

只限教師參閱

FOR TEACHERS' USE ONLY

香港考試及評核局

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

香港中學文憑考試

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION

練習卷

PRACTICE PAPER

數學

必修部分

試卷一

MATHEMATICS COMPULSORY PART PAPER 1

評卷參考（暫定稿）

PROVISIONAL MARKING SCHEME

本評卷參考乃香港考試及評核局專為本科練習卷而編寫，供教師參考之用。教師應提醒學生，不應將評卷參考視為標準答案，硬背死記，活剝生吞。這種學習態度，既無助學生改善學習，學懂應對及解難，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各位教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for teachers' reference. Teachers should remind their students NOT to regard this marking scheme as a set of model answers. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, will not help students to improve their learning nor develop their abilities in addressing and solving problems. The Authority is counting on the co-operation of teachers in this regard.



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Hong Kong Diploma of Secondary Education Examination
Mathematics Compulsory Part Paper 1

General Marking Instructions

1. This marking scheme is the preliminary version before the normal standardisation process and some revisions may be necessary after actual samples of performance have been collected and scrutinised by the HKEAA. Teachers are strongly advised to conduct their own internal standardisation procedures before applying the marking schemes. After standardisation, teachers should adhere to the marking scheme to ensure a uniform standard of marking within the school.
2. It is very important that all teachers should adhere as closely as possible to the marking scheme. In many cases, however, students will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits *all the marks* allocated to that part, unless a particular method has been specified in the question. Teachers should be patient in marking alternative solutions not specified in the marking scheme.
3. In the marking scheme, marks are classified into the following three categories:

'M' marks	awarded for correct methods being used;
'A' marks	awarded for the accuracy of the answers;
Marks without 'M' or 'A'	awarded for correctly completing a proof or arriving at an answer given in a question.

In a question consisting of several parts each depending on the previous parts, 'M' marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, 'A' marks for the corresponding answers should NOT be awarded (unless otherwise specified).
4. For the convenience of teachers, the marking scheme was written as detailed as possible. However, it is still likely that students would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, teachers should exercise their discretion in marking students' work. In general, marks for a certain step should be awarded if students' solution indicated that the relevant concept/technique had been used.
5. Use of notation different from those in the marking scheme should not be penalized.
6. In marking students' work, the benefit of doubt should be given in the students' favour.
7. Marks may be deducted for wrong units (*u*) or poor presentation (*pp*).
 - a. The symbol $(u-1)$ should be used to denote 1 mark deducted for *u*. At most deduct **1 mark** for *u* in each of Section A(1) and Section A(2). Do not deduct any marks for *u* in Section B.
 - b. The symbol $(pp-1)$ should be used to denote 1 mark deducted for *pp*. At most deduct **1 mark** for *pp* in each of Section A(1) and Section A(2). Do not deduct any marks for *pp* in Section B.
 - c. At most deduct 1 mark in each of Section A(1) and Section A(2).
 - d. In any case, do not deduct any marks in those steps where students could not score any marks.
8. In the marking scheme, 'r.t.' stands for 'accepting answers which can be rounded off to' and 'f.t.' stands for 'follow through'. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.

Solution	Marks	Remarks
1. $\frac{(m^5 n^{-2})^6}{m^4 n^{-3}}$ $= \frac{m^{30} n^{-12}}{m^4 n^{-3}}$ $= \frac{m^{30-4}}{n^{12-3}}$ $= \frac{m^{26}}{n^9}$	1M 1M 1A -----(3)	for $(ab)^p = a^p b^p$ or $(a^p)^q = a^{pq}$ for $\frac{a^p}{a^q} = a^{p-q}$ or $\frac{a^p}{a^q} = \frac{1}{a^{q-p}}$
2. $\frac{5+b}{1-a} = 3b$ $5+b = 3b(1-a)$ $5+b = 3b - 3ab$ $3ab = 2b - 5$ $a = \frac{2b-5}{3b}$	1M 1M 1A -----(3)	for $3b(1-a)$ for putting a on one side or equivalent
$\frac{5+b}{1-a} = 3b$ $5+b = 3b(1-a)$ $a = 1 - \frac{5+b}{3b}$ $a = \frac{3b - (5+b)}{3b}$ $a = \frac{2b-5}{3b}$	1M 1M 1A -----(3)	for $3b(1-a)$ for putting a on one side or equivalent
3. (a) $9x^2 - 42xy + 49y^2$ $= (3x - 7y)^2$ (b) $9x^2 - 42xy + 49y^2 - 6x + 14y$ $= (3x - 7y)^2 - 6x + 14y$ $= (3x - 7y)^2 - 2(3x - 7y)$ $= (3x - 7y)(3x - 7y - 2)$	1A 1M 1A -----(3)	or equivalent for using (a) or equivalent

Solution	Marks	Remarks
4. Let \$ x be the marked price of the chair. $x(1 - 20\%) = 360(1 + 30\%)$ $x = \frac{360(1.3)}{0.8}$ $x = 585$ Thus, the marked price of the chair is \$585 .	1M+1M+1A 1A	pp-1 for undefined symbol 1M for $x(1 - 20\%)$ + 1M for $360(1 + 30\%)$ u-1 for missing unit
<div style="border: 1px solid black; padding: 5px;"> The marked price of the chair $= \frac{360(1 + 30\%)}{1 - 20\%}$ $= \\$585$ </div>	1M+1M+1A 1A	1M for $360(1 + 30\%)$ + 1M for dividing by $(1 - 20\%)$ u-1 for missing unit
------(4)		
5. Let x litres and y litres be the capacities of a bottle and a cup respectively. $\begin{cases} \frac{x}{y} = \frac{4}{3} \\ 7x + 9y = 11 \end{cases}$ So, we have $7x + 9\left(\frac{3x}{4}\right) = 11$. Solving, we have $x = \frac{4}{5}$. Thus, the capacity of a bottle is $\frac{4}{5}$ litre.	} 1A+1A 1M 1A	pp-1 for undefined symbol for getting a linear equation in x or y only 0.8 u-1 for missing unit
<div style="border: 1px solid black; padding: 5px;"> Let x litres be the capacity of a bottle. $7x + 9\left(\frac{3x}{4}\right) = 11$ Solving, we have $x = \frac{4}{5}$. Thus, the capacity of a bottle is $\frac{4}{5}$ litre. </div>	1A+1M+1A 1A	pp-1 for undefined symbol 1A for $y = \frac{3x}{4}$ + 1M for $7x + 9y = 11$ 0.8 u-1 for missing unit
------(4)		

Solution	Marks	Remarks
6. (a) $\angle AOC$ $= 337^\circ - 157^\circ$ $= 180^\circ$ Thus, A , O and C are collinear.	1M 1A	for considering $\angle AOC$ f.t.
(b) Note that $BO \perp AC$. The area of $\triangle ABC$ $= \frac{1}{2}(13+15)(14)$ $= 196$	1M 1A -----(4)	
7. Note that $\angle BCD = 90^\circ$. Also note that $\angle CBD = 180^\circ - 90^\circ - 36^\circ = 54^\circ$. Further note that $\angle BAC = \angle BDC = 36^\circ$. Since $AB = AC$, we have $\angle ACB = \angle ABC$. So, we have $\angle ABC = \frac{180^\circ - 36^\circ}{2}$. Therefore, we have $\angle ABC = 72^\circ$.	1A 1M 1M	
$\angle ABD$ $= \angle ABC - \angle CBD$ $= 72^\circ - 54^\circ$ $= 18^\circ$	1A	u-1 for missing unit
Note that $\angle BAC = \angle BDC = 36^\circ$. Since $AB = AC$, we have $\angle ACB = \angle ABC$. So, we have $\angle ACB = \frac{180^\circ - 36^\circ}{2}$. Therefore, we have $\angle ACB = 72^\circ$. Also note that $\angle BCD = 90^\circ$. $\angle ACD$ $= 90^\circ - 72^\circ$ $= 18^\circ$ $\angle ABD$ $= \angle ACD$ $= 18^\circ$	1M 1M 1A 1A -----(4)	u-1 for missing unit

Solution	Marks	Remarks
<p>8. (a) The coordinates of A' $= (3, 4)$</p> <p>The coordinates of B' $= (5, -2)$</p> <p>(b) Let (x, y) be the coordinates of P. $\sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(x-5)^2 + (y-(-2))^2}$ $x^2 - 6x + 9 + y^2 - 8y + 16 = x^2 - 10x + 25 + y^2 + 4y + 4$ $4x - 12y - 4 = 0$ Thus, the required equation is $x - 3y - 1 = 0$.</p>	<p>1A</p> <p>1A</p> <p>1M+1A</p> <p>1A</p>	<p>pp-1 for missing '(' or ')'</p> <p>pp-1 for missing '(' or ')'</p> <p>or equivalent</p>
<p>The coordinates of the mid-point of $A'B'$ $= \left(\frac{3+5}{2}, \frac{4+(-2)}{2} \right)$ $= (4, 1)$</p> <p>The slope of $A'B'$ $= \frac{4 - (-2)}{3 - 5}$ $= -3$</p> <p>So, the required equation is $y - 1 = \frac{1}{3}(x - 4)$. Thus, the required equation is $x - 3y - 1 = 0$.</p>	<p>1M</p> <p>1A</p> <p>1A</p>	<p>or equivalent</p>
------(5)		
<p>9. (a) The least possible value of the inter-quartile range of the distribution $= 5 - 5$ or $2 - 2$ $= 0$</p> <p>The greatest possible value of the inter-quartile range of the distribution $= 5 - 2$ $= 3$</p> <p>(b) Since $r = 9$ and the median of the distribution is 3, we have $9 + 8 > 12 + s$. Therefore, we have $s < 5$. So, we have $s = 1, 2, 3$ or 4. Thus, there are 4 possible values of s.</p>	<p>1M</p> <p>1A</p> <p>1A</p> <p>1M</p> <p>1A</p>	<p>either one</p> <p>f.t.</p>
------(5)		

Solution	Marks	Remarks
<p>10. (a) Note that when $f(x)$ is divided by $x-1$, the remainder is 4.</p> $f(x)$ $= (x-1)(6x^2 + 17x - 2) + 4$ $= 6x^3 + 11x^2 - 19x + 6$ $f(-3)$ $= 6(-3)^3 + 11(-3)^2 - 19(-3) + 6$ $= 0$	<p>1M</p> <p>1M</p> <p>1A</p> <p>------(3)</p>	<p>can be absorbed</p> <p>for $(x-1)(6x^2 + 17x - 2) + r$</p>
<p>(b) $f(x)$</p> $= (x+3)(6x^2 - 7x + 2)$ $= (x+3)(2x-1)(3x-2)$	<p>1M+1A</p> <p>1A</p> <p>------(3)</p>	<p>1M for $(x+3)(ax^2 + bx + c)$</p>
<p>11. (a) Let $C = a + bx^2$, where a and b are non-zero constants.</p> <p>So, we have $a + (20^2)b = 42$ and $a + (120^2)b = 112$.</p> <p>Solving, we have $a = 40$ and $b = \frac{1}{200}$.</p> <p>The required cost</p> $= 40 + \frac{1}{200}(50^2)$ $= \$ 52.5$	<p>1A</p> <p>1M</p> <p>1A</p> <p>1A</p> <p>------(4)</p>	<p>for either substitution</p> <p>for both correct</p> <p>u-1 for missing unit</p>
<p>(b) $40 + \frac{1}{200}x^2 = 58$</p> $x^2 = 3600$ $x = 60$ <p>Thus, the required length is 60 cm.</p>	<p>1M</p> <p>1A</p> <p>------(2)</p>	<p>u-1 for having unit</p>

Solution	Marks	Remarks
12. (a) The required duration $= 63 - 32$ $= 31$ minutes	1M 1A -----(2)	u-1 for missing unit
(b) Suppose Ada and Billy meet at a place which is at a distance of x km from town P . $\frac{x}{78} = \frac{12}{120}$ $x = 7.8$ Thus, Ada and Billy meet at a place which is at a distance of 7.8 km from town P .	1M+1A 1A -----(3)	1M for ratio 78 : 120
(c) The average speed of Ada $= \frac{12}{2}$ $= 6$ km/h The average speed of Billy $= \frac{16-2}{2}$ $= 7$ km/h Note that $7 > 6$. Thus, Billy runs faster.	1M 1A -----(3)	either one f.t.
<div style="border: 1px solid black; padding: 5px;"> During the period, Ada runs 12 km while Billy runs 14 km. Note that $14 > 12$. So, the average speed of Billy is higher than that of Ada. Thus, Billy runs faster. </div>	1M 1A -----(2)	f.t.

Solution	Marks	Remarks
14. (a) $\triangle BCD \sim \triangle OAD$	2A	
	------(2)	
(b) (i) Let $(0, h)$ be the coordinates of C .	1M	
By (b), we have $\left(\frac{CD}{AD}\right)^2 = \frac{16}{45}$.	1M	for using similarity
$\left(\frac{12-h}{\sqrt{6^2+12^2}}\right)^2 = \frac{16}{45}$	1M	for either AD or CD
$h^2 - 24h + 80 = 0$		
$h = 4$ or $h = 20$ (rejected)	1A	
Thus, the coordinates of C are $(0, 4)$.		pp-1 for missing '(' or ')'
(ii) Note that AC is a diameter of the circle $OABC$.	1M	
So, the coordinates of the centre of the circle are $(3, 2)$.	1M	
Also, the radius of the circle is $\sqrt{(3-0)^2 + (4-2)^2} = \sqrt{13}$.		either one
Thus, the equation of the circle $OABC$ is $(x-3)^2 + (y-2)^2 = 13$.	1A	$x^2 + y^2 - 6x - 4y = 0$
Suppose that the equation of the circle $OABC$ is $x^2 + y^2 + k_1x + k_2y + k_3 = 0$, where k_1, k_2 and k_3 are constants. $\begin{cases} 0^2 + 0^2 + k_1(0) + k_2(0) + k_3 = 0 \\ 6^2 + 0^2 + k_1(6) + k_2(0) + k_3 = 0 \\ 0^2 + 4^2 + k_1(0) + k_2(4) + k_3 = 0 \end{cases}$ Solving, we have $k_1 = -6$, $k_2 = -4$ and $k_3 = 0$. Thus, the equation of the circle $OABC$ is $x^2 + y^2 - 6x - 4y = 0$.	1M 1M 1A	for solving system of equations
	------(7)	

Solution	Marks	Remarks
16. (a) The required probability $= \frac{C_4^{18}}{C_4^{30}}$ $= \frac{68}{609}$	1M 1A	for numerator or denominator r.t. 0.112
The required probability $= \left(\frac{18}{30}\right)\left(\frac{17}{29}\right)\left(\frac{16}{28}\right)\left(\frac{15}{27}\right)$ $= \frac{68}{609}$	1M 1A	for $\left(\frac{r}{n}\right)\left(\frac{r-1}{n-1}\right)\left(\frac{r-2}{n-2}\right)\left(\frac{r-3}{n-3}\right)$, $r < n$ r.t. 0.112
------(2)		
(b) The required probability $= 1 - \frac{68}{609} - \frac{C_4^{12}}{C_4^{30}}$ $= \frac{530}{609}$	1M 1A	for $1 - (a) - p_1$ r.t. 0.870
The required probability $= \frac{C_1^{18}C_3^{12} + C_2^{18}C_2^{12} + C_3^{18}C_1^{12}}{C_4^{30}}$ $= \frac{530}{609}$	1M 1A	for considering 3 cases r.t. 0.870
The required probability $= 1 - \frac{68}{609} - \left(\frac{12}{30}\right)\left(\frac{11}{29}\right)\left(\frac{10}{28}\right)\left(\frac{9}{27}\right)$ $= \frac{530}{609}$	1M 1A	for $1 - (a) - p_2$ r.t. 0.870
The required probability $= 4\left(\frac{18}{30}\right)\left(\frac{12}{29}\right)\left(\frac{11}{28}\right)\left(\frac{10}{27}\right) + 6\left(\frac{18}{30}\right)\left(\frac{17}{29}\right)\left(\frac{12}{28}\right)\left(\frac{11}{27}\right) + 4\left(\frac{18}{30}\right)\left(\frac{17}{29}\right)\left(\frac{16}{28}\right)\left(\frac{12}{27}\right)$ $= \frac{530}{609}$	1M 1A	for considering 14 cases r.t. 0.870
------(2)		

Solution	Marks	Remarks
<p>18. (a) By cosine formula, $AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos \angle ACB$ $AB^2 = 20^2 + 12^2 - 2(20)(12)\cos 60^\circ$ $AB = 4\sqrt{19}$ cm</p> <p>(b) By sine formula, $\frac{\sin \angle BAC}{BC} = \frac{\sin \angle ACB}{AB}$ $\frac{\sin \angle BAC}{12} = \frac{\sin 60^\circ}{4\sqrt{19}}$ $\angle BAC \approx 36.58677555^\circ$ Let Q be the foot of the perpendicular from C to AB. $\sin \angle BAC = \frac{CQ}{AC}$ $CQ \approx 20 \sin 36.58677555^\circ$ $CQ \approx 11.92079121$ cm Since $\triangle ABC \cong \triangle ABD$, the required angle is $\angle CQD$. $\sin \frac{\angle CQD}{2} = \frac{\frac{1}{2}CD}{CQ}$ $\sin \frac{\angle CQD}{2} \approx 0.587209345$ $\angle CQD \approx 71.91844786^\circ$ $\angle CQD \approx 71.9^\circ$ Thus, the angle between the plane ABC and the plane ABD is 71.9°.</p>	<p>1M</p> <p>1A</p> <p>------(2)</p> <p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p>r.t. 17.4 cm $AB \approx 17.43559577$ cm</p> <p>for identifying the angle</p> <p>r.t. 71.9°</p>
<p>By sine formula, $\frac{\sin \angle ABC}{AC} = \frac{\sin \angle ACB}{AB}$ $\frac{\sin \angle ABC}{20} = \frac{\sin 60^\circ}{4\sqrt{19}}$ $\angle ABC \approx 83.41322445^\circ$ Let Q be the foot of the perpendicular from C to AB. $\sin \angle ABC = \frac{CQ}{BC}$ $CQ \approx 12 \sin 83.41322445^\circ$ $CQ \approx 11.92079121$ cm Since $\triangle ABC \cong \triangle ABD$, the required angle is $\angle CQD$. $\sin \frac{\angle CQD}{2} = \frac{\frac{1}{2}CD}{CQ}$ $\sin \frac{\angle CQD}{2} \approx 0.587209345$ $\angle CQD \approx 71.91844786^\circ$ $\angle CQD \approx 71.9^\circ$ Thus, the angle between the plane ABC and the plane ABD is 71.9°.</p>	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p>for identifying the angle</p> <p>r.t. 71.9°</p>

Solution	Marks	Remarks
<p>By sine formula,</p> $\frac{\sin \angle BAC}{BC} = \frac{\sin \angle ACB}{AB}$ $\frac{\sin \angle BAC}{12} = \frac{\sin 60^\circ}{4\sqrt{19}}$ $\angle BAC \approx 36.58677555^\circ$ <p>Let Q be the foot of the perpendicular from C to AB.</p> $\sin \angle BAC = \frac{CQ}{AC}$ $CQ \approx 20 \sin 36.58677555^\circ$ $CQ \approx 11.92079121 \text{ cm}$ <p>By symmetry, we have $DQ = CQ$.</p> $DQ \approx 11.92079121 \text{ cm}$ <p>Since $\triangle ABC \cong \triangle ABD$, the required angle is $\angle CQD$.</p> $CD^2 = CQ^2 + DQ^2 - 2(CQ)(DQ) \cos \angle CQD$ $14^2 \approx 11.92079121^2 + 11.92079121^2 - 2(11.92079121)(11.92079121) \cos \angle CQD$ $\angle CQD \approx 71.91844786^\circ$ $\angle CQD \approx 71.9^\circ$ <p>Thus, the angle between the plane ABC and the plane ABD is 71.9°.</p>	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p>for identifying the angle</p> <p>r.t. 71.9°</p>
<p>The area of $\triangle ABC$</p> $= \frac{1}{2}(AC)(BC) \sin \angle ACB$ $= \frac{1}{2}(20)(12) \sin 60^\circ$ $= 60\sqrt{3} \text{ cm}^2$ <p>Let Q be the foot of the perpendicular from C to AB.</p> $\frac{1}{2}(AB)(CQ) = 60\sqrt{3}$ $\frac{1}{2}(4\sqrt{19})(CQ) = 60\sqrt{3}$ $CQ \approx 11.92079121 \text{ cm}$ <p>Since $\triangle ABC \cong \triangle ABD$, the required angle is $\angle CQD$.</p> $\sin \frac{\angle CQD}{2} = \frac{\frac{1}{2}CD}{CQ}$ $\sin \frac{\angle CQD}{2} \approx 0.587209345$ $\angle CQD \approx 71.91844786^\circ$ $\angle CQD \approx 71.9^\circ$ <p>Thus, the angle between the plane ABC and the plane ABD is 71.9°.</p>	<p>1M</p> <p>1M</p> <p>1M</p> <p>1A</p>	<p>for identifying the angle</p> <p>r.t. 71.9°</p>
<p>(c) Let Q be the foot of the perpendicular from C to AB.</p> <p>Note that $\sin \frac{\angle CPD}{2} = \frac{\frac{1}{2}CD}{CP}$.</p> <p>Since $CP \geq CQ$, we have $\angle CPD \leq \angle CQD$.</p> <p>Thus, $\angle CPD$ increases as P moves from A to Q and decreases as P moves from Q to B.</p>	<p>------(4)</p> <p>1M</p> <p>1A</p> <p>------(2)</p>	<p>f.t.</p>

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PRACTICE PAPER

MATHEMATICS COMPULSORY PART PAPER 2

Question No.	Key	Question No.	Key
1.	A	31.	D
2.	C	32.	B
3.	A	33.	C
4.	D	34.	D
5.	D	35.	A
6.	C	36.	B
7.	B	37.	A
8.	D	38.	C
9.	A	39.	A
10.	B	40.	C
11.	D	41.	B
12.	A	42.	A
13.	A	43.	B
14.	B	44.	D
15.	C	45.	C
16.	D		
17.	C		
18.	A		
19.	D		
20.	C		
21.	C		
22.	B		
23.	C		
24.	D		
25.	B		
26.	D		
27.	B		
28.	A		
29.	B		
30.	C		

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