HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2023

MATHEMATICS Compulsory Part
PAPER 2

11:30 am – 12:45 pm (1¼ hours)

INSTRUCTIONS

1. Read carefully the instructions on the Answer Sheet. After the announcement of the start of the examination, you should first stick a barcode label and insert the information required in the spaces provided. No extra time will be given for sticking on the barcode label after the 'Time is up' announcement.

2. When told to open this book, you should check that all the questions are there. Look for the words '**END OF PAPER**' after the last question.

3. All questions carry equal marks.

4. **ANSWER ALL QUESTIONS**. You are advised to use an HB pencil to mark all the answers on the Answer Sheet, so that wrong marks can be completely erased with a clean rubber. You must mark the answers clearly; otherwise you will lose marks if the answers cannot be captured.

5. You should mark only **ONE** answer for each question. If you mark more than one answer, you will receive **NO MARKS** for that question.

6. No marks will be deducted for wrong answers.

Not to be taken away before the end of the examination session

There are 30 questions in Section A and 15 questions in Section B.
The diagrams in this paper are not necessarily drawn to scale.
Choose the best answer for each question.

Section A

1. If $\frac{a+5b}{7a+2b}=\frac{1}{b+3}$, then $a=$

   A. $\frac{4-b}{5b^2+13b}$.

   B. $\frac{4+b}{5b^2+13b}$.

   C. $\frac{5b^2+13b}{4-b}$.

   D. $\frac{5b^2+13b}{4+b}$.

2. $\frac{2}{5-4x}-\frac{1}{5+4x}=$

   A. $\frac{5+4x}{25-16x^2}$.

   B. $\frac{5-4x}{25-16x^2}$.

   C. $\frac{5+12x}{25-16x^2}$.

   D. $\frac{5-12x}{25-16x^2}$.

3. $4^{n+2}3^{2n+4}=$

   A. 6^{2n+4}.

   B. 6^{4n+8}.

   C. 12^{2n+4}.

   D. 12^{3n+6}.

4. $2x^2+xy-y^2+4x+4y=$

A. $(x+y)(2x+y-4)$.

B. $(x+y)(2x-y+4)$.

C. $(x-y)(2x+y-4)$.

D. $(x-y)(2x-y+4)$.

5. If c and d are constants such that $(x+2)(x+c)+12\equiv x(x+d)+6c(x+1)$, then $d=$

A. -13 .

B. -3 .

C. 3 .

D. 17 .

6. The solution of $x-3<-5$ or $\frac{6-x}{4}<2$ is

A. $x<-2$.

B. $x>-2$.

C. $x=-2$.

D. $x\neq -2$.

7. If $y=73.8$ (correct to 3 significant figures), find the range of values of y .

A. $73.7\leq y<73.9$

B. $73.7<y\leq 73.9$

C. $73.75\leq y<73.85$

D. $73.75<y\leq 73.85$

Go on to the next page

8. Let $g(x)=13-5x^2$. If α is a constant, find $g(1-3\alpha)$.

A. $8-45\alpha^2$

B. $8+45\alpha^2$

C. $8-30\alpha+45\alpha^2$

D. $8+30\alpha-45\alpha^2$

9. Let $h(x)=ax^6+16x^3+b$, where a and b are constants. If $h(x)$ is divisible by $2x-3$, find the remainder when $h(x)$ is divided by $2x+3$.

A. -108

B. -54

C. 54

D. 108

10. Which of the following statements about the graph of $y=5+(x-3)^2$ is true?

A. The graph opens downwards.

B. The x-intercept of the graph is 3.

C. The y-intercept of the graph is 5.

D. The graph passes through the point $(3, 5)$.

11. The marked price of a jacket is 60% above its cost. A profit of $\$104$ is made by selling the jacket at a discount of 25% on its marked price. Find the cost of the jacket.

A. $\$416$

B. $\$520$

C. $\$728$

D. $\$832$

12. The scale of a map is $1:50\,000$. If the actual area of an airport is $10\ \text{km}^2$, then the area of this airport on the map is

A. $2\ \text{cm}^2$.

B. $4\ \text{cm}^2$.

C. $20\ \text{cm}^2$.

D. $40\ \text{cm}^2$.

13. It is given that z varies as the square of x and the cube root of y. When $x=12$ and $y=64$, $z=36$. When $x=16$ and $y=729$, $z=$

A. 108.

B. 144.

C. 162.

D. 216.

14. Let a_n be the nth term of a sequence. If $a_6=23$, $a_8=60$ and $a_{n+2}=a_{n+1}+a_n$ for any positive integer n, then $a_3=$

A. 4.

B. 5.

C. 9.

D. 14.

15. The length of a side of a solid cube is 60 cm. The volume of a solid right circular cylinder is equal to the volume of the cube while the curved surface area of the circular cylinder is equal to the total surface area of the cube. Find the base radius of the circular cylinder.

A. 20 cm

B. 30 cm

C. 76 cm

D. 172 cm

Go on to the next page

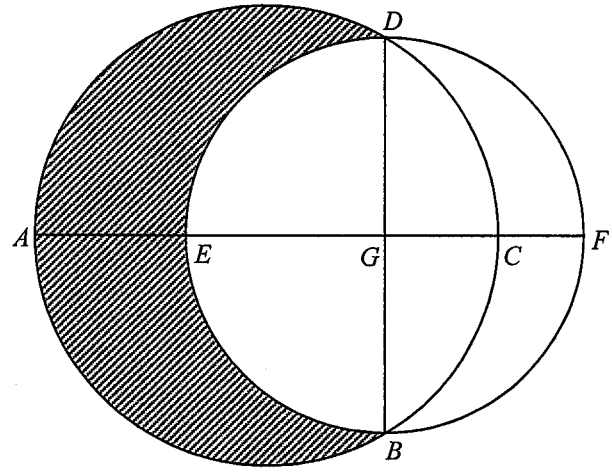
16. In the figure, AC is a diameter of the circle $ABCD$ while BD and EF are diameters of the circle $BEDF$. It is given that C and E lie on AF. Let G be the point of intersection of AF and BD. If $AG = 30$ cm and $CG = 10$ cm, find the area of the shaded region correct to the nearest cm^2.

A. $209\ \text{cm}^2$

B. $367\ \text{cm}^2$

C. $383\ \text{cm}^2$

D. $540\ \text{cm}^2$



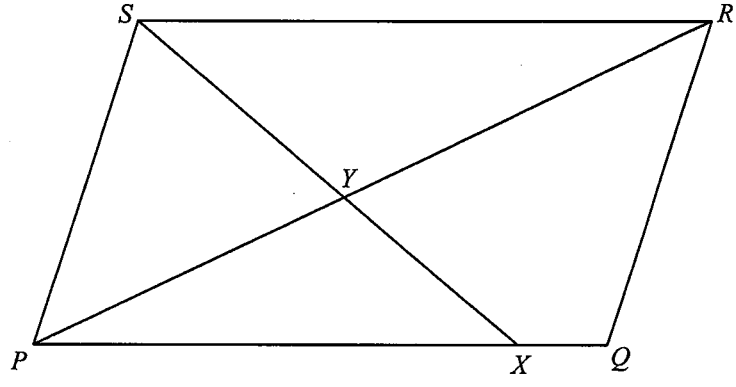
17. In the figure, $PQRS$ is a parallelogram. Let X be a point lying on PQ. Denote the point of intersection of PR and SX by Y. If the area of ΔPXY and the area of the quadrilateral $QRYX$ are $32\ \text{cm}^2$ and $58\ \text{cm}^2$ respectively, then the area of ΔRSY is

A. $40\ \text{cm}^2$.

B. $50\ \text{cm}^2$.

C. $58\ \text{cm}^2$.

D. $72\ \text{cm}^2$.



18. According to the figure, which of the following must be true?

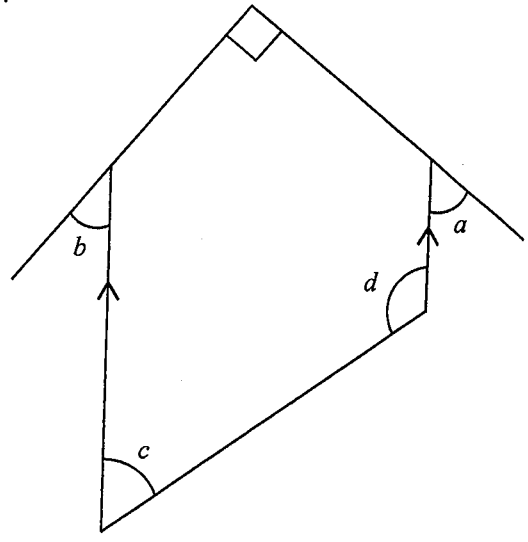
I. $a + b = 90°$
II. $c + d = 180°$
III. $a + b + c = d$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III



19. It is given that $ABCD$ is a rhombus. Denote the point of intersection of AC and BD by E. Which of the following must be true?

I. $AE = BE$

II. $\frac{AE}{AC} = \frac{BE}{BD}$

III. $AE^2 + BE^2 = CD^2$

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

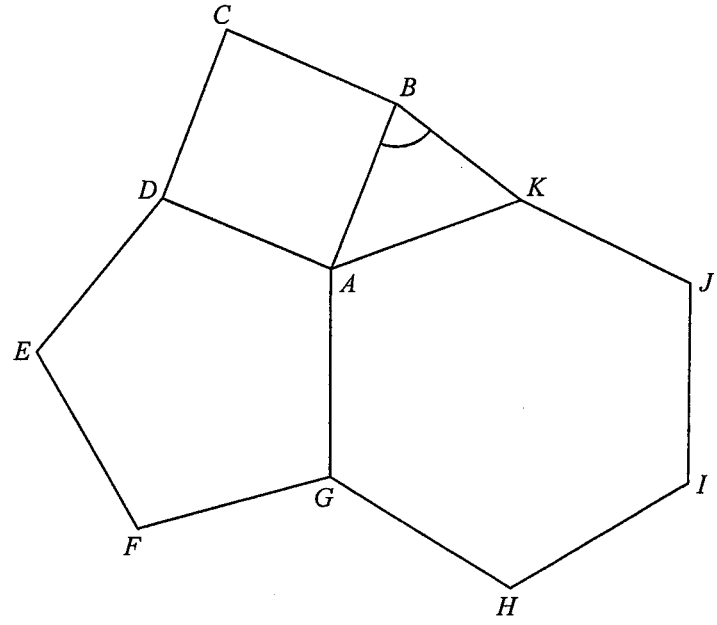
20. The figure shows the square $ABCD$, the regular pentagon $ADEFG$ and the regular hexagon $AGHIJK$. Find $\angle ABK$.

A. 69°

B. 72°

C. 74°

D. 75°



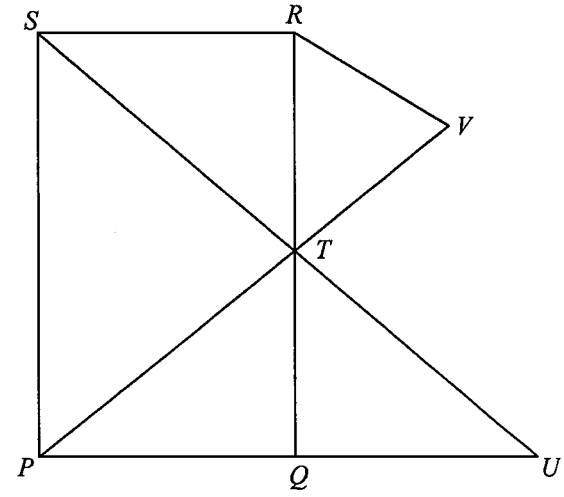
21. In the figure, $PQRS$ is a rectangle. Let T be a point lying on QR such that $\angle PTS = 90°$. PQ produced and ST produced meet at the point U. PT is produced to the point V such that $RT = RV$. Which of the following must be true?

A. $RV // ST$

B. $\angle PTQ = \angle RTS$

C. $\Delta PST \sim \Delta UTQ$

D. $\Delta PQT \cong \Delta TRS$



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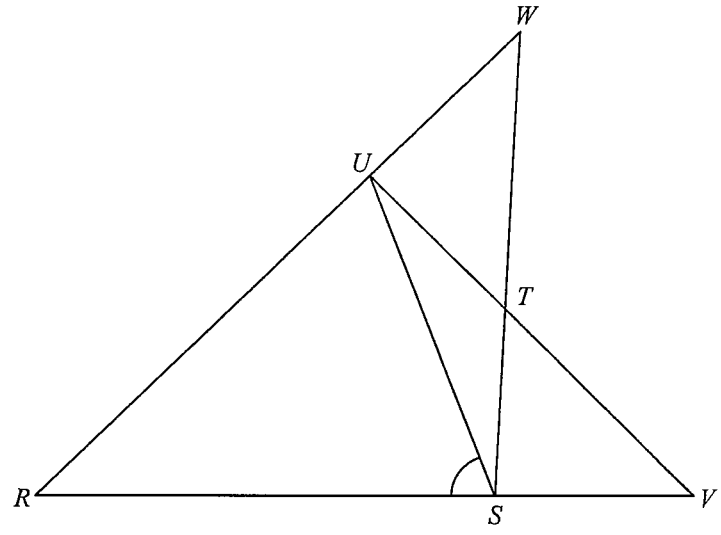
22. The figure shows the cyclic quadrilateral $RSTU$, where $ST = TU$. RS produced and UT produced meet at the point V while RU produced and ST produced meet at the point W. If $\angle RWS = 32°$ and $\angle RVU = 48°$, then $\angle RSU =$

A. $65°$.

B. $73°$.

C. $80°$.

D. $82°$.



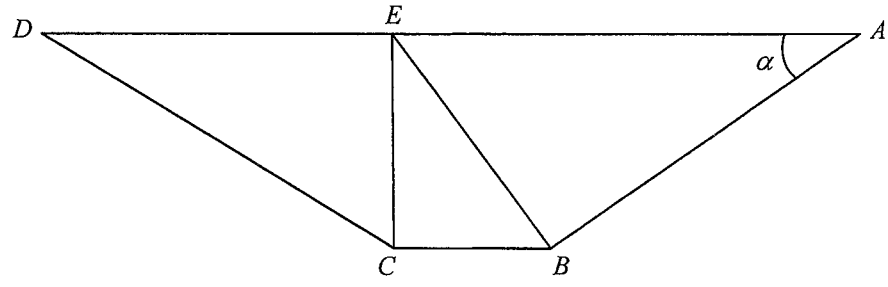
23. In the figure, $ABCD$ is a trapezium with $AD//BC$. Let E be the mid-point of AD. It is given that $\angle ABE = \angle BCE = 90°$. Find $\frac{CE}{DE}$.

A. $\frac{1}{2}$

B. 1

C. $\tan\alpha$

D. $\sin\alpha\cos\alpha$



24. The rectangular coordinates of the point P are $(\sqrt{2}, -\sqrt{2})$. If P is rotated anticlockwise about the origin through $90°$, then the polar coordinates of its image are

A. $(\sqrt{2}, 45°)$.

B. $(\sqrt{2}, 225°)$.

C. $(2, 45°)$.

D. $(2, 225°)$.

25. Find the constant a such that the straight lines $2x+(a+3)y-5=0$ and $ax-4y+1=0$ are perpendicular to each other.

A. -6

B. -5

C. -2

D. 4

26. The equations of the straight lines ℓ and L are $9x+12y-37=0$ and $12x+16y+85=0$ respectively. ℓ cuts the x-axis at the point A while L cuts the y-axis at the point B. Let P be a moving point in the rectangular coordinate plane such that the perpendicular distance from P to ℓ is equal to the perpendicular distance from P to L. Denote the locus of P by Γ. Which of the following are true?

I. Γ is parallel to L.
II. Γ is perpendicular to AB.
III. Γ passes through the mid-point of AB.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

27. The equations of the circles C_1 and C_2 are $x^2+y^2+7x-4y+15=0$ and $2x^2+2y^2-2x-16y-17=0$ respectively. Let G_1 and G_2 be the centres of C_1 and C_2 respectively. Denote the origin by O. Which of the following are true?

I. ΔOG_1G_2 is an equilateral triangle.
II. The line segment OG_1 lies inside C_2.
III. C_1 and C_2 intersect at two distinct points.

A. I and II only

B. I and III only

C. II and III only

D. I, II and III

Go on to the next page

28. A box contains five cards numbered 1 , 2 , 3 , 4 and 5 respectively while another box contains four cards numbered 6 , 7 , 8 and 9 respectively. If a number is randomly drawn from each box, find the probability that the product of the two numbers drawn is divisible by 4 .

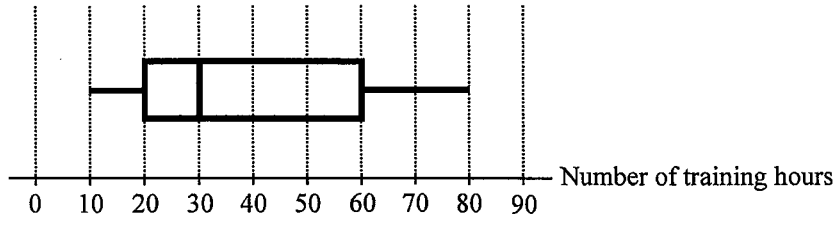
A. $\frac{1}{5}$

B. $\frac{3}{10}$

C. $\frac{7}{20}$

D. $\frac{9}{20}$

29. The box-and-whisker diagram below shows the distribution of the numbers of training hours of some engineers in a year. Find the upper quartile of the distribution.



A. 20

B. 40

C. 60

D. 70

30. There are 14 full-time employees and 56 part-time employees in a company. The mean salary of the full-time employees is \$31 530 while the mean salary of the part-time employees is \$21 525 . Find the mean salary of these employees of the company.

A. \$23 526

B. \$25 527

C. \$27 528

D. \$29 529

Section B

31. $1011001011001011_2 =$

A. $11\times2^{11}+11\times2^5+11$.

B. $11\times2^{12}+11\times2^6+11$.

C. $11\times2^{13}+11\times2^7+11$.

D. $11\times2^{14}+11\times2^8+11$.

32. The L.C.M. of a^4b^2c , a^3b^4c and $a^2b^5c^2$ is

A. a^2b^2c .

B. $a^2b^2c^2$.

C. a^4b^5c .

D. $a^4b^5c^2$.

33. It is given that $\log_8 y$ is a linear function of $\log_4 x$. The intercepts on the vertical axis and on the horizontal axis of the graph of the linear function are 5 and 3 respectively. Which of the following must be true?

A. $x^5y^2=8^{10}$

B. $x^6y^5=8^{20}$

C. $x^{10}y^3=8^{20}$

D. $x^9y^{10}=8^{30}$

Go on to the next page

34. If k is a real number, then the real part of $\frac{i}{k-i}+\frac{2}{k+i}$ is

A. $\frac{2k+1}{k^2-1}$.

B. $\frac{2k-1}{k^2+1}$.

C. $\frac{k+2}{k^2-1}$.

D. $\frac{k-2}{k^2+1}$.

35. Let $f(x)=3x^2+18mx+22m^2$, where m is a real constant. Which of the following statements about the graph of $y=-f(3x)$ must be true?

I. The x-coordinate of the vertex of the graph is m.
II. The y-coordinate of the vertex of the graph is $5m^2$.
III. The equation of the axis of symmetry of the graph is $x+m=0$.

A. I only

B. II only

C. I and III only

D. II and III only

36. Let $T(n)$ be the nth term of an arithmetic sequence. If $T(11)=83$ and $T(25)+T(30)=463$, find the least value of k such that $T(1)+T(2)+T(3)+\cdots+T(k)>4\times10^5$.

A. 299

B. 300

C. 944

D. 945

37. Consider the following system of inequalities:

$$\begin{cases} x+3 \geq 0 \\ 2x+3y-12 \leq 0 \\ 5x-3y+12 \leq 0 \end{cases}$$

Let D be the region which represents the solution of the above system of inequalities. Find the range of values of β such that the greatest value of $\beta x+6y$ is 24, where (x, y) is a point lying in D.

A. $\beta \leq -10$

B. $\beta \geq -10$

C. $\beta \leq 4$

D. $\beta \geq 4$

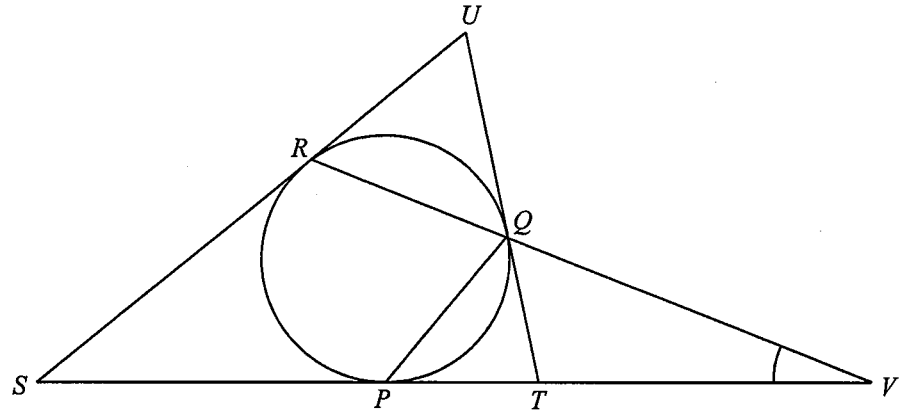
38. In the figure, P, Q and R are points lying on a circle. ST, TU and SU are the tangents to the circle at P, Q and R respectively. RQ produced and ST produced meet at the point V. If $\angle PSR = 34°$ and $\angle QPT = 46°$, then $\angle PVQ =$

A. $17°$.

B. $22°$.

C. $27°$.

D. $28°$.



39. The straight line $hx+ky=6$ and the circle $x^2+y^2-8x-4y-18=0$ intersect at the points M and N, where h and k are constants. If the coordinates of the mid-point of MN are $(1, 0)$, find k.

A. 4

B. 6

C. 9

D. 12

Go on to the next page

40. The base of the right pyramid $VABCD$ is the square $ABCD$. Let θ be the angle between ΔABV and ΔBCV. If $AB:AV=5:4$, then $\cos\theta=$

A. $\frac{-25}{39}$.

B. $\frac{-17}{33}$.

C. $\frac{-9}{16}$.

D. 0.

41. The equations of the straight lines L_1 and L_2 are $3x-4y+k=0$ and $4x+3y-k=0$ respectively, where k is a positive constant. It is given that L_1 cuts the x-axis at the point P. Denote the point of intersection of L_1 and L_2 by Q. If R is a point lying on L_2 such that the in-centre of ΔPQR lies on the x-axis, then the x-coordinate of R is

A. $-7k$.

B. $-k$.

C. k.

D. $7k$.

42. There are 15 teachers in a group. If 5 teachers are selected from the group to form a committee consisting of 1 chairperson and 4 members, how many different committees can be formed?

A. 3 003

B. 15 015

C. 20 475

D. 360 360

43. When a boy throws a dart, the probability that he hits the target is 0.6 . If this boy throws the dart 4 times, find the probability that he hits the target at least 2 times.

A. 0.5248

B. 0.7056

C. 0.8208

D. 0.8464

44. The table below shows the scores (in marks) and the corresponding standard scores of three students in an examination.

Score (marks)	46	x	86
Standard score	−3	1	2

Find x.

A. 64

B. 66

C. 70

D. 78

45. It is given that n is an integer. Let u, v and w be the standard deviation, the median and the range of the group of numbers $\{1-9n, 3-9n, 4-9n, 5-9n, 7-9n\}$ respectively. Which of the following must be true?

I. $u=2$
II. $v<4$
III. $w>6$

A. I only

B. II only

C. I and III only

D. II and III only

END OF PAPER