## Learning Content of the Compulsory Part

## Notes:

1. Learning units are grouped under three strands ("Number and Algebra", "Measures, Shape and Space" and "Data Handling") and a Further Learning Unit.
2. Related learning objectives are grouped under the same learning unit.
3. The learning objectives underlined are the Non-foundation Topics.
4. The notes in the "Remarks" column of the table may be considered as supplementary information about the learning objectives.
5. To aid teachers in judging how far to take a given topic, a suggested lesson time in hours is given against each learning unit. However, the lesson time assigned is for their reference only. Teachers may adjust the lesson time to meet their individual needs.
6. Schools may allocate up to 313 hours (i.e. $12.5 \%$ of the total lesson time) to those students who need more time for learning.

| Learning Unit | Learning Objective | Time | Remarks |  |
| :--- | :--- | :--- | :--- | :--- |
| Number and Algebra Strand |  |  |  |  |
| 1. Quadratic equations in <br> one unknown | 1.1 | solve quadratic equations by the factor method | 19 |  |


| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
|  | 1.4 solve quadratic equations by the quadratic formula |  | The following are not required for students taking only the Foundation Topics: <br> - expressing nonreal roots in the form $a \pm b i$ <br> - simplifying expressions involving surds such as $2 \pm \sqrt{48}$ |
|  | 1.5 understand the relations between the discriminant of a quadratic equation and the nature of its roots |  | When $\Delta<0$, students have to point out that "the equation has no real roots" or "the equation has two nonreal roots" as they are expected to recognise the existence of complex numbers in Learning Objective 1.8. |
|  | 1.6 solve problems involving quadratic equations |  | Teachers should select the problems related to students' experiences. <br> Problems involving complicated equations such as $\frac{6}{x}+\frac{6}{x-1}=5$ are required only in the Non-foundation Topics and dealt with in Learning Objective 5.4. |


| Learning Unit | Learning Objective | Time | Remarks |
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|  | 1.7 understand the relations between the roots and coefficients and form quadratic equations using these relations <br> 1.8 appreciate the development of the number systems including the system of complex numbers <br> 1.9 perform addition, subtraction, multiplication and division of complex numbers |  | The relations between the roots and coefficients include: <br> - $\alpha+\beta=-\frac{b}{a}$ and $\alpha \beta=\frac{c}{a}$, <br> where $\alpha$ and $\beta$ are the roots of the equation $a x^{2}+b x+c=0$ and $a \neq 0$. <br> The topics such as the hierarchy of the number systems and the conversion between recurring decimals and fractions may be discussed. <br> Complex numbers are confined to the form $a$ $\pm b i$. <br> Note: The coefficients of quadratic equations are confined to real numbers. |
| 2. Functions and graphs | 2.1 recognise the intuitive concepts of functions, domains and co-domains, independent and dependent variables | 10 |  |


| Learning Unit | Learning Objective | Time | Remarks |
| :--- | :--- | :--- | :--- |
| 2.2recognise the notation of functions and use tabular, <br> algebraic and graphical methods to represent <br> functions | Representations like <br> 2.3 <br> understand the features of the graphs of quadratic <br> functions | The features of the graphs of quadratic <br> functions include: <br> - the vertex <br> - the axis of symmetry <br> - the direction of opening <br> - relations with the axes <br> Students are required to find the maximum and <br> minimum values of quadratic functions by the <br> graphical method. |  |
| 2.4find the maximum and minimum values of <br> quadratic functions by the algebraic method | The method of completing the square is <br> required. <br> Students are required to solve problems <br> related to maximum and minimum values of <br> quadratic functions. |  |  |


| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
| 3. Exponential and logarithmic functions | 3.1 understand the definitions of rational indices <br> 3.2 understand the laws of rational indices <br> 3.3 understand the definition and properties of logarithms (including the change of base) | 16 | The definitions include $a^{\frac{1}{n}}$ and $a^{\frac{m}{n}}$ <br> The laws of rational indices include: <br> - $a^{p} a^{q}=a^{p+q}$ <br> - $\frac{a^{p}}{a^{q}}=a^{p-q}$ <br> - $\left(a^{p}\right)^{q}=a^{p q}$ <br> - $a^{p} b^{p}=(a b)^{p}$ <br> - $\frac{a^{p}}{b^{p}}=\left(\frac{a}{b}\right)^{p}$ <br> The properties of logarithms include: <br> - $\log _{a} 1=0$ <br> - $\log _{a} a=1$ <br> - $\log _{a} M N=\log _{a} M+\log _{a} N$ <br> - $\log _{a} \frac{M}{N}=\log _{a} M-\log _{a} N$ <br> - $\log _{a} M^{k}=k \log _{a} M$ <br> - $\log _{b} N=\frac{\log _{a} N}{\log _{a} b}$ |


| Learning Unit | Learning Objective | Time | Remarks |
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|  | 3.4 understand the properties of exponential functions and logarithmic functions and recognise the features of their graphs <br> 3.5 solve exponential equations and logarithmic equations <br> 3.6 appreciate the applications of logarithms in real-life situations |  | The properties and features include: <br> - the domains of the functions <br> - the function $f(x)=a^{x}$ and $f(x)=\log _{a} x$ increases (decreases) as $x$ increases for $a>1(0<a<1)$ <br> - $y=a^{x}$ is symmetric to $y=\log _{a} x$ about $y=x$ <br> - the intercepts with the axes <br> - the rate of increasing/the rate of decreasing of the functions (by direct inspection) <br> Equations which can be transformed into quadratic equations such as $4^{x}-3 \cdot 2^{x}-4=$ 0 or $\log (x-22)+\log (x+26)=2$ are dealt with in Learning Objective 5.3. <br> The applications such as measuring earthquake intensity in the Richter Scale and sound intensity level in decibels may be discussed. |


| Learning Unit | Learning Objective | Time | Remarks |
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|  | 3.7 appreciate the development of the concepts of logarithms |  | The topics such as the historical development of the concepts of logarithms and its applications to the design of some past calculation tools such as slide rules and the logarithmic table may be discussed. |
| 4. More about polynomials | 4.1 perform division of polynomials <br> 4.2 understand the remainder theorem <br> 4.3 understand the factor theorem <br> 4.4 understand the concepts of the greatest common divisor and the least common multiple of polynomials <br> 4.5 perform addition, subtraction, multiplication and division of rational functions | 14 | Methods other than long division are also accepted. <br> Students are required to use factor theorem to factorise polynomials such as $x^{3} \pm a^{3}$. <br> The terms "H.C.F." , "gcd", etc. can be used. <br> Computation of rational functions with more than two variables is not required. <br> Rational functions refer to algebraic fractions at Key Stage 3. |
| 5. More about equations | 5.1 use the graphical method to solve simultaneous equations in two unknowns, one linear and one quadratic in the form $y=a x^{2}+b x+c$ | 10 |  |


| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
|  | 5.2 use the algebraic method to solve simultaneous equations in two unknowns, one linear and one quadratic <br> 5.3 solve equations (including fractional equations, exponential equations, logarithmic equations and trigonometric equations) which can be transformed into quadratic equations <br> 5.4 solve problems involving equations which can be transformed into quadratic equations |  | Solutions for trigonometric equations are confined to the interval from $0^{\circ}$ to $360^{\circ}$. <br> Teachers should select the problems related to students' experience. |
| 6. Variations | 6.1 understand direct variations and inverse variations, and their applications to solving real-life problems <br> 6.2 understand the graphs of direct and inverse variations <br> 6.3 understand joint and partial variations, and their applications to solving real-life problems | 7 |  |
| 7. Arithmetic and geometric sequences and their summations | 7.1 understand the concept and the properties of arithmetic sequences | 17 | The properties of arithmetic sequences include: <br> - $T_{n}=1 / 2\left(T_{n-1}+T_{n+1}\right)$ <br> - if $T_{1}, T_{2}, T_{3}, \ldots$ is an arithmetic sequence, then $k T_{1}+a, k T_{2}+a, k T_{3}+$ $a, \ldots$ is also an arithmetic sequence |


| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
|  | 7.2 understand the general term of an arithmetic sequence <br> 7.3 understand the concept and the properties of geometric sequences <br> 7.4 understand the general term of a geometric sequence <br> 7.5 understand the general formulae of the sum to a finite number of terms of an arithmetic sequence and a geometric sequence and use the formulae to solve related problems <br> 7.6 explore the general formulae of the sum to infinity for certain geometric sequences and use the formulae to solve related problems <br> 7.7 solve related real-life problems |  | The properties of geometric sequences include: <br> - $T_{n}^{2}=T_{n-1} \times T_{n+1}$ <br> - if $T_{1}, T_{2}, T_{3}, \ldots$ is a geometric sequence, then $k T_{1}, k T_{2}, k T_{3}, \ldots$ is also a geometric sequence <br> Example: geometrical problems involving the sum of arithmetic or geometric sequences. <br> Example: geometrical problems involving infinite sum of the geometric sequences. <br> Examples: problems about interest, growth or depreciation. |


| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
| 8. Inequalities and linear programming | 8.1 solve compound linear inequalities in one unknown <br> 8.2 solve quadratic inequalities in one unknown by the graphical method <br> 8.3 solve quadratic inequalities in one unknown by the algebraic method <br> 8.4 represent the graphs of linear inequalities in two unknowns in the rectangular coordinate plane <br> 8.5 solve systems of linear inequalities in two unknowns <br> 8.6 solve linear programming problems | 16 | Compound inequalities involving logical connectives "and" or "or" are required. <br> Solving the problems on triangle inequalities is required. |
| 9. More about graphs of functions | 9.1 sketch and compare graphs of various types of functions including constant, linear, quadratic, trigonometric, exponential and logarithmic functions <br> 9.2 solve the equation $f(x)=k$ using the graph of $y=f(x)$ | 11 | Comparison includes domains, existence of maximum or minimum values, symmetry and periodicity. |


| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
|  | 9.3 solve the inequalities $f(x)>k, f(x)<k$, $f(x) \geq k$ and $f(x) \leq k \quad$ using the graph of $y=f(x)$ <br> 9.4 understand the transformations of the function $f(x)$ including $f(x)+k, \quad f(x+k), \quad k f(x)$ and $f(k x)$ from tabular, symbolic and graphical perspectives |  |  |

## Measures, Shape and Space Strand

10. Equations of straight 10.1 understand the equation of a straight line
lines

Students are required to find the equation of a straight line from given conditions such as:

- the coordinates of any two points on the straight line
- the slope of the straight line and the coordinates of a point on it
- the slope and the $y$-intercept of the straight line

Students are required to describe the features of a straight line from its equation. The features include:

- the slope

| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
|  | 10.2 understand the possible intersection of two straight lines |  | - the intercepts with the axes <br> - whether it passes through a given point <br> The normal form is not required. <br> Students are required to recognise the relation between slope and inclination. <br> Students are required to determine the number of intersection points of two straight lines from their equations. <br> At Key Stage 3, students are required to solve simultaneous linear equations in two unknowns. <br> Note: Teachers are suggested to arrange the teaching of this Learning Unit in the first term of S4. |
| 11. Basic properties of circles | 11.1 understand the properties of chords and arcs of a circle | 23 | The properties of chords and arcs of a circle include: <br> - the chords of equal arcs are equal <br> - equal chords cut off equal arcs <br> - the perpendicular from the centre to a chord bisects the chord |

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\begin{array}{|l|l|l|l|}\hline \text { Learning Unit } & \text { Learning Objective } & \text { Time } & \text { Remarks }\end{array}
$$ \left\lvert\, $$
\begin{array}{l}\text { - the straight line joining the centre and the } \\
\text { mid-point of a chord which is not a } \\
\text { diameter is perpendicular to the chord } \\
\text { - the perpendicular bisector of a chord passes } \\
\text { through the centre }\end{array}
$$\right.\right\} $$
\begin{array}{l}\text { - equal chords are equidistant from the centre } \\
\text { - chords equidistant from the centre are equal } \\
\text { Students are required to understand that there }\end{array}
$$\right\} \begin{array}{l}is one and only one circle passing through <br>
given three non-collinear points. <br>
The property that the arcs are proportional to <br>
their corresponding angles at the centre should <br>
be discussed at Key Stage 3 when the formula <br>
for calculating arc lengths is being explicated. <br>

The angle properties of a circle include:\end{array}\right\}\)| - the angle subtended by an arc of a circle at |
| :--- |
| the centre is double the angle subtended by |
| the arc at any point on the remaining part |
| of the circumference |


| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
|  | 11.3 understand the properties of a cyclic quadrilateral <br> 11.4 understand the tests for concyclic points and cyclic quadrilaterals |  | - the angle in a semi-circle is a right angle <br> - if the angle at the circumference is a right angle, then the chord that subtends the angle is a diameter <br> The properties of a cyclic quadrilateral include: <br> - the opposite angles of a cyclic quadrilateral are supplementary <br> - an exterior angle of a cyclic quadrilateral equals its interior opposite angle <br> The tests for concyclic points and cyclic quadrilaterals include: <br> - if $A$ and $D$ are two points on the same side of the line $B C$ and $\angle B A C=\angle B D C$, then $A, B, C$ and $D$ are concyclic <br> - if a pair of opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic <br> - if the exterior angle of a quadrilateral equals its interior opposite angle, then the quadrilateral is cyclic |

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\begin{array}{|l|l|l|l|}\hline \text { Learning Unit } & \text { Learning Objective } & \text { Time } & \begin{array}{l}\text { Remarks }\end{array} \\
\hline 11.5 \begin{array}{l}\text { understand the properties of tangents to a circle and } \\
\text { angles in the alternate segments }\end{array} & \begin{array}{l}\text { The properties include: } \\
\text { - a tangent to a circle is perpendicular to the } \\
\text { radius through the point of contact } \\
\text { - the straight line perpendicular to a radius of } \\
\text { a circle at its external extremity is a tangent } \\
\text { to the circle }\end{array} \\
\begin{array}{ll}\text { - the perpendicular to a tangent at its point of } \\
\text { contact passes through the centre of the } \\
\text { circle }\end{array}
$$ <br>
- if two tangents are drawn to a circle from <br>
an external point, then: <br>
the distances from the external point <br>

to the points of contact are equal\end{array}\right\}\)| the tangents subtend equal angles at |
| :--- |
| the centre |


| Learning Unit | Learning Objective | Time | Remarks |
| :--- | :--- | :--- | :--- |$\quad$| -if a straight line passes through an end point <br> of a chord of a circle so that the angle it <br> makes with the chord is equal to the angle <br> in the alternate segment, then the straight <br> line touches the circle |
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| Learning Unit | Learning Objective | Time | Remarks |
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|  | 12.3 describe the locus of points with algebraic equations |  | Students are required to find the equations of simple loci, which include equations of straight lines, circles and parabolas (in the form of $\left.y=a x^{2}+b x+c\right)$. |
| 13. Equations of circles | 13.1 understand the equation of a circle <br> 13.2 find the coordinates of the intersections of a straight line and a circle and understand the possible intersection of a straight line and a circle | 7 | Students are required to find the equation of a circle from given conditions such as: <br> - the coordinates of the centre and the radius of the circle <br> - the coordinates of any three points on the circle <br> Students are required to describe the features of a circle from its equation. The features include: <br> - the centre <br> - the radius <br> - whether a given point lies inside, outside or on the circle <br> Students are required to find the equations of tangents to a circle. |


| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
| 14. More about trigonometry | 14.1 understand the functions sine, cosine and tangent, and their graphs and properties, including maximum and minimum values and periodicity <br> 14.2 solve the trigonometric equations $a \sin \theta=b$, $a \cos \theta=b, a \tan \theta=b \quad$ (solutions in the interval from $0^{\circ}$ to $360^{\circ}$ ) and other trigonometric equations (solutions in the interval from $0^{\circ}$ to $\underline{360^{\circ} \text { ) }}$ <br> 14.3 understand the formula $1 / 2 a b \sin C$ for areas of triangles <br> 14.4 understand the sine and cosine formulae <br> 14.5 understand Heron's formula <br> 14.6 understand the concept of projection <br> 14.7 understand the angle between a line and a plane, and the angle between 2 planes <br> 14.8 understand the theorem of three perpendiculars | 25 | Simplification of expressions involving sine, cosine and tangent of $-\theta, \quad 90^{\circ} \pm \theta$, $180^{\circ} \pm \theta, \ldots$, etc. is required. <br> Equations that can be transformed into quadratic equations are required only in the Non-foundation Topics and dealt with in Learning Objective 5.3. <br> The concept of inclination is required. |


| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
|  | 14.9 solve related 2 -dimensional and 3-dimensional problems |  | 3-dimensional problems include finding the angle between two lines, the angle between a line and a plane, the angle between two planes, the distance between points, the distance between a point and a line, and the distance between a point and a plane. |
| Data Handling Strand |  |  |  |
| 15. Permutations and combinations | 15.1 understand the addition rule and multiplication rule in the counting principle <br> 15.2 understand the concept and notation of permutation <br> 15.3 solve problems on the permutation of distinct objects without repetition <br> 15.4 understand the concept and notation of combination | 11 | Notations such as " $P_{r}^{n} ", ~ "{ }_{n} P_{r}$ ", " ${ }^{n} P_{r} "$, etc. can be used. <br> Problems such as "permutation of objects in which three particular objects are put next to each other" are required. <br> Circular permutation is not required. <br> Notations such as " $C_{r}^{n} ",{ }_{n} C_{r} ",{ }^{n} C_{r}$ ", " $\binom{n}{r}$ ", etc. can be used. |


| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
|  | 15.5 solve problems on the combination of distinct objects without repetition |  |  |
| 16. More about probability | 16.1 recognise the notation of set language including union, intersection and complement <br> 16.2 understand the addition law of probability and the concepts of mutually exclusive events and complementary events <br> 16.3 understand the multiplication law of probability and the concept of independent events <br> 16.4 recognise the concept and notation of conditional probability <br> 16.5 use permutation and combination to solve problems related to probability | 10 | The concept of Venn Diagram is required. <br> The addition law of probability refers to $" P(A \cup B)=P(A)+P(B)-P(A \cap B) "$. <br> The multiplication law of probability refers to " $P(A \cap B)=P(A) \times P(B)$, where $A$ and $B$ are independent events". <br> The rule " $P(A \cap B)=P(A) \times P(B \mid A)$ " is required. <br> Bayes' Theorem is not required. |
| 17. Measures of dispersion | 17.1 understand the concept of dispersion <br> 17.2 understand the concepts of range and inter-quartile range | 13 |  |


| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
|  | 17.3 construct and interpret the box-and-whisker diagram and use it to compare the distributions of different sets of data <br> 17.4 understand the concept of standard deviation for both grouped and ungrouped data sets <br> 17.5 compare the dispersions of different sets of data using appropriate measures <br> 17.6 understand the applications of standard deviation to real-life problems involving standard scores and the normal distribution <br> 17.7 understand the effect of the following operations on the dispersion of the data: <br> (i) adding a common constant to each item of the set of data <br> (ii) multiplying each item of the set of data by a common constant |  | A box-and-whisker diagram can also be called a "boxplot". <br> Students are required to recognise the term "variance" and that variance equals to the square of standard deviation. <br> Students are required to understand the following formula for standard deviation: $\sigma=\sqrt{\frac{\left(x_{1}-\mu\right)^{2}+\ldots+\left(x_{N}-\mu\right)^{2}}{N}}$ |


| Learning Unit | Learning Objective | Time | Remarks |
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| 18. Uses and abuses of statistics | 18.1 recognise different techniques in survey sampling and the basic principles of questionnaire design <br> 18.2 discuss and recognise the uses and abuses of statistical methods in various daily-life activities or investigations <br> 18.3 assess statistical investigations presented in different sources such as news media, research reports, etc. | 4 | Students are required to recognise the concepts of "populations" and "samples". <br> Students are required to recognise probability sampling and non-probability sampling. <br> Students are required to recognise that, in constructing questionnaires, factors such as the types, wording and ordering of questions and response options influence their validity and reliability. |
| Further Learning Unit |  |  |  |
| 19. Further applications | Solve more sophisticated real-life and mathematical problems that may require students to search the information for clues, to explore different strategies, or to integrate various parts of mathematics which they have learned in different areas | 14 | Examples: <br> - solve simple financial problems in areas such as taxation and instalment payment <br> - analyse and interpret data collected in surveys |


| Learning Unit | Learning Objective | Time | Remarks |
| :---: | :---: | :---: | :---: |
|  | The main focuses are: <br> (a) to explore and solve more sophisticated real-life problems <br> (b) to appreciate the connections between different areas of mathematics |  | - explore and interpret graphs related to reallife situations <br> - explore Ptolemy's Theorem and its applications <br> - model the relation between two sets of data which show a strong linear correlation and explore how to reduce simple non-linear relations such as $y=m \sqrt{x}+c$ and $y=k a^{x}$ to linear relations <br> - explore the relation between the Fibonacci sequence and the Golden Ratio <br> - appreciate the applications of cryptography <br> - explore the Ceva's Theorem and its applications <br> - analyse mathematical games (e.g. explore the general solution of the water puzzle) |
| 20. Inquiry and investigation | Through various learning activities, discover and construct knowledge, further improve the ability to inquire, communicate, reason and conceptualise mathematical concepts | 10 | This is not an independent and isolated learning unit. The time is allocated for students to engage in learning activities from different learning units. |

Grand total: $\mathbf{2 5 0}$ hours

