X=x)$	a	0.15	0.15	b	0.05	0.25

It is given that $E(5X+1)=10$.

(a) Find a and b.

(b) Let C be the event that $X>0$ and D be the event that $X \le 2$. Find $P(C \mid D)$.

(6 marks)

2021 DSE Q2

The probability that a person has disease D is 0.12. Test T is used to show whether a person has disease D or not. For a person who has disease D, the probability that test T shows that the person has disease D is 0.97. For a person who does not have disease D, the probability that test T shows that the person does not have disease D is 0.89.

(a) Find the probability that test T shows a correct result.

(b) Find the probability that test T shows that a person has disease D.

(c) Given that a person is shown to have disease D by test T, is the probability that the person actually has disease D less than 0.6? Explain your answer.

(6 marks)

2021 DSE Q3

In an examination, there are 10 questions. For each question, the probability that Peter knows how to do the question is 0.8. For each question that Peter knows how to do, the probability that he carelessly answers the question wrongly is 0.1; otherwise, Peter will answer the question correctly for the question that he knows how to do. For questions that Peter does not know how to do, he will answer them wrongly. Peter gets grade A if he answers 8 or more questions correctly.

(a) Find the probability that Peter gets grade A.

(b) Find the probability that Peter knows how to do all the questions and gets grade A.

(c) Given that Peter gets grade A, find the probability that he knows how to do all the questions.

(7 marks)

7. Further Probability

Set notation

1. (2018 DSE-MATH-M1 Q1)

2. (2017 DSE-MATH-M1 Q2)

(a) $P(B \mid A)$		
$=\frac{P(A \mid B)P(B)}{P(A)}$		
$=\frac{P(A \mid B)(1-P(B'))}{P(A)}$		
$=\frac{0.6(1-0.7)}{0.2}$	1M	
$=0.9$	1A	
(b) $P(A \cap B)$		
$=P(A \mid B)P(B)$		
$=P(A \mid B)(1-P(B'))$		
$=0.6(1-0.7)$		
$=0.18$		
$\neq 0$	1M	
Thus, A and B are not mutually exclusive.	1A	f.t.
(c) Note that $P(A \mid B)=0.6 \neq 0.2=P(A)$.	1M	
Thus, A and B are not independent.	1A	f.t.
Note that $P(A \cap B)=0.18 \neq 0.06=P(A)P(B)$.	1M	
Thus, A and B are not independent.	1A	f.t.
	----------(6)	

(a)	Very good. Over 90% of the candidates were able to find the value of $P(B \mid A)$ by using Bayes' Theorem.
(b)	Very good. Most candidates were able to conclude that A and B are not mutually exclusive events.
(c)	Very good. About 80% of the candidates were able to conclude that A and B are not independent events.

3. (2016 DSE-MATH-M1 Q1)

(a) $P(Y\mid X)$ $=0.5$ $\neq 0.7$ $=P(Y)$	1M	
Thus, X and Y are not independent.	1A	f.t.
$P(X)P(Y)$ $=(0.4)(0.7)$ $=0.28$ $P(X\cap Y)$ $=P(Y\mid X)P(X)$ $=(0.5)(0.4)$ $=0.2$ $P(X\cap Y)\neq P(X)P(Y)$	1M	
Thus, X and Y are not independent.	1A	f.t.
(b) $P(X\cap Y)$ $=P(Y\mid X)P(X)$ $=(0.5)(0.4)$ $=0.2$	1M	
$P(X\cup Y)$ $=P(X)+P(Y)-P(X\cap Y)$ $=0.4+0.7-0.2$	1M	
$=0.9$	1A	
	-----------(5)	

(a)	Very good. More than 70% of the candidates were able to mention $P(Y\mid X)\neq P(Y)$ or $P(X\cap Y)\neq P(X)P(Y)$ to conclude that A and B were not independent events. Only some candidates were unable to show their numerical values in comparison.
(b)	Very good. A very high proportion of the candidates were able to use the identity $P(X\cup Y)=P(X)+P(Y)-P(X\cap Y)$ to find the value of $P(X\cup Y)$ while a few candidates were unable to find the value of $P(X\cap Y)$.

4. (2015 DSE-MATH-M1 Q2)

(a) $P(A'\cap B')$ $=P(B'\mid A')P(A')$ $=0.6(1-0.3)$	1M	
$=0.42$	1A	
$P(A'\cap B)$ $=P(A')-P(A'\cap B')$ $=1-0.3-0.42$	1M	
$=0.28$	1A	
(b) Note that $P(B)=P(A\cap B)+P(A'\cap B)$. Since $P(B)=P(A'\cap B)=0.28$, we have $P(A\cap B)=0$.	1M	
Thus, A and B are mutually exclusive.	1A	f.t.
	-----------(6)	

(a)	Very good. Most candidates were able to find the value of $P(A'\cap B')$ while a few candidates failed to find the value of $P(A'\cap B)$ properly.
(b)	Fair. Many candidates mixed up mutually exclusive events with independent events. Only some candidates were able to mention $P(A\cap B)=0$ to conclude that A and B are mutually exclusive events.

5. (2014 DSE-MATH-M1 Q7)

(a) $P(A\cap B)=P(B)P(A\mid B)$ $=(1-0.75)\times 0.4$	1M
$=0.1$	1A
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $0.45=P(A)+(1-0.75)-0.1$	1M
$P(A)=0.3$	1A
(b) $P(A\mid B)=0.4$ $\neq P(A)$	1M
Alternative Solution $P(A)P(B)=0.3\times 0.25$ $=0.075$ $\neq P(A\cap B)$	1M
Hence A and B are not independent events.	1
	(6)

(a) (b)	Excellent. Good. Few candidates wrote $P(A\cap B)=0.1$ for (a) and then $P(A)\cdot P(B)=\cdots\neq 0.1\neq P(A\cap B)$ for (b); while others tried to make conclusion by comparing $P(A\cap B)$ with 0, or $P(A\mid B)$ with $P(A)\cdot P(B)$.

6. (PP DSE-MATH-M1 Q9)

(a) $P(A)=P(A\cap B)+P(A\cap B')$	
$=0.12+k$	1A
$P(A\mid B')=\dfrac{P(A\cap B')}{P(B')}$	
$0.6=\dfrac{k}{1-P(B)}$	
$P(B)=1-\dfrac{5k}{3}$	1A
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$	
$=(0.12+k)+\left(1-\dfrac{5k}{3}\right)-0.12$	1M
$=1-\dfrac{2k}{3}$	1A
(b) If A and B are independent, $P(A)P(B)=P(A\cap B)$.	
$(0.12+k)\left(1-\dfrac{5k}{3}\right)=0.12$	1M
$0.8k-\dfrac{5k^2}{3}=0$	
$k=0.48$ or 0 (rejected)	1A
Alternative solution 1	
If A and B are independent, $P(A)=P(A\mid B')$.	
$0.12+k=0.6$	1M
$k=0.48$	1A
Alternative solution 2	
If A and B are independent, $P(A)P(B')=P(A\cap B')$.	
$(0.12+k)\left(\dfrac{5k}{3}\right)=k$	1M
$\dfrac{5k^2}{3}-0.8k=0$	
$k=0.48$ or 0 (rejected)	1A
Alternative solution 3	
If A and B are independent, $P(A\mid B)=P(A\mid B')$.	
$\therefore \dfrac{P(A\cap B)}{P(B)}=P(A\mid B')$	
$\dfrac{0.12}{1-\dfrac{5k}{3}}=0.6$	1M
$k=0.48$	1A
	(6)

(a)	平平。部分學生忽略了 $P(A)=P(A\cap B)+P(A\cap B')$ 及誤以爲 $P(A\cup B)=P(A)+P(B)$ 。
(b)	平平。少數學生不懂如何利用 A 與 B 爲獨立事件這個條件。

7. (2013 ASL-M&S Q5)

(a) $P(A\cap B')=P(B'\mid A)\cdot P(A)$	
$=\dfrac{27}{32}a$	1A
(b) $P(A\cap B')=P(A\mid B')\cdot P(B')$	
$\dfrac{27}{32}a=\dfrac{27}{31}\cdot[1-P(B)]$	1M
$P(B)=1-\dfrac{31}{32}a$	1A
(c) (i) $P(A)=P(A\cap B)+P(A\cap B')$	
$a=0.1+\dfrac{27}{32}a$	1M
$a=0.64$	1A
(ii) $P(A)\cdot P(B)=(0.64)\left[1-\dfrac{31}{32}(0.64)\right]$	
$=0.2432$	1A
$\neq P(A\cap B)$	
Hence A and B are not independent.	1
	(7)

Good.
Some candidates were not able to apply the definition of independent events and some mixed up the events B and its complement B' .

8. (2012 ASL-M&S Q5)

Solution	Marks	Remarks
(a) $P(A\cap B)=P(A)P(B\mid A)$		
$=\dfrac{3a}{8}$	1A	
(b) $P(A\cap B)=P(B)P(A\mid B)$		
$\dfrac{3a}{8}=\dfrac{3}{4}P(B)$	1M	
$P(B)=\dfrac{a}{2}$	1A	
(c) (i) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$		
$1-\dfrac{7}{16}=a+\dfrac{a}{2}-\dfrac{3a}{8}$	1M	
$a=\dfrac{1}{2}$	1A	
(ii) $P(A\mid B')=\dfrac{P(A\cap B')}{P(B')}$		
$=\dfrac{P(A)-P(A\cap B)}{1-P(B)}$		
$=\dfrac{\frac{1}{2}-\frac{3}{8}\times\frac{1}{2}}{1-\frac{1}{2}\times\frac{1}{2}}$	1M	For numerator
$=\dfrac{5}{12}$	1A	
	(7)	

Very good.
Nevertheless, some candidates were not familiar with the operations of complement and intersection of events.

9. (2010 ASL-M&S Q4)

Solution	Marks	Remarks
(a) $\because\ P(A\cup B)=P(A)+P(B)-P(A\cap B)$		
$\therefore\ 1=P(A)+b-c$	1A	For $P(A\cup B)=1$
i.e. $P(A)=1+c-b$	1A	
(b) (i) $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}$		
$\dfrac{1}{2}=\dfrac{c}{b}$	1M	For conditional probability
$b=2c$ ---------- (1)	1A	For either one
$P(B\mid A)=\dfrac{P(A\cap B)}{P(A)}$		
$\dfrac{2}{3}=\dfrac{c}{1+c-b}$		
$c=2-2b$ ---------- (2)		For either one
Solving (1) and (2), we have $b=0.8$ and $c=0.4$.	1A+1A	
(ii) $P(A)P(B)=(0.6)(0.8)\neq 0.4=P(A\cap B)$		
Alternative Solution 1: $P(A\mid B)=0.5\neq 0.6=P(A)$		
Alternative Solution 2: $P(B\mid A)=\dfrac{2}{3}\neq 0.8=P(B)$		
Hence the events A and B are not independent.	1	Follow through
	(7)	

Poor. Many candidates were not familiar with the basic concept of exhaustive events. The definition and concept of independent events were also not well mastered.

10. (2009 ASL-M&S Q4)

Solution	Marks	Remarks
(a) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$		
$\therefore\ \dfrac{5}{12}=a+\dfrac{1}{4}-P(A\cap B)$	1A	
i.e. $P(A\cap B)=a-\dfrac{1}{6}$	1A	
(b) $P(B')P(A\mid B')=P(A\cap B')$		
$[1-P(B)]\cdot P(A\mid B')=P(A)-P(A\cap B)$		OR $\cdots=P(A\cup B)-P(B)$
$\therefore\ \left(1-\dfrac{1}{4}\right)k=a-\left(a-\dfrac{1}{6}\right)$	1M+1M+1M	OR $\cdots=\dfrac{5}{12}-\dfrac{1}{4}$
i.e. $k=\dfrac{2}{9}$	1A	
(c) Since A and B are independent, $P(A\cap B)=P(A)\times P(B)$.		
$\therefore\ a-\dfrac{1}{6}=a\times\dfrac{1}{4}$	1M	
$a=\dfrac{2}{9}$	1A	
Alternative Solution: Since A and B are independent, $P(A\mid B')=P(A)$.		
$\therefore\ a=k$	1M	
$=\dfrac{2}{9}$	1A	
	(8)	

Very good. Most candidates were able to master the basic rules of probability.

11. (2008 ASL-M&S Q4)

Solution	Marks	Remarks
(a) $P(A\cap B)=P(A\mid B)\cdot P(B)$		
$=\dfrac{k}{6}$	1A	
(b) $\because P(A\cup B)=P(A)+P(B)-P(A\cap B)$		
$\therefore \dfrac{9}{20}=\dfrac{1}{5}+k-\dfrac{k}{6}$	1M	
i.e. $k=\dfrac{3}{10}$	1A	
(c) $P(A'\cap B)=P(B)-P(A\cap B)$		
$=\left(\dfrac{3}{10}\right)-\dfrac{1}{6}\left(\dfrac{3}{10}\right)$	1M	
Alternative Solution: $P(A'\cap B)=P(A\cup B)-P(A)$ $=\dfrac{9}{20}-\dfrac{1}{5}$	1M	
$=\dfrac{1}{4}$	1A	
(d) $P(A'\cap B')=1-P(A\cup B)$ $=1-\dfrac{9}{20}$		
Alternative Solution 1: $P(A'\cap B')=P(A')-P(A'\cap B)$ $=\left(1-\dfrac{1}{5}\right)-\left(\dfrac{1}{4}\right)$		
Alternative Solution 2: $P(A'\cap B')=P(A')+P(B')-P(A'\cup B')$ $=P(A')+P(B')-P((A\cap B)')$ $=\left(1-\dfrac{1}{5}\right)+\left(1-\dfrac{3}{10}\right)-\left(1-\dfrac{1}{6}\cdot\dfrac{3}{10}\right)$		
$=\dfrac{11}{20}$		
$\neq 0$	1M	For $P(A'\cap B')\neq 0$
Hence A' and B' are not mutually exclusive.	1	Follow through
Alternative Solution: $P(A')+P(B')=\left(1-\dfrac{1}{5}\right)+\left(1-\dfrac{3}{10}\right)$ $=\dfrac{3}{2}$		
$\neq P(A'\cup B')$ since $P(A'\cup B')\leq 1$	1M	For $P(A')+P(B')\neq P(A'\cup B')$
Hence A' and B' are not mutually exclusive.	1	Follow through
	(7)	

Very good, except part (d) where many candidates were not sure of the definition of "mutually exclusive" events and confused with that of "independent" events.

12. (2007 ASL-M&S Q5)

Solution	Marks	Remarks
(a) $P(A'\cap B)$ $=P(B\mid A')P(A')$ $=0.3(1-a)$	1M	either one
$P(A'\cap B)$ $=P(A'\mid B)P(B)$ $=0.6b$		
Hence, we have $0.6b=0.3(1-a)$.		
Thus, we have $a+2b=1$.	1	
(b) $P(A\cap B')$ $=P(B'\mid A)P(A)$ $=0.7a$		
$P(A\cup B')$ $=1-P(A'\cap B)$ $=1-0.6b$	1M for complementary events	
Note that $P(A\cup B')=P(A)+P(B')-P(A\cap B')$.		
Hence, we have $1-0.6b=a+(1-b)-0.7a$	1M	
So, we have $3a=4b$.		
Solving $a+2b=1$ and $3a=4b$, we have $a=0.4$ and $b=0.3$.	1A for both correct	
Alternative: $P(A\cap B')$ $=P(B'\mid A)P(A)$ $=0.7a$		
$P(A\cup B')$ $=1-P(A'\cap B)$ $=1-0.3(1-a)$ $=0.7+0.3a$	1M for complementary events	
Note that $P(A\cup B')=P(A)+P(B')-P(A\cap B')$.		
Hence, we have $0.7+0.3a=a+1-b-0.7a$	1M	
So, we have $b=0.3$. By (a), we have $a+2(0.3)=1$. Thus, we have $a=0.4$.	1A	both correct
(c) Since $P(A\cap B)=P(A)-P(A\cap B')$, $P(A)=0.4$ and $P(A\cap B')=0.28$, we have $P(A\cap B)=0.12=(0.4)(0.3)=P(A)P(B)$.	1M for relating $P(A\cap B)$ and $P(A)P(B)$	
Thus, A and B are independent events.	1A f.t.	
Alternative: Since $P(A)=a$, we have $P(A\cap B)=P(A)-P(A\cap B')=a-0.7a=0.3a$. With the help of $P(B)=0.3$, we have $P(A\cap B)=P(A)P(B)$.	1M for relating $P(A\cap B)$ and $P(A)P(B)$	
Thus, A and B are independent events.	1A f.t.	
Alternative: Since $P(A'\mid B)=0.6$, we have $P(A\mid B)=1-P(A'\mid B)=1-0.6=0.4$. With the help of $P(A)=0.4$, we have $P(A\mid B)=P(A)$.	1M for relating $P(A\mid B)$ and $P(A)$	
Thus, A and B are independent events.	1A f.t.	
	(7)	

Fair. Some candidates could not apply the laws of probability especially when complementary events are also involved.

13. (2006 ASL-M&S Q5)

Solution	Marks
(a) $P(A\mid B')=\dfrac{P(A\cap B')}{P(B')}$	
$0.5=\dfrac{P(A\cap B')}{1-b}$	1M
$P(A\cap B')=0.5(1-b)$	1A accept $0.5-0.5b$
$P(A)$	
$=P(A\cap B)+P(A\cap B')$	
$=0.2+0.5(1-b)$	1M
$=0.7-0.5b$	1A
(b) $P(A\cap B)=P(A)P(B)$	
$0.2=(0.7-0.5b)b$	1M for using (a) + 1M for using independence
$5b^2-7b+2=0$	
$b=0.4$ or $b=1$ (rejected)	1A
Thus, we have $b=0.4$.	
Since A and B are independent events, we have $P(A\cap B)=P(A)P(B)$.	
So, we have $P(A\mid B')P(B')=P(A\cap B')=P(A)-P(A)P(B)=P(A)P(B')$.	
Since $P(B')\neq 0$, we have $P(A\mid B')=P(A)$.	
By (a), we have $0.5=0.7-0.5b$.	1M for using (a) + 1M for using independence
Therefore, we have $0.5b=0.2$.	
Thus, we have $b=0.4$.	1A
	-----------(7)

Good. Some candidates confused the definition of 'mutually exclusive events' with the definition of 'independent events'.

14. (2005 ASL-M&S Q5)

Solution	Marks
(a) $P(A\cap B')$	
$=P(A\mid B')P(B')$	1M can be absorbed (either one)
$=(\frac{5}{12})(1-\frac{2}{5})$	
$=\frac{1}{4}$	1A (either one)
(b) $P(A'\cap B)$	
$=P(B\mid A')P(A')$	
$=(\frac{8}{15})(1-a)$	1A or equivalent
(c) $P(A'\cup B)$	
$=1-P(A\cap B')$	
$=1-\frac{1}{4}$ (by (a))	1M accept $P(A)+P(A'\cap B)=P(B)+P(A\cap B')$
$=\frac{3}{4}$	
Note that $P(A'\cup B)=P(A')+P(B)-P(A'\cap B)$.	
Hence, we have $\frac{3}{4}=(1-a)+\frac{2}{5}-(\frac{8}{15})(1-a)$ (by (b)).	1M for using (b)
Thus, we have $a=\frac{1}{4}$.	1A
(d) $\because$ $P(A)=P(A\cap B)+P(A\cap B')$, $P(A)=\frac{1}{4}$ (by (c))	
and $P(A\cap B')=\frac{1}{4}$ (by (a))	
$\therefore$ $P(A\cap B)=0$	
Thus, A and B are mutually exclusive.	1A must show reasons
	-----------(7)

Good. Some candidates confused the definition of 'mutually exclusive events' with the definition of 'independent events'.

15. (2004 ASL-M&S Q1)

(a) $P(A\cup B)$	
$=P(A)+P(B)-P(A\cap B)$	
$=0.75+0.8-k$	1M can be absorbed
$=1.55-k$	1A
(b) (i) $\because P(A\cup B)\le 1$	1M can be absorbed (either one)
$\therefore 1.55-k\le 1$ (by (a))	
Therefore, we have $k\ge 0.55$.	1 (either one)
$\because P(A\cap B)\le 1$	
$\therefore k\le 1$	
Thus, we have $0.55\le k\le 1$.	1
(ii) $P(A'\cup B')$	
$=1-P(A\cap B)$ (since $A'\cup B'$ is the complementary event of $A\cap B$)	
$=1-k$	1M accept $k=1-P(A'\cup B')$
$\le 1-0.55$ (by (b)(i))	
$=0.45$	1
Thus, we have $P(A'\cup B')\le 0.45$.	

$P(A')$	
$=1-P(A)$	
$=1-0.75$	1M (either one)
$=0.25$	
$P(B')$	either one
$=1-P(B)$	
$=1-0.8$	
$=0.2$	
$P(A'\cup B')$	
$=P(A')+P(B')-P(A'\cap B')$	
$=0.25+0.2-P(A'\cap B')$	
$=0.45-P(A'\cap B')$	
≤ 0.45 [redacted]	1

----------(7)

Good. Most candidates knew that any probability is between 0 and 1 but some could not state the fact that $k\le 1$.

16. (2003 ASL-M&S Q4)

(a) $P(A\cap B)=P(A)\,P(B\mid A)$	1M can be absorbed (either one)
$P(A\cap B)=0.4P(A)$	1A
$P(A\cap B)=0.4a$	
$P(A\cap B)=P(B)\,P(A\mid B)=0.5P(B)$	
$0.4a=0.5P(B)$	
$P(B)=\dfrac{0.4}{0.5}a$	
$P(B)=0.8a$	1A

$P(A\cap B)=P(A)\,P(B\mid A)$	
$P(A\cap B)=0.4P(A)$	1M can be absorbed
$P(A\cap B)=0.4a$	1A
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$	
$0.84=a+P(B)-0.4a$	1M can be absorbed
$P(B)=0.84-0.6a$	1A

$P(A\cap B)=P(B)\,P(A\mid B)$	
$P(B)=2\,P(A\cap B)$	1M can be absorbed
$P(A\cup B)=P(A)+P(B)-P(A\cap B)$	
$0.84=a+2\,P(A\cap B)-P(A\cap B)$	1M can be absorbed
$P(A\cap B)=0.84-a$	1A accept $P(A\cap B)=1.8a-0.84$
$P(B)=1.68-2a$	1A accept $P(B)=0.8a$

(b) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$	
$0.84=a+0.8a-0.4a$ (by (a))	1M can be absorbed
$0.84=1.4a$	
$a=0.6$	1A

$P(A\cap B)=P(B)\,P(A\mid B)=0.5P(B)$	
$0.4a=0.5P(B)$	
$P(B)=\dfrac{0.4}{0.5}a$	
$P(B)=0.8a$	
$0.8a=0.84-0.6a$ (by (a))	
$1.4a=0.84$	
$a=0.6$	1A

$P(A\cap B)=P(A)\,P(B\mid A)$	
$P(A\cap B)=0.4\,P(A)$	
$0.84-a=0.4a$	
$a=0.6$	1A

(c) $\because P(A)\,P(B)=(0.6)(0.8)(0.6)=0.288\ne 0.24=(0.4)(0.6)=P(A\cap B)$	1M
$\therefore$ A and B are not independent.	1A

$\because P(A\mid B)=0.5\ne 0.6=P(A)$	1M
$\therefore$ A and B are not independent.	1A

---------(7)

Fair. Concepts of independent events and mutually exclusive events have to be strengthened. Some candidates were unable to distinguish the formulas for intersection and union under general and specific conditions.

17. (2001 ASL-M&S Q1)

Since $P(A\cup B)=P(A)+P(B)-P(A\cap B)$	1M
and $P(A\cap B)=P(A)P(B)$	1M
$\therefore$ $0.7=0.4+P(B)-0.4P(B)$	1A
$P(B)=0.5$	1A

---------(4)

Tree Diagram, Conditional Probability and Bayes' Theorem

18. (2015 DSE-MATH-M1 Q2)

(a) The required probability

$$=\frac{\left(\frac{1}{6}\right)^3(3)}{\left(\frac{1}{6}\right)^3(3+3!+3+3)}$$ — 1M+1M+1M (1M for $\left(\frac{1}{6}\right)^3$ + 1M for numerator + 1M for denominator)

$$=\frac{1}{5}$$ — 1A

(b) The required probability

$$=\frac{\left(\frac{1}{5}\right)^3(3)}{\left(\frac{1}{5}\right)^3(3+3!+3+3)}$$ — 1M

$$=\frac{1}{5}$$

Thus, the required probability will not change. — 1A f.t.

---------(6)

(a)	Good. Some candidates were unable to count the correct number of relevant outcomes for the sum of 7, hence unable to work out the denominator of the required probability properly.
(b)	Fair. Although many candidates guessed correctly that the required conditional probability remains unchanged, they were unable to provide a mathematical argument to justify the guess.

19. (2015 DSE-MATH-M1 Q3)

(a) The required probability

$$=\frac{2}{7}p+\frac{5}{7}(1-p)$$ — 1M for $rs+(1-r)(1-s)$

$$=\frac{5-3p}{7}$$ — 1A

(b) (i) $\frac{5-3p}{7}=\frac{50}{91}$ — 1M for using (a)

$p=\frac{5}{13}$ — 1A

$p\approx 0.384615384$

$p\approx 0.3846$ — r.t. 0.3846

(ii) The required probability

$$=\frac{\frac{2}{7}\left(1-\frac{5}{13}\right)}{1-\frac{50}{91}}$$ — 1M for numerator using (b)(i)

$$=\frac{16}{41}$$ — 1A

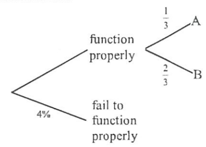
≈ 0.3902439024

≈ 0.3902 — r.t. 0.3902

---------(6)

(a)	Good. Some candidates did not simplify the answer and some candidates failed to give an answer as an expression in terms of p.
(b) (i)	Very good. Most candidates were able to set up an equation by using the result of (a).
(ii)	Good. Some candidates failed to use the result of (b)(i) to find the required probability.

20. (2014 DSE-MATH-M1 Q8)



(a) P(the selected microwave oven is produced by line B and functions properly)

$=(1-0.04)\times\frac{2}{3}$

$=0.64$ — 1A

(b) $P(A)$ P(functions properly | A) = P(functions properly) $P(A$ | functions properly)

$P(A)(1-0.02)=(1-0.04)\left(1-\frac{2}{3}\right)$ — 1M

$P(A)=\frac{16}{49}$ — 1A OR 0.3265

(c) P(functions properly | B) $=\frac{0.64}{1-\frac{16}{49}}$ — 1M

$=\frac{784}{825}$ — 1A OR 0.9503

(5)

(a)	Very good.
(b)	Poor. Most candidates did not understand the meanings of the conditional probabilities given.
(c)	Fair. Many candidates used correct methods but obtained wrong answers, because their answers to (a) or (b) were wrong.

21. (2013 DSE-MATH-M1 Q8)

(a) P(get a prize | Mabel) $= 0.6+0.4\times0.6+0.4^2\times0.6$	1M	OR $1-(1-0.6)^3$
$= 0.936$	1A	OR $(0.6)^3+C_1^3(0.6)^2(0.4)+C_1^3(0.6)(0.4)^2$
(b) P(get a prize | Owen) $= 0.5+0.5\times0.5+0.5^2\times0.5$ $= 0.875$		OR $1-(1-0.5)^3$
$\therefore$ P(win) $= 0.7\times0.936+0.3\times0.875$	1M	
$= 0.9177$	1A	
(c) P(Owen | does not get a prize) $= \dfrac{0.3\times(1-0.875)}{1-0.9177}$	1M	OR $\dfrac{0.3\times(1-0.5)^3}{1-0.9177}$
$= \dfrac{375}{823}$	1A	OR 0.4557
	(6)	

(a)	Fair. Many candidates actually found P(Mabel is selected and Team A gets a prize) instead of the required probability.
(b)	Good.
(c)	Satisfactory. Quite a number of candidates were weak in applying Bayes' theorem.

22. (PP DSE-MATH-M1 Q7)

(a) P(a player is rewarded) $= \frac{1}{2}\cdot\frac{2}{5}+\frac{1}{2}\cdot\frac{1}{5}$ $= 0.3$	1A	
(b) P(both players are rewarded | one player is rewarded) $= \dfrac{0.3\times0.3}{0.3\times0.3+0.3\times0.7\times2}$	1M	OR $\dfrac{0.3\times0.3}{1-0.7\times0.7}$
$= \dfrac{3}{17}$	1A	OR 0.1765
(c) E(no. of players having drawn a blue ball from A) $= 60\times\dfrac{\frac{1}{2}\times\frac{2}{5}}{0.3}$	1M	
$= 40$	1A	
	(5)	

(a)	甚佳。
(b)	尚可。 部分學生忘記條件性概率的公式。
(c)	平平。 大部分學生不懂如何求期望值。

23. (SAMPLE DSE-MATH-M1 Q4)

(a) The required probability $= 0.9\times0.05+0.1\times(1-0.35)$	1M	For $p_1p_2+(1-p_1)p_3$
$= 0.11$	1A	
(b) The required probability $= \dfrac{0.1\times(1-0.35)}{0.11}$	1M	For denominator using (a)
$= \dfrac{13}{22}$	1A	OR 0.5909
	(4)	

24. (2002 ASL-M&S Q5)

(a) The required probability $= \frac{6}{8}\times\frac{1}{3}+\frac{4}{7}\times\frac{1}{3}+\frac{2}{7}\times\frac{1}{3}$	1A
$= \frac{15}{28}$ ($\approx$ 0.5357)	1A a−1 for r.t. 0.536
(b) P(the boy is selected from group A | a boy is selected) $= \dfrac{\frac{6}{8}\times\frac{1}{3}}{\frac{15}{28}}$	1M + 1A (1A for numerator)
$= \frac{7}{15}$ ($\approx$ 0.4667)	1A a−1 for r.t. 0.467
	----------(5)

25. (2001 ASL-M&S Q7)

(a) The required probability $= \dfrac{0.39\times0.58}{0.48\times0.65+0.39\times0.58+0.13\times0.5}$	1A numerator 1A denominator
$= 0.375$ (p)	1A
(b) The required probability $= C_2^5(0.375)^2(1-0.375)^3$	1M binomial, for any p 1M $C_2^5p^2(1-p)^3$, for p in (a)
$= 0.3433$	1A a−1 for r.t. 0.343
	---------(6)

26. (2000 ASL-M&S Q8)

(a) The probability of a customer winning a prize in 1 trial $= 0.3\left(\frac{5}{9}\right)+0.7\left(\frac{5}{6}\right)$	1A
$= 0.75$	1A
(b) Let x and y be the probabilities of generating games A and B respectively. Then $\frac{5}{9}x+\frac{5}{6}y=\frac{2}{3}$	1M
and $x+y=1$	1M
$\therefore$ $\frac{5}{9}x+\frac{5}{6}(1-x)=\frac{2}{3}$	
$\frac{5}{6}-\frac{5}{18}x=\frac{2}{3}$	
$x=\frac{3}{5}$ (or 0.6)	1A or $y=\frac{2}{5}$, $y=0.4$
$\therefore$ The probabilities of generating game A and game B are $\frac{3}{5}$ (or 0.6) and $\frac{2}{5}$ (or 0.4) respectively.	1A
	-----(6)

27. (1999 ASL-M&S Q7)

Let E_X be the event that cable X is operative,
E_Y be the event that cable Y is operative,
E_Z be the event that cable Z is operative, and
F be the event that A and B are not able to make contact.

Solution	Marks	Remarks
(a) (i) $P(E_X' \cap E_Z') = (0.015)(0.030)$ $= 0.00045$ (p_1)	1A	a–1 for r.t. 0.0005 (method must be shown)
(ii) $P(E_X' \cap E_Y' \cap E_Z') = (0.015)(0.025)(0.030)$ $= 0.00001125$ (p_2)	1A	a–1 for r.t. 0.0000 (method must be shown)
(iii) $P(F) = P(E_X' \cap E_Z') + P(E_Y' \cap E_Z') - P(E_X' \cap E_Y' \cap E_Z')$ $= 0.00045 + (0.025)(0.030) - 0.00001125$	1M	$p_1 + (0.025)(0.030) - p_2$
$= 0.00118875$ ≈ 0.001189 (p_3)	1A	r.t. 0.001189, a–1 for r.t. 0.0012 (method must be shown)
(b) $P(F \mid E_X') = P(E_Z')$ $= 0.030$ (p_4)	1A	or 0.03
(c) $P(E_X' \mid F) = \dfrac{P(E_X')P(F \mid E_X')}{P(F)}$ $= \dfrac{(0.015)(0.030)}{0.00118875}$	1M	$\dfrac{(0.015)p_4}{p_3}$
≈ 0.3785489 ≈ 0.3785	1A (7)	r.t. 0.3785

Remark on (a)(iii):

Note that A and B are not able to make contact when **Z is not operative** and **either X or Y is not cooperative**.

$$F = E_Z' \cap (E_X' \cup E_Y')$$
$$= (E_Z' \cap E_X') \cup (E_Z' \cap E_Y')$$

So we have

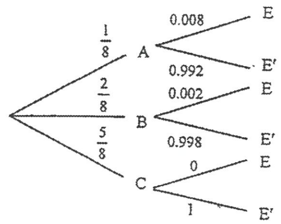
$$P(F) = P(E_Z' \cap E_X') + P(E_Z' \cap E_Y') - P(E_Z' \cap E_X' \cap E_Y')$$

Alternatively, we have

$$P(F) = P(E_Z' \cap (E_X' \cup E_Y'))$$
$$= P(E_Z') \times P(E_X' \cup E_Y')$$
$$= 0.030 \times (P(E_X') + P(E_Y') - P(E_X' \cap E'))$$
$$= 0.030 \times (0.015 + 0.025 - 0.015 \times 0.025)$$
$$= 0.00118875$$

28. (1998 ASL-M&S Q6)

Let E be the event that an ice-cream bar is contaminated.



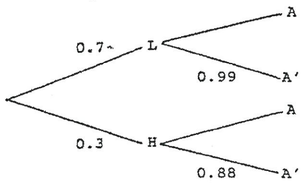
Solution	Marks	Remarks
(tree diagram)	1A	For the tree diagram or all parts in (a) being correct
(a) (i) $P(A)P(E' \mid A) = \dfrac{1}{8} \times 0.992$ $= 0.124$	1A	(p_1)
(ii) $P(E') = P(A)\,P(E' \mid A) + P(B)\,P(E' \mid B) + P(C)\,P(E' \mid C)$ $= 0.124 + \dfrac{2}{8} \times 0.998 + \dfrac{5}{8} \times 1$	1M	for $p_1 + \dfrac{2}{8}p_2 + \dfrac{5}{8}p_3$
$= 0.9985$	1A	a–1 for r.t. 0.999
(b) $P(A \mid E) = \dfrac{P(A)P(E \mid A)}{1 - P(E')}$ $= \dfrac{\frac{1}{8} \times 0.008}{1 - 0.9985}$ $\left(\text{or } \dfrac{\frac{1}{8} \times 0.8\%}{\frac{1}{8} \times 0.8\% + \frac{2}{8} \times 0.2\%}\right)$	1M	
≈ 0.6667	1A (6)	a–1 for r.t. 0.667

29. (1996 ASL-M&S Q6)

Solution	Marks	Remarks
(a) The probability that a lot will be accepted $= (0.5)[(0.99)^2 + (0.96)^2]$	1M+1M	1M for $(0.99)^2 + (0.96)^2$; 1M for $0.5p$
≈ 0.9509 (or 0.95085)	1A	
(b) The probability that a lot came from supplier A $= \dfrac{(0.5)(0.96)^2}{0.95085}$	1A+1M	1A for the numerator; 1M for the denominator
≈ 0.4846	1A (6)	

30. (1995 ASL-M&S Q5)

(a) Let A' be the event of having no accident within a year.



1A

Alternatively,
$P(L) = 0.7,\quad P(H) = 0.3$
$P(A'|L) = 0.99,\quad P(A'|H) = 0.88$ — 1A

$P(H|A') = \dfrac{P(A' \cap H)}{P(A')}$

$= \dfrac{P(A'|H)P(H)}{P(A'|H)P(H) + P(A'|L)P(L)}$

$= \dfrac{0.88 \times 0.3}{0.88 \times 0.3 + 0.99 \times 0.7}$ — 1M+1M+1A (1M for the numerator; 1M for the denominator)

$\approx 0.2759 \quad (\text{or } \dfrac{8}{29})$ — 1A

(b) $P(L|A') = 1 - P(H|A')$
$= 1 - 0.2759$ — 1M
$= 0.7241 \quad (\text{or } \dfrac{21}{29})$ — 1A

Alternatively,
$P(L|A') = \dfrac{P(A'|L)P(L)}{P(A'|H)P(H) + P(A'|L)P(L)}$

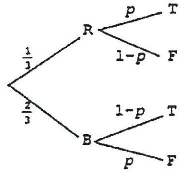
$= \dfrac{0.99 \times 0.7}{0.88 \times 0.3 + 0.99 \times 0.7}$ — 1M

$= 0.7241 \quad (\text{or } \dfrac{21}{29})$ — 1A

(7)

31. (1994 ASL-M&S Q7)

(a) R: red card is drawn T: response 'True'
B: black card is drawn F: response 'False'
p: percentage of persons who are homosexual



1A

$\dfrac{790}{1200} = \dfrac{1}{3}p + \dfrac{2}{3}(1-p)$ — 1M + 1A

$\dfrac{790}{1200} = \dfrac{2}{3} - \dfrac{1}{3}p$

$p = 2.5\%$ — 1A

Alternatively
Let x, y be the no. of interviewees who are homosexual and not homosexual respectively, then

$\frac{1}{3}x + \frac{2}{3}y = 790$ — 1A

$\frac{2}{3}x + \frac{1}{3}y = 410$ — 1A

Solving the equations, we have
$x = 30,\ y = 1170$ — 1M (For reducing into one unknown)

$\therefore$ The percentage required $= \dfrac{30}{1200}$
$= 2.5\%$ — 1A

(b) $P(R|T) = \dfrac{P(T|R)P(R)}{P(T)}$

$= \dfrac{(0.025)(\frac{1}{3})}{\frac{79}{120}}$ — 1M + 1A

$= 0.0127 \quad (\text{or } \dfrac{1}{79})$ — 1A

7

Section B

32. (2009 ASL-M&S Q10)

Solution	Marks	Remarks
(a) (i) The required probability $=\frac{1}{C_3^9}=\frac{1}{84}$	1A	OR 0.0119
(ii) The required probability $=\frac{C_3^4}{C_3^9}=\frac{1}{21}$	1A	OR 0.0476
	(2)	
(b) $(1-p)\left(\frac{1}{84}\right)+p\left(\frac{1}{21}\right)\le\frac{1}{60}$	1M+1A	(pp-1) for using "="
$5-5p+20p\le 7$		
$p\le\frac{2}{15}$		
Hence, the largest value of p should be $\frac{2}{15}$.	1A	OR 0.1333
	(3)	
(c) (i) The required probability $=\frac{2}{15}\cdot\frac{1}{C_4^9}$	1M	For using (b)
$=\frac{1}{945}$	1A	OR 0.0011
(ii) The required probability $=\left(1-\frac{2}{15}\right)\cdot\frac{C_3^3C_1^6}{C_4^9}+\frac{2}{15}\cdot\frac{C_3^4C_1^5}{C_4^9}$	1M+1A	
$=\frac{59}{945}$	1A	OR 0.0624
(iii) The probability of exactly 2 logos are found on 1 card		
$=\left(1-\frac{2}{15}\right)\cdot\frac{C_2^3C_2^6}{C_4^9}+\frac{2}{15}\cdot\frac{C_2^4C_2^5}{C_4^9}$	1M	
$=\frac{47}{126}$	1A	OR 0.3730
Hence, the required probability		
$=\left(\frac{1}{945}+\frac{59}{945}\right)^2+2\left(\frac{1}{945}+\frac{59}{945}\right)\left(1-\frac{1}{945}-\frac{59}{945}\right)+\left(\frac{47}{126}\right)^2$	1M+1A	
$=\frac{1387}{5292}$	1A	OR 0.2621
	(10)	

(a) (i)		Very good.
(ii)		Very good.
(b)		Good. Some candidates were unfamiliar with handling inequalities.
(c) (i)		Fair. Many candidates could not analyse the situation and in fact a simple tree diagram would be helpful.
(ii)		Poor. Many candidates had difficulties in counting and exhausting the relevant cases.
(iii)		Very poor. Again many candidates encountered difficulties in counting the relevant cases when the situation was complicated in having two cards.

33. (2010 ASL-M&S Q11)

Solution	Marks	Remarks
(a) (i) P(getting 3 points | Gold VIP)		
$=2(0.4)(0.3)$		
$=0.24$	1A	
(ii) P(getting 3 points)		
$=(0.25)(0.2)+(0.6)(0.24)+(0.15)(0.4)^3$	1M	
$=0.2036$	1A	
(iii) P(Gold VIP | 3 points are obtained)		
$=\frac{(0.6)(0.24)}{0.2036}$	1M	
≈ 0.7073	1A	
	(5)	
(b) P(\$ 20 cash rebate)		
$=(0.25)(0.4)+[(0.25)(0.3)+(0.6)(0.4)^2]+0.2036$	1M	
$=0.4746$	1A	
	(2)	
(c) (i) P(getting 10 points)		
$=(0.15)[3(0.3)(0.1)^2+3(0.2)^2(0.1)]$		
$=0.00315$	1A	
(ii) P(\$ 200 cash rebate)		
$=0.00315+(0.15)[3(0.2)(0.1)^2+(0.1)^3]$	1M	
$=0.0042$	1A	
	(3)	
(d) (i) Expected cash rebate using the online game		
$=\$\{0.7[20(0.4746)+50(1-0.4746-0.0042)+200(0.0042)]+(1-0.7)(0)\}$	1M	
$=\$25.4744$		
The minimum cash rebate under the 4% direct cash rebate plan		
$>\$[(0.25)(400)+(0.6)(800)+(0.15)(1000)](0.04)$	1M	
$=\$29.2$		
Since $29.2>25.4744$, Winnie is agreed with.	1	Follow through
(ii) The maximum cash rebate under the 2% direct cash rebate plan		
$=\$[(0.25)(800)+(0.6)(1000)+(0.15)(3000)](0.02)$	1M	
$=\$25$		
Since $25<25.4744$, John is agreed with.	1	Follow through
	(5)	

(a)		Very good.
(b)		Good. Some candidates did not exhaust all possible cases in their counting.
(c)		Fair. Many candidates made some errors in counting the relevant events.
(d)		Poor. Many candidates were not familiar with expected values.

34. (2000 ASL-M&S Q13)

(a) Possible teams: B_1G_1, B_1G_2, B_2G_1 and B_2G_2. 1A

(b) The probability that B_1G_1 can enter the second round of the contest
$= 0.9 \times 0.8$
$= 0.72$ 1A

(c) Probability required $= \frac{1}{4}(0.9\times0.8+0.9\times0.6+0.7\times0.8+0.7\times0.6)$ 1M 4 cases
$= 0.56$ 1A

(d) Suppose B_1G_1 and B_2G_2 are formed to represent the school.

(i) The probability that exactly one team can enter the second round
$= (0.9\times0.8)(1-0.7\times0.6)+(0.7\times0.6)(1-0.9\times0.8)$ 1A probability of 1 case; 1M summation of 2 cases

or $1-(0.9\times0.8)(0.7\times0.6)-(1-0.9\times0.8)(1-0.7\times0.6)$

$= 0.5352$ 1A

(ii) The probability that at least one team can enter the second round
$= 0.5352 + 0.9\times0.8\times0.7\times0.6$ 1M
or $1-(1-0.9\times0.8)(1-0.7\times0.6)$
or $0.9\times0.8+0.7\times0.6-0.9\times0.8\times0.7\times0.6$
$= 0.8376$ 1A

(e) (i) If the two teams are formed randomly, the probability that exactly one team can enter the second round

$= \frac{1}{2}\times0.5352+\frac{1}{2}[(0.9\times0.6)(1-0.7\times0.8)+(0.7\times0.8)(1-0.9\times0.6)]$ 1M the combination B_1G_2, B_2G_1; 1M multiplying by $\frac{1}{2}$

$= \frac{1}{2}(0.5352+0.4952)$
$= 0.5152$ 1A

(ii) If B_1G_2 and B_2G_1 are formed to represent the school, the probability that at least one team can enter the second round
$= 0.4952 + 0.9\times0.8\times0.7\times0.6$ 1M
or $1-(1-0.9\times0.6)(1-0.7\times0.8)$
or $0.9\times0.6+0.7\times0.8-0.9\times0.6\times0.7\times0.8$
$= 0.7976$ 1A

From (d)(ii), the combination B_1G_1 and B_2G_2 will have a better chance of having at least one team that can enter the second round of the contest. 1M